

# Quantum Origin of Turbulence: Wheeler's Spacetime Foam within Penrose's Time Cyclic Cosmology

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## Abstract

Classical turbulence theory, as formulated by Kolmogorov in 1941, assumes an irreversible forward cascade of energy and monotonic entropy growth. This framework, though powerful, conflicts with the cyclic time hypothesis in which all physical variables recur exactly over a fundamental period. In this paper, we propose a unification of turbulence with cyclic spacetime dynamics by embedding Kolmogorov scaling within a two-phase process: a forward cascade followed by a restorative collapse. Using Fourier expansions in both space and cyclic time, we demonstrate that turbulent modes must satisfy strict recurrence conditions, enforced by an end-phase restorative potential. This mechanism resolves the entropy paradox by generating entropy hysteresis loops rather than unbounded growth, and it introduces spectral breathing, nonlinear mode locking, and fractal intermittency as natural features of cyclic turbulence. Furthermore, by drawing analogies with holography, spacetime foam, and quantum cosmology, we argue that turbulence may represent the macroscopic manifestation of Planck-scale fluctuations subject to recurrence. The proposed framework bridges fluid turbulence, statistical mechanics, and quantum gravity, providing a pathway for testing cyclic time principles in both laboratory turbulence and cosmological observations.

## 1 Introduction

Kolmogorov's 1941 theory of turbulence, commonly referred to as K41 [9], has long been considered one of the cornerstones of statistical fluid mechanics. It provides a phenomenological framework for describing turbulence at high Reynolds numbers, in which the inertial subrange is characterized by universal scaling independent of viscosity. The central result is the celebrated  $k^{-5/3}$  energy spectrum for the distribution of turbulent kinetic energy across wavenumbers. However, K41 rests on fundamental assumptions that may not be valid in a universe governed by time-periodicity, as proposed in the framework of Fourier-expanded physical laws in cyclic spacetime [10].

The classical assumptions of Kolmogorov include statistical stationarity, homogeneity, isotropy, and most crucially, irreversibility of the energy cascade. Energy injected at large scales passes through intermediate scales and is ultimately dissipated into heat at small scales, with dissipation rate  $\varepsilon$  assumed to be finite and positive in the infinite Reynolds number limit. In contrast, the time-periodic approach demands that all physical variables repeat exactly with a period  $T$ , and that dissipation and entropy growth must reverse in the second half of each cycle. This raises a critical question: can Kolmogorov’s framework be reconciled with a cyclic universe, or does it fundamentally fail under such constraints?

In this paper, we carefully examine the incompatibility between K41 and time-periodic dynamics. We show how Fourier-based recurrence conditions modify turbulence scaling laws and propose a generalized cyclic turbulence theory that incorporates alternating phases of forward and reverse cascades. Throughout the discussion, we provide explicit mathematical formulations of energy spectra, dissipation, and entropy evolution under cyclic boundary conditions. Our analysis builds upon the foundational work of Kolmogorov [9], Loschmidt’s paradox [0], and the time-periodic framework developed by Modgil [10].

The study of turbulence has remained one of the central unsolved problems of classical physics since the pioneering work of Richardson and Kolmogorov. The Kolmogorov 1941 (K41) theory established a scaling law for the inertial range, predicting an energy spectrum of the form

$$E(k) \sim k^{-5/3}, \tag{1}$$

where  $k$  is the wavenumber. While this framework has been extremely successful, it rests on the assumption of irreversibility, monotonic entropy growth, and an infinite forward cascade of energy. These assumptions place turbulence in tension with cyclic models of time, in which all physical variables, including velocity fields and their derivatives, recur exactly over a fundamental cycle.

Earlier work [10] introduced the concept of Fourier recurrence for physical variables, showing that cyclicity imposes strict conditions on derivatives of dynamical quantities. More recently, the idea of restorative forces was developed [1], whereby a diverging potential near the end of the cycle enforces exact recurrence by collapsing dynamical variables back to their seed states. Together, these principles suggest a new framework for turbulence in cyclic time: a forward cascade p This completes the idea presented in this paragraph.

Furthermore, we argue that turbulence may be understood as a macroscopic avatar of quantum gravity fluctuations. Holographic dualities demonstrate that Einstein’s equations reduce to Navier–Stokes dynamics on null surfaces, suggesting that turbulence and spacetime fluctuations share a common origin. Within cyclic time, both turbulence and spacetime foam obey the same recurrence principle: chaotic growth through forward cascades followed by restorative collapse into seed configurations. This unification of This completes the idea presented in this paragraph.

The purpose of this paper is therefore threefold. First, we formulate turbulence in cyclic time using Fourier recurrence and restorative potentials. Second, we reinterpret Kolmogorov scaling and intermittency through this lens, showing that entropy growth and reduction balance across cycles. Finally, we explore the broader implications of this framework, linking turbulence to holography, quantum gravity, and cyclic cosmology. In doing so, we aim to construct a coherent picture in which turbulence is no lo This completes the idea presented in this paragraph.

## 2 Kolmogorov's 1941 Turbulence Theory

Kolmogorov's 1941 framework assumes that in the inertial subrange, the statistical properties of turbulence depend only on the dissipation rate  $\varepsilon$  and the wavenumber  $k$ . By dimensional analysis, the energy spectrum takes the form

$$E(k) = C\varepsilon^{2/3}k^{-5/3}, \quad (2)$$

where  $C$  is the Kolmogorov constant. The dissipation rate  $\varepsilon$  is defined as

$$\varepsilon = \nu \int_0^\infty k^2 E(k) dk, \quad (3)$$

with  $\nu$  denoting the kinematic viscosity. In the high Reynolds number limit,  $\varepsilon$  remains finite and positive, ensuring that turbulence is sustained in a statistically stationary state.

Kolmogorov's assumptions are deeply tied to irreversibility. The cascade is inherently one-directional, with kinetic energy transferred from large scales to small scales, eventually dissipated into thermal motion. This unidirectional nature is consistent with the second law of thermodynamics, where entropy monotonically increases in time. As pointed out by Loschmidt [0], however, irreversibility poses challenges in reconciling microscopic time-reversal symmetry with macroscopic entropy growth. This tension becomes particularly significant in a universe with time-periodicity, where entropy must return to its original state after each cycle.

## 3 Time-Periodic Constraints on Physical Variables

In a time-periodic universe with period  $T$ , every physical observable  $A(t)$  can be represented by a Fourier series expansion [10]:

$$A(t) = \sum_{m=0}^{\infty} \left( C_m^{(1)} \cos \frac{2\pi mt}{T} + C_m^{(2)} \sin \frac{2\pi mt}{T} \right). \quad (4)$$

The recurrence condition requires

$$A(t+T) = A(t), \quad (5)$$

and for higher-order derivatives,

$$\int_0^T A^{(n)}(t) dt = 0, \quad n = 1, 2, 3, \dots \quad (6)$$

These constraints ensure that the net evolution of any physical variable over one cycle is zero. Entropy  $S(t)$  must also obey such constraints, implying that

$$\int_0^T \frac{dS}{dt} dt = 0. \quad (7)$$

Consequently, entropy must increase during half the cycle and decrease during the other half. This stands in stark contradiction to Kolmogorov's irreversibility assumption.

For turbulence, the dissipation rate  $\varepsilon(t)$  must satisfy

$$\int_0^T \varepsilon(t) dt = 0, \quad (8)$$

which necessitates alternating positive and negative dissipation phases. In practical terms, turbulence must not only dissipate energy but also regenerate it in the reverse cycle, effectively undoing the energy cascade.

## 4 A new look at Kolmogorov's Assumptions in Cyclic Time

The first point of conflict is entropy evolution. Kolmogorov assumes a monotonically increasing entropy due to irreversible energy transfer, whereas cyclic time enforces entropy oscillations. The second conflict arises in the definition of dissipation rate. In classical turbulence,  $\varepsilon$  is strictly positive. In a cyclic universe, however, dissipation must alternate in sign, implying a fundamentally different physical mechanism. Finally, the statistical stationarity assumed by K41 is incompatible with cyclicity, since the turbulence spectrum must evolve and return to its initial configuration after every period.

Mathematically, let  $E(k, t)$  denote the turbulent energy spectrum. In cyclic time, the recurrence condition demands

$$E(k, t + T) = E(k, t), \quad (9)$$

and

$$\int_0^T \frac{\partial E(k, t)}{\partial t} dt = 0. \quad (10)$$

This implies that any growth of  $E(k)$  in one phase must be exactly compensated by decay in the reverse phase. Hence, Kolmogorov's universal inertial subrange scaling cannot be valid across the entire cycle, but only during the forward cascade phase. In the reverse phase, one would require an "inverse cascade" law, potentially scaling as

$$E(k) \sim k^{+5/3}, \quad (11)$$

to regenerate large-scale structures. This is in sharp contrast with classical turbulence theory.

## 5 Towards a Cyclic Turbulence Theory

To reconcile turbulence with time-periodicity, we propose a generalized framework of cyclic turbulence. In this picture, turbulence alternates between forward and reverse cascades. The dissipation rate is modeled as a time-antisymmetric function:

$$\varepsilon(t) = -\varepsilon(T - t). \quad (12)$$

During the forward phase, energy cascades as in classical turbulence, and the Kolmogorov spectrum is recovered. During the reverse phase, energy flows backward from small scales to large scales, regenerating coherent structures.

The energy spectrum can then be expressed as

$$E(k, t) = E_f(k) H(t) + E_r(k) H(T - t), \quad (13)$$

where  $E_f(k)$  denotes the forward spectrum,  $E_r(k)$  denotes the reverse spectrum, and  $H(t)$  is the Heaviside function ensuring phase separation. This cyclic turbulence model respects the Fourier recurrence conditions and resolves the entropy paradox by allowing entropy to increase and decrease in alternating phases.

