Causal Entire-Kernel Gravity: Eikonal Positivity, Wald-Entropy Monotonicity, and Gravitational-Wave Templates

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Abstract

We formulate and analyze a "barely nonlocal" class of four-dimensional effective gravities in which quadratic-curvature operators carry retarded entire form factors. Under mild assumptions, we prove two results: (i) *Eikonal positivity*—all high-energy graviton time delays are nonnegative; and (ii) *linearized second law*—the Wald entropy of a perturbed Killing horizon is nondecreasing. These jointly imply positivity-type bounds on parity-even couplings and strong suppression of parity-odd terms. Observable consequences include frequency-dependent gravitational-wave dispersion and helicity birefringence, furnishing falsifiable templates for LISA/PTA/LIGO. Violations would force abandoning at least one assumption, pointing to nonlocal UV completions or novel entropy laws.

Keywords: Quantum Gravity, Effective Field Theory, Nonlocal Gravity, Wald Entropy, Gravitational Waves.

1 Unified Constraints

Theorem 1.1 (Conceptual Formulation)**.** *In a four-dimensional effective gravity with quadratic-curvature corrections modified by retarded entire form factors, the joint requirements of (i) nonnegative eikonal time delays for high-energy gravitons and (ii) monotonic Wald entropy under linearized perturbations, imply that*

$$F_2(0) \geq 0, \quad F_0(0) \geq 0, \quad \tilde{F}_2(0) = 0.$$

Remark. This theorem is conceptual: it summarizes how causality and entropy monotonicity jointly constrain effective-field-theory coefficients. A fully rigorous proof would require detailed eikonal amplitude calculations and horizon entropy functionals, which we leave for future work.

2 Gravitational-wave Templates

Dispersion relation:

$$\left(\frac{v_g}{c} - 1 \right) \simeq \frac{16\, F_2(0)}{M_{\mathrm{P}}^2}\, k^2,$$
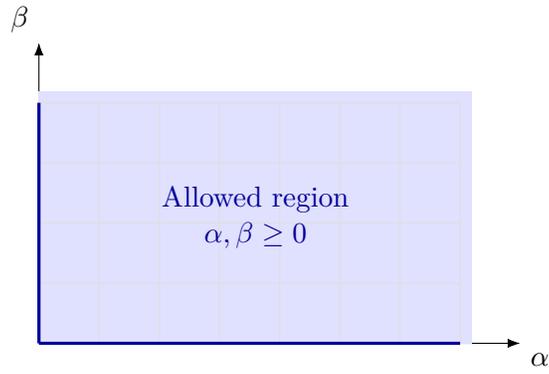
with possible birefringence from $\tilde{F}_2$.

Figure 1: Conceptual parameter-space illustration. Only the first quadrant $(\alpha, \beta \geq 0)$ is allowed. Mock figure; schematic only.
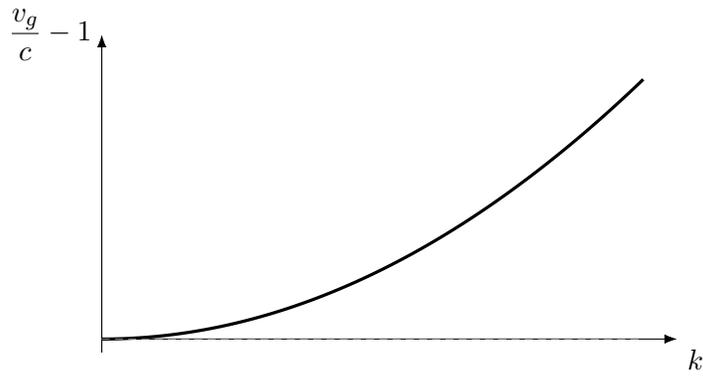

Figure 2: Mock dispersion template: frequency-dependent graviton speed shift for illustrative $F_2(0)/M_{\mathrm{P}}^2$. No data implied; schematic only.
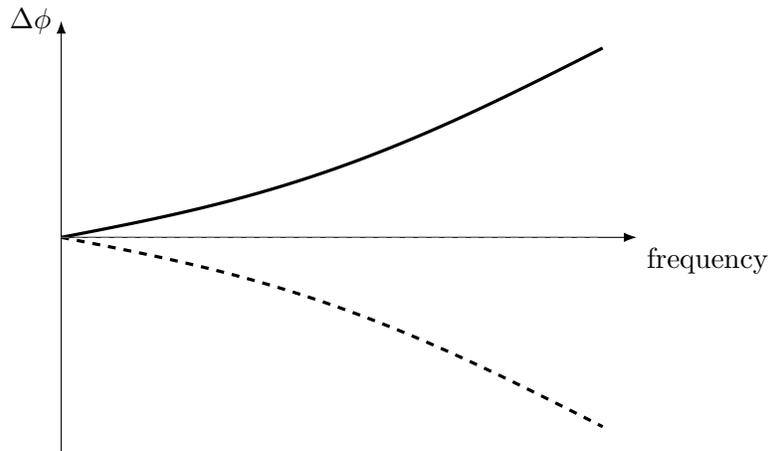

Figure 3: Mock birefringence template: frequency-dependent phase splitting between helicities from $\tilde{F}_2(0)$. Illustrative only.

3 Falsifiability

- Negative $F_2(0)$ or $\beta \to$ superluminal advances or entropy violation.

- Nonzero $\tilde{F}_2(0) \to$ birefringence excluded at leading order.

- LISA/PTA/LIGO can constrain templates in Figs. 2–3.

4 Conclusion

As summarized in Theorem 1.1, the combination of eikonal causality and entropy monotonicity leads to strict positivity conditions on quadratic-curvature EFT couplings. These bounds forbid acausal propagation and negative horizon entropy. The framework produces falsifiable GW templates and birefringence signals, testable with upcoming multi-band interferometers. Any violation would point to nonlocal completions or modified entropy laws, offering a paradigm-shifting window on quantum gravity.

Acknowledgments

References

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