## 6 Kolmogorov's 1941 Turbulence Theory

Kolmogorov's 1941 theory of turbulence, commonly referred to as K41 [9], has long been considered one of the cornerstones of statistical fluid mechanics. It provides a phenomenological framework for describing turbulence at high Reynolds numbers, in which the inertial subrange is characterized by universal scaling independent of viscosity. The central result is the celebrated  $k^{-5/3}$  energy spectrum for the distribution of turbulent kinetic energy across wavenumbers. However, K41 res This completes the idea presented in this paragraph. The classical assumptions of Kolmogorov include statistical stationarity, homogeneity, isotropy, and most crucially, irreversibility of the energy cascade. Energy injected at large scales passes through intermediate scales and is ultimately dissipated into heat at small scales, with dissipation rate  $\varepsilon$  assumed to be finite and positive in the infinite Reynolds number limit. In contrast, the time-periodic approach demands that all physical variables repeat exactly with a period  $T$ , and that di This completes the idea presented in this paragraph. In this paper, we carefully examine the incompatibility between K41 and time-periodic dynamics. We show how Fourier-based recurrence conditions modify turbulence scaling laws and propose a generalized cyclic turbulence theory that incorporates alternating phases of forward and reverse cascades. We then integrate the restorative potential framework recently introduced in [1], where entropy reduction and cascade reversal are concentrated in a singular phase towards the end of the cycle, ther This completes the idea presented in this paragraph.

Kolmogorov's 1941 framework assumes that in the inertial subrange, the statistical properties of turbulence depend only on the dissipation rate  $\varepsilon$  and the wavenumber  $k$ . By dimensional analysis, the energy spectrum takes the form

$$E(k) = C\varepsilon^{2/3}k^{-5/3}, \quad (14)$$

where  $C$  is the Kolmogorov constant. The dissipation rate  $\varepsilon$  is defined as

$$\varepsilon = \nu \int_0^\infty k^2 E(k) dk, \quad (15)$$

with  $\nu$  denoting the kinematic viscosity. In the high Reynolds number limit,  $\varepsilon$  remains finite and positive, ensuring that turbulence is sustained in a statistically stationary state.

Kolmogorov's assumptions are deeply tied to irreversibility. The cascade is inherently one-directional, with kinetic energy transferred from large scales to small scales, eventually dissipated into thermal motion. This unidirectional nature is consistent with the second law of thermodynamics, where entropy monotonically increases in time. As pointed out by Loschmidt [0], however, irreversibility poses challenges in reconciling microscopic time-reversal symmetry with macroscopic entropy g This completes the idea presented in this paragraph.

## 7 Time-Periodic Constraints on Physical Variables

In a time-periodic universe with period  $T$ , every physical observable  $A(t)$  can be represented by a Fourier series expansion [10]:

$$A(t) = \sum_{m=0}^{\infty} \left( C_m^{(1)} \cos \frac{2\pi mt}{T} + C_m^{(2)} \sin \frac{2\pi mt}{T} \right). \quad (16)$$

The recurrence condition requires

$$A(t + T) = A(t), \quad (17)$$

and for higher-order derivatives,

$$\int_0^T A^{(n)}(t) dt = 0, \quad n = 1, 2, 3, \dots \quad (18)$$

These constraints ensure that the net evolution of any physical variable over one cycle is zero. Entropy  $S(t)$  must also obey such constraints, implying that

$$\int_0^T \frac{dS}{dt} dt = 0. \quad (19)$$

Consequently, entropy must increase during half the cycle and decrease during the other half. This stands in stark contradiction to Kolmogorov's irreversibility assumption.

For turbulence, the dissipation rate  $\varepsilon(t)$  must satisfy

$$\int_0^T \varepsilon(t) dt = 0, \quad (20)$$

which necessitates alternating positive and negative dissipation phases. In practical terms, turbulence must not only dissipate energy but also regenerate it in the reverse cycle, effectively undoing the energy cascade.

## 8 Kolmogorov's Model and the Assumptions in Cyclic Time

The first point of conflict is entropy evolution. Kolmogorov assumes a monotonically increasing entropy due to irreversible energy transfer, whereas cyclic time enforces entropy oscillations. The second conflict arises in the definition of dissipation rate. In classical turbulence,  $\varepsilon$  is strictly positive. In a cyclic universe, however, dissipation must alternate in sign, implying a fundamentally different physical mechanism. Finally, the statistical stationarity assumed by K41 is incompatible. This completes the idea presented in this paragraph. Mathematically, let  $E(k, t)$  denote the turbulent energy spectrum. In cyclic time, the recurrence condition demands

$$E(k, t + T) = E(k, t), \quad (21)$$

and

$$\int_0^T \frac{\partial E(k, t)}{\partial t} dt = 0. \quad (22)$$

This implies that any growth of  $E(k)$  in one phase must be exactly compensated by decay in the reverse phase. Hence, Kolmogorov's universal inertial subrange scaling cannot be valid across the entire cycle, but only during the forward cascade phase. In the reverse phase, one would require an "inverse cascade" law, potentially scaling as

$$E(k) \sim k^{+5/3}, \quad (23)$$

to regenerate large-scale structures. This is in sharp contrast with classical turbulence theory.

## 9 Restorative Dynamics and End-Phase Corrections

In order to reconcile turbulent dynamics with the time-periodic structure of the universe, Modgil [1] introduced the notion of restorative forces that act in a concentrated interval near the end of each cycle. The guiding principle is that forward turbulent evolution is allowed to proceed unimpeded for the majority of the cycle, including entropy growth and energy cascade from large to small scales. However, as  $t \rightarrow T^-$ , a singular corrective mechanism ensures that all dynamical variables This completes the idea presented in this paragraph. One central equation describing the restorative potential is

$$\Phi(q, \dot{q}, \ddot{q}, \dots, t) = \sum_{k=0}^n \frac{1}{k!} \frac{(q^{(k)}(t) - q^{(k)}(0))^2}{(T-t)^{2(k+1)}}, \quad (24)$$

where  $q^{(k)}(t)$  denotes the  $k$ -th derivative of the generalized coordinate  $q(t)$ . The denominator ensures that as  $t$  approaches  $T$ , the restorative term diverges, forcing the system back towards its initial conditions. For turbulence, this implies that small-scale fluctuations, which would ordinarily decay irreversibly, are instead regenerated and merged back into coherent large-scale eddies in this terminal interval.

This corrective mechanism also modifies the dissipation rate. While for most of the cycle  $\varepsilon(t)$  behaves as in Kolmogorov's forward cascade, near the end phase it transitions rapidly, enforcing

$$\varepsilon(t) \rightarrow -\varepsilon(t), \quad \text{as } t \rightarrow T^-. \quad (25)$$

This sign reversal ensures compliance with the Fourier recurrence condition

$$\int_0^T \varepsilon(t) dt = 0. \quad (26)$$

## 10 Cyclic Turbulence and Fourier-Based Restoration

The Fourier representation of turbulence provides an effective tool for visualizing how energy and entropy evolve across the cycle. By expanding the velocity field  $u(x, t)$  as

$$u(x, t) = \sum_{m=-\infty}^{\infty} \hat{u}_m(x) e^{i2\pi mt/T}, \quad (27)$$

the recurrence condition demands that each Fourier mode  $\hat{u}_m(x)$  must return to its initial amplitude and phase at  $t + T$ . In Kolmogorov turbulence, forward cascade leads to decay of large-scale modes and amplification of small-scale modes until dissipation dominates. However, the restorative dynamics framework requires that as  $t \rightarrow T$ , the high-wavenumber modes collapse back into the low-wavenumber spectrum, thus completing the cyclic restoration.

We can mathematically express this collapse as

$$\hat{u}_m(x, T) = \hat{u}_m(x, 0), \quad \forall m, \quad (28)$$

with the understanding that for  $0 < t < T - \delta$ , the Kolmogorov cascade applies, while for  $T - \delta < t < T$ , the restorative potential  $\Phi$  dominates. Here  $\delta$  denotes a small corrective interval where singular restorative dynamics force the recurrence. This two-phase model of turbulence is fully consistent with cyclic time and provides a novel extension of Kolmogorov's theory into the domain of cosmological time topology.

# 11 Relating Fourier Expansions in Cyclic Time to Turbulence Fourier Modes in the Restorative Force Framework

The problem of turbulence in a cyclic universe requires a consistent framework in which the recurrence of physical variables over a fundamental time cycle is reconciled with the seemingly irreversible nature of Kolmogorov turbulence [9]. In the cyclic time model, all physical variables are required to satisfy strict recurrence conditions, which can be naturally formulated using Fourier expansions in time [10]. Meanwhile, turbulence is traditionally described in terms of a Fou This completes the idea presented in this paragraph.

In the cyclic setting, let a generic physical variable  $A(t)$  be represented as a Fourier expansion over the fundamental time cycle  $T$ :

$$A(t) = \sum_{m=-\infty}^{\infty} \hat{A}_m e^{i2\pi mt/T}. \quad (29)$$

The recurrence condition requires

$$A(t + T) = A(t), \quad (30)$$

which enforces periodicity of  $A(t)$  and ensures that the system returns to its initial state after each cycle. Importantly, this requirement applies not only to the variable itself but also to all its dynamical derivatives, so that

$$\frac{d^n A}{dt^n}(T) = \frac{d^n A}{dt^n}(0), \quad n = 1, 2, 3, \dots \quad (31)$$

In turbulence theory, the velocity field  $u(x, t)$  is expressed in terms of spatial Fourier modes:

$$u(x, t) = \sum_{k=-\infty}^{\infty} \hat{u}_k(t) e^{ik \cdot x}, \quad (32)$$

where  $\hat{u}_k(t)$  represents the amplitude of the mode corresponding to wavevector  $k$ . Non-linear triadic interactions couple these modes and lead to the forward energy cascade, where energy flows from large-scale eddies ( $k$  small) to small-scale eddies ( $k$  large), as formalized by Kolmogorov [9].

In the cyclic framework, each turbulent spatial Fourier coefficient  $\hat{u}_k(t)$  itself must evolve periodically in time. This can be expressed as a secondary Fourier expansion:

$$\hat{u}_k(t) = \sum_{m=-\infty}^{\infty} U_{k,m} e^{i2\pi mt/T}. \quad (33)$$

This construction yields a double Fourier representation, where the turbulent field is simultaneously decomposed in space and in cyclic time. The condition of recurrence then imposes that

$$\hat{u}_k(T) = \hat{u}_k(0), \quad \forall k. \quad (34)$$

In other words, the time evolution of each turbulent spatial mode is constrained by the temporal harmonics of the cycle. This coupling between spatial and temporal Fourier modes represents the fundamental modification to turbulence in a cyclic universe.

The restorative force framework developed in Modgil [1] enforces the return of all dynamical variables to their initial values as  $t \rightarrow T^-$ . The restorative potential is given by

$$\Phi(q, \dot{q}, \ddot{q}, \text{This completes the idea presented in this paragraph.}, t) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{(q^{(n)}(t) - q^{(n)}(0))^2}{(T-t)^{2(n+1)}}. \quad (35)$$

This potential diverges as  $t$  approaches  $T$ , which ensures that every derivative of  $q(t)$ , and by analogy every Fourier coefficient  $\hat{u}_k(t)$  of turbulence, is driven back to its initial state. Therefore, while most of the cycle is governed by the forward cascade with positive dissipation, the closing interval is dominated by the restorative potential, which forces  $\hat{u}_k(T) = \hat{u}_k(0)$  and resets turbulence to its seed state.

The dissipation rate  $\varepsilon(t)$  provides an explicit example of this dual behavior. In conventional turbulence, dissipation is strictly positive, leading to the irreversible spectrum

$$E(k) = C\varepsilon^{2/3}k^{-5/3}. \quad (36)$$

In cyclic turbulence, however, we must impose the condition

$$\int_0^T \varepsilon(t) dt = 0, \quad (37)$$

which requires that dissipation be positive during most of the cycle but negative during the restorative phase. This alternating behavior is naturally captured by the symmetry

$$\varepsilon(t) = -\varepsilon(T-t). \quad (38)$$

The forward cascade dominates the interval  $0 < t < T - \delta$ , while in the closing window  $T - \delta < t < T$ , dissipation becomes negative and the energy cascade reverses, reconstructing large scales.

Entropy dynamics follow a similar pattern. While entropy  $S(t)$  increases monotonically in conventional turbulence, in the cyclic setting it must satisfy

$$\int_0^T \frac{dS}{dt} dt = 0. \quad (39)$$

Thus, entropy production during the forward cascade is compensated by entropy reduction during the restorative phase, a phenomenon directly enforced by the divergent behavior of the restorative potential.

In summary, the Fourier expansion of variables in cyclic time provides the foundation for strict recurrence conditions, while the Fourier decomposition of turbulence encodes the cascade dynamics in spatial modes. The restorative force framework unifies the two by enforcing that the time evolution of each turbulent Fourier mode is itself periodic and returns to its seed value. This produces a two-phase turbulence model: a Kolmogorov-like forward cascade with entropy growth followed by a restorative phase. This completes the idea presented in this paragraph.

## 12 Spectral Breathing of Turbulence

In classical turbulence theory, the inertial subrange is described by Kolmogorov's universal scaling law, in which the energy spectrum takes the form

$$E(k) = C\varepsilon^{2/3}k^{-5/3}, \quad (40)$$

where  $C$  is the Kolmogorov constant and  $\varepsilon$  is the mean dissipation rate [9]. This law is inherently stationary, meaning that the energy spectrum maintains the same scaling form irrespective of time, provided that the Reynolds number is sufficiently high. However, in a cyclic time universe as proposed in [1, 10], this assumption cannot hold. The demand of recurrence and the introduction of a restorative force at the end of each cycle imply that the energy This completes the idea presented in this paragraph.

We introduce the concept of spectral breathing, by which we mean the cyclic expansion and contraction of the turbulent energy spectrum over the course of a cycle of duration  $T$ . Mathematically, this can be expressed as

$$E(k, t) = E(k, 0)f\left(\frac{t}{T}\right), \quad (41)$$

where  $E(k, 0)$  is the reference spectrum at the beginning of the cycle and  $f(t/T)$  is a modulation function that encodes the breathing dynamics. The function  $f(t/T)$  must satisfy the recurrence condition

$$f\left(\frac{t+T}{T}\right) = f\left(\frac{t}{T}\right), \quad (42)$$

ensuring that the spectrum returns to its seed configuration at  $t = T$ .

During the forward cascade phase,  $0 < t < T - \delta$ , the modulation function should enforce spectral broadening. This corresponds to an increase of energy at high wavenumbers due to dissipation. In this phase, we can write

$$f\left(\frac{t}{T}\right) \sim \left(\frac{t}{T}\right)^\alpha, \quad \alpha > 0, \quad (43)$$

which represents monotonic entropy growth and positive dissipation. In contrast, during the restorative end-phase  $T - \delta < t < T$ , the function  $f(t/T)$  must enforce contraction of the spectrum, with small scales collapsing back into large scales. This requires

$$f\left(\frac{t}{T}\right) \sim \left(1 - \frac{t}{T}\right)^\beta, \quad \beta > 0, \quad (44)$$

where the exponent  $\beta$  governs the rate at which energy is transferred back to large scales.

The dissipation rate in this framework must also oscillate, with

$$\int_0^T \varepsilon(t) dt = 0, \quad (45)$$

implying that dissipation is positive in the forward phase and negative in the restorative phase. We may define the dissipation as

$$\varepsilon(t) = \varepsilon_0 g\left(\frac{t}{T}\right), \quad (46)$$

where  $g(t/T)$  is antisymmetric with respect to  $t = T/2$ , so that

$$g\left(\frac{t}{T}\right) = -g\left(1 - \frac{t}{T}\right). \quad (47)$$

This antisymmetry ensures that positive dissipation in the first half of the cycle is exactly cancelled by negative dissipation in the closing phase, a mechanism made possible by the restorative potential described in [1].

Entropy evolution follows naturally from these considerations. The rate of entropy production must satisfy

$$\int_0^T \frac{dS}{dt} dt = 0, \quad (48)$$

with

$$\frac{dS}{dt} \propto \varepsilon(t). \quad (49)$$

Thus entropy increases during the forward cascade and decreases during the restorative phase, tracing out a closed hysteresis-like loop over the course of a cycle. This hysteresis reflects the breathing of the turbulent spectrum, which expands and contracts in synchrony with entropy oscillations.

A further implication of spectral breathing is the modification of the inertial subrange. Instead of a static  $k^{-5/3}$  law, the energy spectrum becomes explicitly time-dependent:

$$E(k, t) = C\varepsilon(t)^{2/3}k^{-5/3}f\left(\frac{t}{T}\right). \quad (50)$$

This equation generalizes Kolmogorov's law by incorporating the modulation function  $f(t/T)$ , which encodes the breathing of the spectrum. For  $0 < t < T - \delta$ ,  $f(t/T)$  increases, leading to a forward cascade consistent with classical turbulence. For  $T - \delta < t < T$ ,  $f(t/T)$  decreases, causing the spectrum to contract and large eddies to be reconstituted.

Spectral breathing therefore provides a direct physical interpretation of how turbulence can satisfy both Kolmogorov's scaling locally and cyclic recurrence globally. It resolves the paradox between irreversibility and recurrence by confining irreversibility to part of the cycle and reversing it in the restorative phase. This model provides a pathway for extending turbulence theory into the domain of cyclic cosmology, where spectral breathing of fluctuations may also apply to cosmological structures.

## 13 Nonlinear Mode Locking between Space and Time Harmonics

A distinctive feature of turbulence in cyclic time is the possibility of nonlinear coupling between spatial Fourier modes and temporal harmonics. In conventional turbulence theory [9], the velocity field  $u(x, t)$  is expanded in spatial Fourier modes as

$$u(x, t) = \sum_{k=-\infty}^{\infty} \hat{u}_k(t) e^{ik \cdot x}, \quad (51)$$

where  $\hat{u}_k(t)$  are time-dependent amplitudes. These amplitudes evolve due to nonlinear interactions, leading to the energy cascade across scales. In cyclic turbulence, however, each spatial Fourier amplitude must itself satisfy recurrence conditions in time, which are naturally expressed using a secondary Fourier expansion [10]:

$$\hat{u}_k(t) = \sum_{m=-\infty}^{\infty} U_{k,m} e^{i\omega_m t}, \quad \omega_m = \frac{2\pi m}{T}. \quad (52)$$

This double Fourier representation creates the possibility of nonlinear resonances between spatial and temporal modes.

Consider the condition for resonance between a spatial mode  $k$  and a temporal harmonic  $m$ . The interaction requires that

$$k \cdot u \sim \omega_m, \quad (53)$$

meaning that the characteristic turnover frequency of an eddy at scale  $k$  locks onto a discrete temporal harmonic  $\omega_m$ . When such a resonance occurs, the eddy is constrained to repeat its structure identically at integer multiples of the cycle period  $T/m$ . This phenomenon is referred to as nonlinear mode locking, and it implies that turbulence in cyclic time can acquire quasi-crystalline structure in spacetime.

The physical implication is that not all turbulent eddies evolve freely; rather, certain classes of eddies synchronize with temporal harmonics of the cycle. To formalize this, we consider the Navier–Stokes equations in Fourier space, which take the form

$$\frac{\partial \hat{u}_k}{\partial t} + \nu k^2 \hat{u}_k = -i \sum_{p+q=k} (k \cdot \hat{u}_p) \hat{u}_q. \quad (54)$$

By substituting the temporal expansion of  $\hat{u}_k(t)$  into this equation, one finds that quadratic interactions between spatial and temporal harmonics generate terms of the form

$$U_{k,m} U_{p,n} e^{i(\omega_m + \omega_n)t}, \quad (55)$$

which enforce conservation conditions of the type

$$\omega_m + \omega_n = \omega_\ell, \quad k + p = q. \quad (56)$$

These are analogous to resonance conditions in nonlinear wave theory, where only specific triads of modes can interact constructively. In the cyclic turbulence context, this implies that temporal harmonics act as discrete frequency channels, forcing the turbulence to select harmonically locked configurations.

An important consequence is the formation of quasi-periodic eddy structures. While classical turbulence is characterized by continuous spectra of fluctuations, cyclic turbulence can generate discrete spectral peaks corresponding to locked spatio-temporal modes. These structures may be interpreted as quasi-crystals in spacetime, where the turbulence exhibits long-range temporal order in addition to spatial complexity. The analogy with quasi-crystals is appropriate because mode locking prevents ergodic cover. This completes the idea presented in this paragraph.

Entropy and dissipation are also influenced by mode locking. The dissipation rate  $\varepsilon(t)$ , which alternates between positive and negative values in cyclic time [1], becomes synchronized with temporal harmonics when nonlinear locking occurs. This synchronization ensures that entropy production and reduction occur at discrete frequencies, creating a structured oscillatory entropy profile rather than a smooth oscillation. Thus, entropy dynamics in cyclic turbulence can be understood as. This completes the idea presented in this paragraph.

The presence of nonlinear mode locking provides a mechanism for reconciling Kolmogorov scaling with cyclic recurrence. While the inertial subrange continues to exhibit the  $k^{-5/3}$  spectrum locally, the global temporal dynamics are constrained by harmonic locking, which enforces periodicity and recurrence. This dual behavior underscores the hybrid nature of cyclic turbulence: it retains the chaotic cascades of classical turbulence while embedding them in a deterministic, harmonic temporal scaffold.

## 14 Fractal Zeros and Intermittency

One of the central challenges in turbulence theory is the phenomenon of intermittency, which manifests as deviations from the classical Kolmogorov  $k^{-5/3}$  scaling law [9]. Intermittency reflects the irregular, burst-like concentration of dissipation in space and time, producing corrections to the scaling of structure functions. In the cyclic time model, where all physical variables and their derivatives are subject to recurrence, intermittency may find a natural explanation in terms of This completes the idea presented in this paragraph.

In the cyclic time hypothesis formulated in [10], higher-order derivatives of physical variables are constrained by recurrence conditions. For example, for a dynamical variable  $A(t)$  we require

$$\frac{d^n A}{dt^n}(T) = \frac{d^n A}{dt^n}(0), \quad n = 1, 2, 3, \dots \quad (57)$$

The zeros of these higher derivatives, particularly in turbulent velocity fields, are distributed across the cycle in patterns that may possess fractal structure. If  $u(t)$  is a component of the velocity field at a fixed spatial point, then its higher derivatives  $u^{(n)}(t)$  possess zeros whose density may scale as a fractal measure. Denoting the set of zeros of  $u^{(n)}(t)$  by  $Z_n$ , one may write

$$N_n(\Delta t) \sim (\Delta t)^{D_n}, \quad (58)$$

where  $N_n(\Delta t)$  is the number of zeros of  $u^{(n)}(t)$  in a time interval  $\Delta t$  and  $D_n$  is the fractal dimension associated with the distribution of zeros of order  $n$ . This connection between derivative zeros and fractal dimensions was outlined in [10], and in the turbulence context it provides a direct link to intermittency statistics.

Intermittency corrections in turbulence are often quantified through anomalous scaling of structure functions. For the velocity increment  $\delta u(r, t)$  across separation  $r$ , the  $p$ -th order structure function is defined as

$$S_p(r) = \langle |\delta u(r, t)|^p \rangle. \quad (59)$$

Kolmogorov's 1941 theory predicts

$$S_p(r) \sim r^{p/3}. \quad (60)$$

However, experiments show deviations of the form

$$S_p(r) \sim r^{\zeta_p}, \quad \zeta_p \neq \frac{p}{3}, \quad (61)$$

where the anomalous exponents  $\zeta_p$  encode intermittency effects. In the cyclic turbulence framework, these deviations can be attributed to the fractal distribution of derivative zeros, since bursts of dissipation correspond to clustering of zeros of higher derivatives of  $u(t)$ .

The restorative force framework described in [1] ensures that as  $t \rightarrow T^-$ , the system is driven back to its initial state. This implies that any clustering of zeros of  $u^{(n)}(t)$  that produced intermittency during the forward cascade must be systematically undone in the restorative phase. Formally, this may be expressed as

$$Z_n(t) = Z_n(T - t), \quad (62)$$

indicating that the fractal set of zeros in the forward phase is mirrored in the restorative phase. This symmetry ensures that while intermittency produces entropy growth during the cascade, the restorative force generates entropy reduction by redistributing zeros in a time-reflected manner.

Morse theory provides a natural mathematical framework for analyzing the topology of these zeros. In Morse theory, the critical points of a function correspond to changes in its topology, and the distribution of zeros of derivatives  $u^{(n)}(t)$  can be mapped onto a Morse-theoretic landscape. This suggests that intermittency statistics in turbulence may be reinterpreted as a manifestation of the topology of fractal zeros in cyclic time. Specifically, the anomalous exponents  $\zeta_p$  may be linked to the This completes the idea presented in this paragraph.

This approach unifies two seemingly distinct aspects of turbulence: intermittency and recurrence. The fractal zeros provide the mechanism for intermittency during forward cascades, while the restorative force ensures that these patterns are erased and reconstituted in reversed form at  $t \rightarrow T^-$ . The cyclic turbulence framework thus predicts that intermittency is not merely a random fluctuation phenomenon but an organized fractal process that is constrained by recurrence and symmetry.

## 15 Entropy Hysteresis Curves

In classical turbulence theory, entropy evolves monotonically due to the irreversible dissipation of kinetic energy into heat, and this monotonic increase of entropy is intimately tied to the second law of thermodynamics [9]. However, in a cyclic universe where recurrence is enforced, entropy cannot increase indefinitely. Instead, the entropy production that occurs during the forward cascade must be precisely cancelled by entropy reduction during the restorative phase.

In cyclic turbulence, the entropy  $S(t)$  must satisfy the recurrence condition

$$S(T) = S(0). \quad (63)$$

This implies that over one complete cycle, the net entropy change must vanish,

$$\Delta S_{\text{total}} = \int_0^T \frac{dS}{dt} dt = 0. \quad (64)$$

Consequently, entropy growth in the forward cascade phase must be exactly compensated by entropy reduction in the restorative phase,

$$\Delta S_{\text{forward}} = -\Delta S_{\text{restorative}}. \quad (65)$$

This condition transforms entropy evolution from a monotonic trajectory into a closed loop, reminiscent of hysteresis in thermodynamic systems.

The entropy production rate is closely tied to the dissipation rate  $\varepsilon(t)$ , so that

$$\frac{dS}{dt} \propto \varepsilon(t). \quad (66)$$

In the forward cascade phase  $0 < t < T - \delta$ , we have  $\varepsilon(t) > 0$ , leading to entropy production. In the restorative phase  $T - \delta < t < T$ , the restorative potential enforces  $\varepsilon(t) < 0$ , causing entropy reduction [1]. Plotting  $S(t)$  against  $\varepsilon(t)$  produces a closed

hysteresis loop, much like the pressure-volume diagram of a thermodynamic cycle. This analogy suggests that turbulence in cyclic time functions as a thermodynamic oscillator.

To make this analogy more precise, we define an entropy functional based on dissipation,

$$S(t) = S(0) + \int_0^t \alpha \varepsilon(\tau) d\tau, \quad (67)$$

where  $\alpha$  is a proportionality constant dependent on system parameters. Because of the antisymmetric property of dissipation,

$$\varepsilon(t) = -\varepsilon(T - t), \quad (68)$$

it follows that

$$S(t) = S(T - t). \quad (69)$$

Thus, entropy traces out a symmetric loop over each cycle, rising during the forward cascade and falling during the restorative phase. This symmetry guarantees the closure of entropy trajectories, producing a hysteresis curve.

The area enclosed by this entropy-dissipation loop represents the irreversibility of each half-phase of the cycle. In classical turbulence, entropy production is unidirectional, and no such loop exists. In cyclic turbulence, however, the restorative force guarantees that this irreversibility is temporary and confined to one half of the cycle. The loop structure therefore encodes both the local irreversibility of turbulence and the global reversibility of cyclic time. This duality resolves the paradox raised. This completes the idea presented in this paragraph.

The hysteresis analogy also suggests that cyclic turbulence can be studied using the mathematical machinery of dynamical systems theory, where limit cycles in phase space represent stable periodic oscillations. In this context, entropy  $S(t)$  and dissipation  $\varepsilon(t)$  form conjugate variables whose dynamics trace out a closed orbit. The turbulence system thus behaves like a thermodynamic oscillator, in which entropy alternately increases and decreases in a self-sustained cycle. This interpretation. This completes the idea presented in this paragraph.

In conclusion, entropy hysteresis curves provide a powerful conceptual framework for understanding turbulence in cyclic time. They show that entropy does not simply rise and fall, but instead traces out closed loops that encode the irreversible dynamics of forward cascades and the restorative reversal at  $t \rightarrow T^-$ . This framework links turbulence to thermodynamic cycles and demonstrates that cyclic turbulence can be viewed as a form of oscillatory thermodynamics embedded within a broader recurrence principle. This completes the idea presented in this paragraph.

## 16 Quantum Analogy: Turbulence as a Wavefunction on Cyclic Time

The recurrence of turbulence in cyclic time suggests a profound analogy with quantum mechanics. In classical turbulence theory, the velocity field is treated as a stochastic field obeying the Navier–Stokes equations. However, when Fourier expansions are applied both in space and in time, the turbulent field acquires a structure that can be interpreted as a wavefunction in a Hilbert space with cyclic boundary conditions. This interpretation is

motivated by the requirement that turbulent variables recur af This completes the idea presented in this paragraph.

Let  $\psi(u, t)$  represent the state functional of turbulence, which encodes the probability amplitude for velocity configurations  $u$  at time  $t$ . In cyclic time,  $\psi(u, t)$  must satisfy the periodic boundary condition

$$\psi(u, t + T) = \psi(u, t). \quad (70)$$

This condition is directly analogous to the requirement of periodic wavefunctions in systems with compactified time dimensions in quantum mechanics. It ensures that turbulence evolves as a unitary process across the cycle, returning to its original state at  $t = T$ .

The time evolution of  $\psi(u, t)$  can be expressed in a Schrödinger-like form,

$$i \frac{\partial \psi}{\partial t} = \hat{H} \psi, \quad (71)$$

where  $\hat{H}$  is a turbulence Hamiltonian operator. Unlike in quantum mechanics, where  $\hat{H}$  is well defined, here it represents the nonlinear dynamics of the Navier–Stokes equations, including dissipation and forcing. Nevertheless, the analogy is fruitful because it places turbulence in the formal setting of Hilbert space evolution with recurrence.

The restorative potential introduced in [1] provides a striking parallel with quantum measurement collapse. As  $t \rightarrow T^-$ , the restorative potential diverges,

$$\Phi(q, \dot{q}, \ddot{q}, \text{This completes the idea presented in this paragraph.}, t) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{(q^{(n)}(t) - q^{(n)}(0))^2}{(T - t)^{2(n+1)}}, \quad (72)$$

forcing all variables back to their initial states. In the quantum analogy, this corresponds to a collapse of the turbulence wavefunction, which projects  $\psi(u, t)$  back onto its seed state  $\psi(u, 0)$  at the end of each cycle. Thus, while turbulence evolves unitarily for most of the cycle, it undergoes a collapse-like correction at  $t = T$ , mirroring quantum measurement.

Another connection arises with Poincaré recurrence, which states that a system with finite energy in a bounded phase space will, after a sufficiently long time, return arbitrarily close to its initial state. In cyclic turbulence, recurrence is exact and occurs after a finite time  $T$ , not just approximately after very long times. The wavefunction formalism makes this exact recurrence natural, since  $\psi(u, t)$  evolves with compactified time, guaranteeing closure of its trajectory in Hilbert space. This pr This completes the idea presented in this paragraph.

The entropy dynamics of turbulence can also be reinterpreted in this quantum framework. The von Neumann entropy of the turbulence wavefunction can be defined as

$$S_{\text{vN}}(t) = -\text{Tr}(\rho(t) \ln \rho(t)), \quad (73)$$

where  $\rho(t) = |\psi(u, t)\rangle\langle\psi(u, t)|$  is the density matrix. In the forward cascade phase, turbulence entropy grows due to spreading of the wavefunction over velocity configurations. During the restorative phase, the collapse potential reduces this entropy, driving the wavefunction back to its initial configuration. This cyclic entropy law mirrors the oscillatory entropy framework developed in [1, 10].

In summary, turbulence in cyclic time can be interpreted as a wavefunction evolving under a Schrödinger-like equation with periodic boundary conditions. The restorative potential acts as a collapse mechanism, enforcing recurrence at the end of the cycle.

This quantum analogy unifies turbulence theory with concepts from statistical mechanics and quantum recurrence, highlighting deep structural similarities between chaotic fluid dynamics and quantum systems evolving in compactified time. It suggests that this completes the idea presented in this paragraph.

## 17 Numerical Experiment Proposal

The theoretical framework of cyclic turbulence, constructed from Fourier recurrence and restorative dynamics, requires empirical and numerical validation. One way to test this theory is through direct numerical simulations (DNS) of the Navier–Stokes equations, modified to include an artificial restorative forcing term that becomes significant near the end of the cycle. This section outlines a concrete proposal for such an experiment, motivated by the need to verify predictions such as spectral breathing. This completes the idea presented in this paragraph.

The incompressible Navier–Stokes equations for a velocity field  $u(x, t)$  are given by

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \nu \nabla^2 u + F(t), \quad (74)$$

with the incompressibility condition

$$\nabla \cdot u = 0. \quad (75)$$

Here,  $F(t)$  is an external forcing term that sustains turbulence. In conventional simulations,  $F(t)$  is designed to inject energy at large scales, thereby maintaining statistical stationarity. In the cyclic turbulence framework, however, we propose a modification of  $F(t)$  by introducing a restorative force  $F_{\text{rest}}(t)$ , which is negligible for most of the cycle but diverges as  $t \rightarrow T^-$ .

The proposed form of the restorative force is

$$F_{\text{rest}}(t) \sim \frac{u(t) - u(0)}{(T - t)^2}. \quad (76)$$

This expression ensures that as  $t$  approaches the cycle endpoint  $T$ , the velocity field is driven back to its initial condition  $u(0)$ . The divergence of the denominator guarantees that deviations from recurrence are corrected with increasing strength as the cycle closes, mirroring the theoretical restorative potential introduced in [1].

The full simulation equation becomes

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \nu \nabla^2 u + F_{\text{inj}}(t) + F_{\text{rest}}(t), \quad (77)$$

where  $F_{\text{inj}}(t)$  is the usual large-scale energy injection and  $F_{\text{rest}}(t)$  is the restorative correction. Numerical experiments can then test whether turbulence subject to this modified forcing reproduces the recurrence conditions predicted by the cyclic time framework.

Key observables in such a simulation include the energy spectrum  $E(k, t)$ , the dissipation rate  $\varepsilon(t)$ , and the entropy  $S(t)$ . Spectral breathing can be tested by measuring the time-dependent spectrum,

$$E(k, t) = \frac{1}{2} \langle |\hat{u}_k(t)|^2 \rangle, \quad (78)$$

and verifying whether it expands during the forward cascade and contracts during the restorative phase. The dissipation rate should satisfy

$$\int_0^T \varepsilon(t) dt = 0, \quad (79)$$

which would confirm the alternating positive and negative dissipation predicted in [1, 10]. Likewise, entropy should trace out hysteresis loops,

$$\Delta S_{\text{forward}} = -\Delta S_{\text{restorative}}, \quad (80)$$

providing numerical evidence of thermodynamic oscillator behavior in turbulence.

Implementing such a simulation requires careful numerical handling of the singularity in  $F_{\text{rest}}(t)$  as  $t \rightarrow T^-$ . Regularization techniques can be employed, for example replacing  $(T - t)^{-2}$  with  $(T - t + \epsilon)^{-2}$  for a small parameter  $\epsilon$ , to avoid numerical instability while preserving the divergence required for recurrence. A range of cycle lengths  $T$  can be tested to examine the robustness of cyclic recurrence across scales.

If successful, this numerical experiment would provide evidence that turbulence can indeed be constrained by restorative forces into a recurrent cycle. It would also allow the quantitative study of cyclic turbulence phenomena, such as mode locking, spectral breathing, and entropy hysteresis, in a controlled computational setting. This approach offers a direct bridge between the theoretical framework of cyclic time and measurable, testable turbulence statistics.

## 18 Cosmological Turbulence

The extension of cyclic turbulence concepts to cosmology opens a new avenue for interpreting large-scale structure formation, plasma dynamics, and fluctuations in the early universe. In the classical picture, cosmic turbulence has been invoked in the study of galaxy formation, the intergalactic medium, and plasma behavior in the early universe [9]. These processes are usually treated as irreversible, with turbulence dissipating energy through cascades in the same manner as terrestrial. This completes the idea presented in this paragraph.

If spacetime itself is cyclic, as proposed in [1, 10], then turbulence at cosmological scales must also obey recurrence conditions. This implies that plasma turbulence in the early universe, galaxy-scale flows, and even turbulence in the cosmic microwave background (CMB) may undergo spectral breathing across cycles. In this framework, the turbulent energy spectrum is written as

$$E(k, t) = E(k, 0) f\left(\frac{t}{T}\right), \quad (81)$$

where  $T$  is the cosmological cycle period,  $k$  denotes the wavenumber associated with cosmic structures, and  $f(t/T)$  encodes spectral breathing. During expansion phases of the universe, forward cascades dominate, transferring energy from large-scale modes to smaller-scale plasma turbulence. As  $t \rightarrow T^-$ , the restorative dynamics reverse this process, collapsing small-scale fluctuations back into coherent structures, ensuring that cosmological turbulence resets at each cycle.

A natural implication of this model is the periodicity of entropy at cosmological scales. The entropy associated with cosmic turbulence, including both plasma processes and gravitational clustering, must satisfy

$$\int_0^T \frac{dS}{dt} dt = 0, \quad (82)$$

so that entropy production in one half of the cosmological cycle is exactly compensated by entropy reduction in the restorative phase. This property resolves the puzzle of entropy accumulation across cycles in cosmological models, since turbulence-driven entropy growth is counteracted by restorative collapse.

The cosmic microwave background (CMB) provides a unique observational window into cosmological turbulence. Fluctuations in the CMB can be analyzed in terms of power spectra  $C_\ell$ , where  $\ell$  plays the role of a wavenumber. If turbulence in the primordial plasma obeyed spectral breathing, then we expect modulations in  $C_\ell$  consistent with recurrence conditions. Specifically, the anisotropy spectrum should satisfy

$$C_\ell(t + T) = C_\ell(t), \quad (83)$$

with oscillatory corrections corresponding to breathing behavior. Observational data from the Planck mission and other CMB experiments could be reinterpreted in this light, providing a potential test for the cyclic turbulence framework.

Similarly, turbulence in the intergalactic medium (IGM) may provide evidence of recurrence. The IGM is a turbulent plasma where energy cascades from galactic outflows and cluster mergers drive fluctuations across megaparsec scales. In cyclic turbulence, these fluctuations are not purely dissipative but are constrained to recur. The governing condition may be expressed as

$$u(x, t + T) = u(x, t), \quad (84)$$

where  $u(x, t)$  is the velocity field of the IGM plasma. The restorative phase ensures that small-scale magnetic and velocity fluctuations collapse back into coherent fields, potentially providing a mechanism for the regeneration of magnetic fields across cycles.

Cosmological turbulence in cyclic spacetime therefore unifies local Kolmogorov-like cascades with global recurrence principles. It suggests that galaxy-scale plasma turbulence, primordial fluctuations in the CMB, and intergalactic flows all undergo spectral breathing, entropy hysteresis, and restorative collapse. This model links turbulence theory not only with fluid mechanics but also with cosmological observations, offering a pathway to test cyclic time models using astrophysical data.

In summary, the hypothesis that spacetime is cyclic implies that turbulence at cosmic scales is also cyclic. This turbulence undergoes forward cascades during expansion phases and restorative collapses near cycle endpoints, ensuring recurrence of structures and entropy. Observational signatures in the CMB, galaxy formation statistics, and IGM turbulence spectra may provide evidence for such cyclic turbulence. This perspective integrates turbulence with cyclic cosmology, demonstrating that the principles of This completes the idea presented in this paragraph.

## 19 Spacetime Foam and Turbulence

The notion of spacetime foam, introduced by John Wheeler, is one of the most striking ideas in quantum gravity. At the Planck scale, Wheeler suggested that spacetime is not

smooth but highly irregular, undergoing constant fluctuations in topology and geometry [2]. These fluctuations are stochastic, nonlinear, and chaotic, making them closely analogous to turbulence in classical fluids. Just as turbulence represents a breakdown of smooth laminar flow into chaotic, multiscale eddy structures, This completes the idea presented in this paragraph.

In order to formalize this analogy, one can consider spacetime fluctuations  $\delta g_{\mu\nu}(x, t)$  of the metric tensor around a smooth background  $g_{\mu\nu}^{(0)}$ . At the Planck scale  $\ell_P$ , these fluctuations are expected to be of order unity. One may write

$$g_{\mu\nu}(x, t) = g_{\mu\nu}^{(0)}(x, t) + \delta g_{\mu\nu}(x, t), \quad (85)$$

with the variance of fluctuations scaling as

$$\langle \delta g_{\mu\nu}^2(\ell) \rangle \sim \ell^{2/3}, \quad (86)$$

where  $\ell$  is a length scale larger than the Planck length. This scaling law, reminiscent of Kolmogorov's two-thirds law for velocity increments in turbulence [9], suggests that spacetime at the smallest scales may indeed behave like a turbulent medium. The similarity between Kolmogorov turbulence and Wheeler's spacetime foam has been explored in multiple studies of quantum gravity phenomenology.

In the cyclic framework developed in [1, 10], spacetime itself is required to satisfy recurrence conditions. This means that foam-like fluctuations cannot accumulate indefinitely, but must undergo periodic spectral breathing. During the forward phase of the cycle, fluctuations grow chaotic, corresponding to a forward cascade in turbulence. Near the end of the cycle  $t \rightarrow T^-$ , the restorative dynamics reverse this process, collapsing small-scale fluctuations back into coherence. F This completes the idea presented in this paragraph.

The entropy associated with spacetime foam fluctuations must also respect recurrence. If  $S_{\text{foam}}(t)$  denotes the entropy generated by spacetime turbulence, then we require

$$S_{\text{foam}}(T) = S_{\text{foam}}(0), \quad (87)$$

which implies

$$\int_0^T \frac{dS_{\text{foam}}}{dt} dt = 0. \quad (88)$$

Thus, entropy growth caused by forward cascades of metric fluctuations is exactly compensated by entropy reduction during the restorative phase. This parallels the thermodynamic oscillator behavior described in cyclic turbulence, where entropy traces out closed hysteresis-like loops [1].

A wavefunction formalism provides an additional bridge between turbulence and spacetime foam. If  $\psi(g_{\mu\nu}, t)$  denotes the wavefunctional of spacetime geometry, then cyclic recurrence requires

$$\psi(g_{\mu\nu}, t + T) = \psi(g_{\mu\nu}, t). \quad (89)$$

The restorative potential in this context acts like a quantum collapse operator, projecting  $\psi(g_{\mu\nu}, t)$  back to its seed state as  $t \rightarrow T^-$ . This makes the cyclic recurrence of spacetime foam exact rather than approximate, in contrast with Poincaré recurrence theorems which only guarantee return after extremely long times [0].

In summary, Wheeler's concept of spacetime foam finds a natural description in terms of turbulence, with fluctuations obeying Kolmogorov-like scaling and exhibiting intermittency. In cyclic time, these fluctuations undergo spectral breathing, expanding into

chaotic foam and contracting back into coherence, thereby avoiding entropy divergence across cycles. This framework unites ideas from turbulence, quantum gravity, and cyclic cosmology, suggesting that the same principles that govern chaotic fluid dynamics complete the idea presented in this paragraph.

## 20 Navier–Stokes $\leftrightarrow$ Einstein Equations

One of the most remarkable insights of modern theoretical physics is the deep connection between fluid dynamics and gravity. It has been shown by Damour, Policastro, Son, Starinets, and others that Einstein’s equations, when projected onto a null hypersurface such as the event horizon of a black hole, reduce to equations identical in form to the incompressible Navier–Stokes equations for a viscous fluid [3–5]. This correspondence has become a central element of the holographic paradigm. This completes the idea presented in this paragraph.

To illustrate the connection, consider the incompressible Navier–Stokes equations for a velocity field  $u^i(x, t)$ ,

$$\partial_t u^i + u^j \nabla_j u^i = -\nabla^i p + \nu \nabla^2 u^i, \quad (90)$$

together with the incompressibility condition

$$\nabla_i u^i = 0. \quad (91)$$

These equations govern the evolution of a viscous fluid in three spatial dimensions. In holographic duality, the same equations arise as constraints on perturbations of the Einstein equations projected onto a null surface. The kinematic viscosity  $\nu$  in this correspondence is related to gravitational couplings, with the ratio of shear viscosity to entropy density taking the universal value

$$\frac{\eta}{s} = \frac{1}{4\pi}, \quad (92)$$

as established in the AdS/CFT correspondence [4].

In cyclic turbulence, this correspondence implies that fluid turbulence may be viewed as the boundary manifestation of deeper spacetime dynamics in quantum gravity. The forward cascade of turbulence, in which energy is transferred from large to small scales, can be interpreted as the gravitational dynamics of horizon perturbations spreading across degrees of freedom. Conversely, the restorative collapse at  $t \rightarrow T^-$  corresponds to the reorganization of spacetime fluctuations into a coherent initial state. This completes the idea presented in this paragraph.

The recurrence condition central to cyclic time,

$$u(x, t + T) = u(x, t), \quad (93)$$

finds a natural parallel in holographic gravity. The wavefunctional of spacetime geometries  $\psi(g_{\mu\nu}, t)$  must also satisfy periodic boundary conditions in cyclic models,

$$\psi(g_{\mu\nu}, t + T) = \psi(g_{\mu\nu}, t). \quad (94)$$

Thus, the recurrence of turbulence fields is a macroscopic signature of the recurrence of spacetime itself. The holographic map ensures that recurrence in boundary fluid dynamics mirrors recurrence in the bulk gravitational system. This correspondence allows us to interpret turbulence not as an isolated chaotic process, but as a macroscopic avatar of quantum gravity subject to cyclic recurrence.

Entropy considerations further strengthen this analogy. In fluid turbulence, entropy grows during forward cascades due to dissipation, and decreases during restorative phases, satisfying

$$\Delta S_{\text{forward}} = -\Delta S_{\text{restorative}}. \quad (95)$$

In the gravitational dual, this corresponds to entropy oscillations of the black hole horizon or cosmological horizon, which must also satisfy recurrence in cyclic cosmologies. The celebrated Bekenstein–Hawking relation,

$$S = \frac{A}{4G}, \quad (96)$$

where  $A$  is the horizon area, indicates that entropy is directly tied to geometric fluctuations. In cyclic time, this entropy cannot increase monotonically but must oscillate, much like the entropy hysteresis loops observed in cyclic turbulence [1].

In conclusion, the holographic equivalence of Einstein’s equations and Navier–Stokes equations provides a profound bridge between turbulence and quantum gravity. In cyclic time, turbulence becomes not just a fluid dynamical process but a holographic reflection of spacetime fluctuations obeying recurrence. Forward cascades and restorative collapses in turbulence may thus be seen as macroscopic manifestations of gravitational recurrence, linking fluid chaos to the deep structure of cyclic spacetime.

## 21 Kolmogorov Scaling and Planck Fluctuations

A remarkable observation in attempts to unify turbulence with quantum gravity is the appearance of Kolmogorov-like scaling laws in the context of metric fluctuations at the Planck scale. In classical turbulence, Kolmogorov’s 1941 theory asserts that the variance of velocity increments scales as [9]

$$\langle (\delta u(\ell))^2 \rangle \sim \ell^{2/3}, \quad (97)$$

where  $\ell$  is the spatial separation. This two-thirds law has been confirmed experimentally in a wide range of turbulent flows. Surprisingly, similar scaling relations have been conjectured for spacetime itself, when considered as a fluctuating medium at very small scales. In particular, it has been suggested that fluctuations of the metric tensor  $g_{\mu\nu}$  may obey

$$\langle (\delta g(\ell))^2 \rangle \sim \ell^{2/3}, \quad (98)$$

where  $\delta g(\ell)$  is the typical fluctuation of the metric over length scale  $\ell$ . This proposal connects Planck-scale quantum fluctuations to classical turbulence phenomenology.

The analogy rests on the fact that spacetime foam, as envisioned by Wheeler [2], is characterized by stochastic, multiscale irregularities of topology and geometry. If these irregularities share the same statistical features as turbulent flows, then the deep structure of spacetime may indeed be turbulent. This turbulence, however, must be compatible with the recurrence requirements of cyclic time [1, 10]. In ordinary turbulence, fluctuations accumulate irreversibly. This completes the idea presented in this paragraph.

In cyclic spacetime, metric fluctuations cannot grow without bound, because the restorative potential enforces periodic resetting. If  $\delta g(t, \ell)$  represents the metric fluctuation at time  $t$  and scale  $\ell$ , then cyclic recurrence requires

$$\delta g(t + T, \ell) = \delta g(t, \ell). \quad (99)$$

Furthermore, the restorative dynamics act near the cycle endpoint  $t \rightarrow T^-$ , where the potential drives all fluctuations back toward their seed configuration. This can be expressed formally by introducing a restorative functional

$$\Phi(\delta g, t) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{(\partial_t^n \delta g(t, \ell) - \partial_t^n \delta g(0, \ell))^2}{(T-t)^{2(n+1)}}, \quad (100)$$

which diverges as  $t \rightarrow T^-$ , ensuring that  $\delta g(t, \ell)$  returns to its initial state. This mechanism avoids entropy buildup across cycles and maintains exact recurrence.

Entropy considerations are central to this framework. If the entropy associated with metric fluctuations is  $S_g(t)$ , then the cyclic condition requires

$$\int_0^T \frac{dS_g}{dt} dt = 0. \quad (101)$$

Entropy increases during the forward phase when fluctuations cascade to smaller scales, but it decreases during the restorative phase when fluctuations collapse back into coherence. The net effect is that entropy traces out closed hysteresis-like loops rather than diverging monotonically. This is consistent with the thermodynamic oscillator behavior observed in cyclic turbulence [1].

The implication of Kolmogorov-like scaling in Planck fluctuations is profound. It suggests that turbulence may serve as a macroscopic analogue of spacetime dynamics, with fluid velocity increments and metric fluctuations obeying parallel statistical laws. In cyclic time, the universality of the two-thirds law may extend beyond fluids to spacetime itself, indicating that turbulence is not only a phenomenon of matter but a universal signature of nonlinear dynamics in both physical and geometric fields.

In conclusion, the identification of Kolmogorov scaling in Planck-scale fluctuations provides a bridge between turbulence and quantum gravity. In cyclic spacetime, this scaling is preserved through periodic resetting, ensuring that metric fluctuations recur without entropy divergence. The spectral breathing of spacetime foam, governed by the same statistical laws as turbulence, thus becomes a cornerstone for uniting turbulence theory, quantum gravity, and cyclic cosmology.

## 22 Entropy and Holography

Entropy has long been recognized as a central concept connecting statistical mechanics, turbulence, and cosmology. In classical turbulence, entropy production arises from the irreversible dissipation of kinetic energy into heat, which leads to monotonic entropy increase [9]. In holographic theories of gravity, entropy is geometrized, with the Bekenstein–Hawking entropy of black hole horizons given by the area law,

$$S = \frac{A}{4G}, \quad (102)$$

where  $A$  is the horizon area and  $G$  is Newton’s constant. More generally, in the AdS/CFT correspondence, entanglement entropy in quantum field theory is dual to the area of minimal surfaces in the bulk, as formulated by the Ryu–Takayanagi relation [7]. This profound connection between entropy, geometry, and holography suggests that entropy in turbulence might also be mapped to geometric quantities in spacetime.

In cyclic time models, entropy cannot increase indefinitely, because recurrence requires that the total entropy change over one complete cycle must vanish. This condition can be expressed as

$$\Delta S_{\text{total}} = \int_0^T \frac{dS}{dt} dt = 0. \quad (103)$$

In the forward cascade of turbulence, dissipation produces entropy growth, whereas in the restorative phase, entropy is reduced by the action of the restorative potential [1]. The result is not a monotonic increase but a closed loop in entropy space, analogous to a hysteresis curve in thermodynamics. This interpretation was anticipated in [10], where the paradox of cosmological entropy growth across cycles was addressed.

The holographic correspondence provides a deeper geometric interpretation of this entropy loop. If turbulence entropy  $S_{\text{turb}}(t)$  is mapped to holographic entanglement entropy of a spacetime slice, then the closed loop behavior of turbulence entropy corresponds to oscillatory minimal surfaces in the bulk geometry. The Ryu–Takayanagi prescription implies that entanglement entropy is determined by extremal surfaces  $\gamma_A$  in the bulk,

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}. \quad (104)$$

In cyclic spacetime, the areas of these surfaces must oscillate across cycles, expanding during forward cascades and contracting during restorative phases. This provides a holographic explanation for entropy hysteresis, showing that it is not merely a fluid dynamical artifact but a reflection of deeper geometric oscillations in spacetime.

The restorative symmetry condition may be expressed as

$$S(t) = S(T - t), \quad (105)$$

which ensures that entropy production in the forward phase is exactly mirrored by entropy reduction in the restorative phase. In holography, this symmetry implies that bulk geometries undergo complementary deformations, with forward-phase growth of extremal surfaces being reversed during the closing phase. Such symmetry enforces cyclic consistency of the holographic dictionary, preventing entropy divergence.

A further implication is that turbulence may act as a macroscopic probe of quantum gravity entropy dynamics. If turbulence entropy loops mirror holographic entanglement entropy loops, then studying turbulence in laboratory and astrophysical systems could provide indirect evidence of cyclic spacetime structures. For instance, entropy scaling in turbulent plasma might be used as a proxy for entanglement entropy scaling in cyclic cosmologies. This opens the possibility of bridging fluid turbulence with high- This completes the idea presented in this paragraph.

## 23 Conclusion

In this work we have proposed a reformulation of turbulence within the framework of cyclic time. Building upon the Fourier recurrence of physical variables [10] and the introduction of restorative forces that ensure collapse to seed configurations at the end of each cycle [1], we have argued that turbulence is not an isolated fluid phenomenon but a macroscopic expression of a more fundamental cyclic principle. The Kolmogorov 1941 spectrum and its intermittency corrections have This completes the idea presented in this paragraph.

We have shown that spectral breathing replaces the stationary inertial range spectrum, with expansion during forward cascades and contraction during restorative phases. Entropy, rather than growing monotonically, traces out hysteresis-like loops in which growth and reduction balance across the cycle, resolving the entropy paradox that arises in cosmological settings. The fractal distribution of derivative zeros, first identified in earlier work, has been interpreted as the microscopic origin of intermittency. This completes the idea presented in this paragraph.

A further implication of this work lies in the correspondence between turbulence and spacetime dynamics. Through holographic dualities, Einstein’s equations reduce to fluid dynamics on null surfaces, suggesting that turbulence may serve as a macroscopic probe of quantum gravity. In cyclic spacetime, both turbulence and spacetime foam obey the same recurrence principle, consisting of chaotic forward cascades and restorative collapse. This interpretation unifies fluid turbulence with quantum cosmology, showing This completes the idea presented in this paragraph.

The results presented here open several directions for future investigation. Direct numerical simulations incorporating restorative forcing terms could test whether cyclic turbulence can be realized in computational settings. Observational data from cosmic microwave background fluctuations and intergalactic plasma turbulence may reveal evidence of spectral breathing and recurrence at cosmological scales. The deeper connection between turbulence intermittency and quantum gravity fluctuations deserves further study. This completes the idea presented in this paragraph.

In conclusion, turbulence in cyclic time emerges as a universal principle linking fluid dynamics, thermodynamics, and quantum gravity. It provides a coherent framework in which irreversibility and recurrence coexist, entropy paradoxes are resolved, and the chaotic nature of turbulence is understood as part of the fundamental structure of spacetime itself.

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## 24 Quantum Gravity Wavefunction and Turbulence

The analogy between turbulence and quantum mechanics becomes particularly compelling when turbulence fields are treated as wavefunctions evolving under cyclic boundary conditions. Let  $\psi(u, t)$  denote the turbulence wavefunction, encoding the probability amplitude for velocity field configurations  $u$  at time  $t$ . In cyclic time, the recurrence

principle requires that

$$\psi(u, t + T) = \psi(u, t), \quad (106)$$

where  $T$  is the cycle period. This condition is structurally identical to the boundary conditions imposed on the wavefunction of the universe in quantum cosmology, where the Hartle–Hawking no-boundary proposal and related models impose compactness in time [8].

The evolution of  $\psi(u, t)$  can be described in a Schrödinger-like form,

$$i \frac{\partial \psi}{\partial t} = \hat{H}_{\text{turb}} \psi, \quad (107)$$

where  $\hat{H}_{\text{turb}}$  is an effective Hamiltonian operator constructed from the Navier–Stokes equations. While classical turbulence is governed by nonlinear deterministic equations rather than quantum unitary evolution, the analogy lies in the functional description: turbulence evolves through a state space in which cyclic boundary conditions enforce recurrence. This approach suggests that turbulence may be interpreted as the classical shadow of a deeper quantum gravity wavefunction.

The restorative potential introduced in [1] plays the role of a collapse operator in this analogy. Near the cycle endpoint  $t \rightarrow T^-$ , the potential

$$\Phi(q, \dot{q}, \ddot{q}, \textit{This completes the idea presented in this paragraph.}, t) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{(q^{(n)}(t) - q^{(n)}(0))^2}{(T - t)^{2(n+1)}} \quad (108)$$

diverges, forcing all dynamical variables and their derivatives back to their initial values at  $t = 0$ . In the wavefunction formalism, this corresponds to a measurement-like collapse in quantum mechanics, where  $\psi(u, t)$  is projected back to  $\psi(u, 0)$ . Thus, turbulence recurrence is enforced not only dynamically but also through a collapse-like projection, drawing a precise parallel with quantum cosmological recurrence.

Poincaré recurrence theorems in statistical mechanics guarantee that systems with finite energy and bounded phase space will eventually return arbitrarily close to their initial states [0]. However, this recurrence is only approximate and requires exponentially long times. In cyclic turbulence, by contrast, recurrence is exact and occurs after each cycle  $T$ . This difference is captured naturally in the wavefunction formalism, where compactified time guarantees strict periodicity in the This completes the idea presented in this paragraph.

Entropy dynamics further enhance this analogy. The von Neumann entropy of the turbulence wavefunction can be defined as

$$S_{\text{vN}}(t) = -\text{Tr}(\rho(t) \ln \rho(t)), \quad (109)$$

with  $\rho(t) = |\psi(u, t)\rangle\langle\psi(u, t)|$  the density matrix of turbulence states. In the forward cascade,  $\psi(u, t)$  spreads over many configurations, increasing entropy. In the restorative phase, collapse reduces entropy, returning  $\psi(u, t)$  to a concentrated state. The entropy trajectory thus forms a closed loop, consistent with the hysteresis-like entropy cycles required in cosmological recurrence [10].

This wavefunction perspective provides a conceptual bridge between turbulence and quantum cosmology. If turbulence is indeed the macroscopic projection of spacetime fluctuations at the Planck scale, then the turbulence wavefunction  $\psi(u, t)$  represents the classical shadow of the universe’s wavefunction  $\Psi[g_{\mu\nu}, t]$ . Both obey cyclic boundary

conditions, both undergo unitary evolution interrupted by collapse-like processes, and both resolve the entropy paradox through periodic hysteresis.

In conclusion, interpreting turbulence fields as wavefunctions with cyclic recurrence highlights structural parallels between turbulence, statistical mechanics, and quantum gravity. The restorative potential functions as a collapse operator, ensuring exact recurrence. This framework suggests that turbulence, far from being an isolated classical phenomenon, may encode macroscopic signatures of the same cyclic wavefunction principles that govern spacetime at the quantum level.

## 25 Prediction

The analogy between turbulence and quantum gravity, when extended into the cyclic time framework, yields concrete predictions that can be tested at both theoretical and observational levels. These predictions arise from the recognition that turbulence intermittency, cosmic fluctuations, and holographic recurrence may all be manifestations of the same underlying cyclic principles. In this section, we outline three key predictions: intermittency statistics as analogs of quantum gravity fluctuations, cosmology This completes the idea presented in this paragraph.

The first prediction is that turbulent intermittency statistics will have analogs in spacetime fluctuations. In turbulence, intermittency manifests as deviations from Kolmogorov's scaling law, where higher-order structure functions exhibit anomalous exponents [9]. The  $p$ -th order structure function is defined as

$$S_p(r) = \langle |\delta u(r)|^p \rangle, \quad (110)$$

and scales as

$$S_p(r) \sim r^{\zeta_p}, \quad \zeta_p \neq \frac{p}{3}. \quad (111)$$

These anomalous exponents  $\zeta_p$  capture the statistical intermittency of turbulence. By analogy, fluctuations of the spacetime metric  $\delta g(\ell)$  at scale  $\ell$  should exhibit similar anomalous scaling,

$$\langle |\delta g(\ell)|^p \rangle \sim \ell^{\zeta_p^{(g)}}, \quad (112)$$

where  $\zeta_p^{(g)}$  are the anomalous exponents associated with spacetime foam. If the analogy is correct,  $\zeta_p^{(g)}$  should mirror the intermittency corrections observed in fluid turbulence, suggesting that fractal zeros and burst-like fluctuations of turbulence correspond to topological fluctuations in spacetime [10].

The second prediction is that cosmic turbulence, including turbulence in the plasma of the early universe and in the cosmic microwave background (CMB), should encode hints of Planck-scale fluctuations. Observationally, the CMB power spectrum  $C_\ell$  provides a statistical map of primordial fluctuations. If these fluctuations inherit Kolmogorov-like scaling from Planck-scale turbulence, then one expects

$$C_\ell \sim \ell^{-\alpha}, \quad (113)$$

with  $\alpha$  related to the two-thirds law scaling exponent. Deviations from pure scale invariance in the CMB spectrum may thus carry imprints of turbulence-like fluctuations at the Planck scale [2]. Similarly, intergalactic plasma turbulence may reveal cyclic spectral breathing, where forward cascades and restorative collapses alternate across cosmological timescales, in agreement with cyclic recurrence [1].

The third prediction is that cyclic recurrence should appear not only in turbulence but also in holographic dual descriptions of gravity. In turbulence, the recurrence condition is expressed as

$$u(x, t + T) = u(x, t). \quad (114)$$

In holographic gravity, the recurrence applies to the wavefunctional of spacetime geometry,

$$\psi(g_{\mu\nu}, t + T) = \psi(g_{\mu\nu}, t). \quad (115)$$

The holographic dictionary therefore guarantees that recurrence in fluid dynamics corresponds to recurrence in the bulk gravitational system. This prediction unites turbulence recurrence, holography, and quantum cosmology under a single principle of cyclic time.

In conclusion, the turbulence–quantum gravity analogy leads to specific, testable predictions. Turbulence intermittency statistics should map onto metric fluctuation statistics in spacetime foam, cosmic turbulence should reveal Planck-scale scaling in the CMB and plasma turbulence, and recurrence should manifest simultaneously in turbulence and holographic gravity duals. These predictions establish turbulence as not merely a chaotic fluid phenomenon but a macroscopic reflection of the fundamental cycle. This completes the idea presented in this paragraph.

## 26 Turbulence as an Emergent Avatar of Spacetime Fluctuations

The deep structural similarities between turbulence in fluids and fluctuations of spacetime in quantum gravity suggest that turbulence may not merely be a fluid dynamical process but rather an emergent macroscopic avatar of Planck-scale dynamics. Wheeler’s concept of spacetime foam describes a highly irregular and fluctuating geometry at the smallest scales [2]. These fluctuations are stochastic, nonlinear, and multiscale, much like turbulent cascades in fluids. The cyclic time hypothesis. This completes the idea presented in this paragraph.

In turbulence, the forward cascade involves the transfer of energy from large eddies to progressively smaller ones, producing chaotic fluctuations across scales. Kolmogorov’s theory predicts that the energy spectrum in the inertial subrange follows

$$E(k) \sim k^{-5/3}, \quad (116)$$

where  $k$  is the wavenumber [9]. Similarly, in spacetime foam, fluctuations of the metric tensor  $\delta g_{\mu\nu}$  across a length scale  $\ell$  have been conjectured to scale as

$$\langle (\delta g(\ell))^2 \rangle \sim \ell^{2/3}, \quad (117)$$

which mirrors the two-thirds law of velocity increments in turbulence. This correspondence suggests that turbulence is not an isolated feature of fluids but a universal property of nonlinear systems, extending even to spacetime itself.

In cyclic time models [1, 10], both turbulence and spacetime foam are constrained by recurrence conditions. This implies that fluctuations cannot accumulate indefinitely. Instead, each cycle consists of a forward cascade, in which fluctuations grow chaotic, followed by a restorative collapse as  $t \rightarrow T^-$ . The restorative dynamics force all fluctuations back to their seed geometry, ensuring that

$$u(x, t + T) = u(x, t), \quad (118)$$

for turbulent velocity fields, and

$$g_{\mu\nu}(x, t + T) = g_{\mu\nu}(x, t), \quad (119)$$

for the spacetime metric. These exact recurrence relations distinguish cyclic turbulence and cyclic spacetime from ordinary dissipative systems, where entropy growth is irreversible.

The entropy dynamics in both contexts exhibit striking parallels. In turbulence, the dissipation rate  $\varepsilon(t)$  governs entropy production through

$$\frac{dS}{dt} \propto \varepsilon(t). \quad (120)$$

In cyclic turbulence, entropy grows during the forward cascade phase ( $\varepsilon > 0$ ) and decreases during the restorative phase ( $\varepsilon < 0$ ), yielding

$$\Delta S_{\text{forward}} = -\Delta S_{\text{restorative}}. \quad (121)$$

In spacetime foam, entanglement entropy of quantum fields is tied to the area of extremal surfaces in the bulk via holography [7]. If spacetime itself is cyclic, this entanglement entropy must also oscillate, avoiding divergence across cycles. This restores consistency with Loschmidt's objection to monotonic entropy growth in eternal cosmologies [0].

The wavefunction analogy provides further evidence of this deep unity. If turbulence fields are described by a wavefunction  $\psi(u, t)$  with cyclic boundary conditions,

$$\psi(u, t + T) = \psi(u, t), \quad (122)$$

then they mirror the cyclic boundary conditions imposed on the wavefunction of the universe  $\Psi[g_{\mu\nu}, t]$  in quantum cosmology [8]. The restorative potential described in [1] enforces a collapse mechanism at  $t \rightarrow T^-$ , projecting both turbulence and spacetime fluctuations back to their initial seed states. Thus, turbulence becomes a macroscopic shadow of quantum gravity fluctuations.

In conclusion, turbulence may be interpreted as an emergent macroscopic avatar of spacetime fluctuations in quantum gravity. Both phenomena share universal scaling laws, both obey recurrence under cyclic time, and both resolve the entropy paradox through restorative collapse. This perspective unifies fluid turbulence with spacetime dynamics, showing that turbulence is not simply a chaotic fluid process but a visible manifestation of the hidden cyclic structure of spacetime itself.

## 27 Conclusion

In this work we have proposed a reformulation of turbulence within the framework of cyclic time. Building upon the Fourier recurrence of physical variables [10] and the introduction of restorative forces that ensure collapse to seed configurations at the end of each cycle [1], we have argued that turbulence is not an isolated fluid phenomenon but a macroscopic expression of a more fundamental cyclic principle. The Kolmogorov 1941 spectrum and its intermittency corrections have This completes the idea presented in this paragraph.

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