Entropic Barrier vs. Geometry:
From Singularities and the Information Paradox to Clarity

A. G. Schubert

Abstract

This study provide a purely classical, thermodynamic/entropic interpretation of gravitational collapse in which a thin, high-entropy shell (entropic barrier) forms before curvature invariants blow up. The shell is modeled as a timelike thin layer separating an effective de Sitter interior from a Schwarzschild exterior. If, after a short transient, no further rest mass crosses inward, the exterior mass parameter M (hence the Schwarzschild radius $R_S = 2M$) remains constant, while the entropic boundary can expand under interior negative pressure. In this setting no geometric singularity forms and no classical information paradox arises. We stay within GR and standard black-hole thermodynamics and avoid holography or quantum-gravity assumptions.

1 Classical identities and interpretation

We work in natural units $G = \hbar = c = k_B = 1$. For Schwarzschild black holes,

$$R_S = 2M, \quad A = 4\pi R_S^2, \quad S = \tfrac{A}{4} = \pi R_S^2, \quad T_H = \frac{1}{4\pi R_S}, \quad dM = T_H\, dS. \tag{1}$$

Differentials give $dR_S = 2\, dM$ and $dS = 2\pi R_S\, dR_S$, hence

$$T_H dS = \frac{1}{4\pi R_S}(2\pi R_S\, dR_S) = \tfrac{1}{2}\, dR_S = dM \;\Rightarrow\; dR_S \sim dM \sim dS. \tag{2}$$

After the transient, we impose $dM \simeq 0$ (no further rest mass crosses inward). Then R_S remains constant while the entropy we assign to the system is carried by the entropic boundary (thin shell) with radius $R(\tau)$ and

$$S_{\rm EB} = \pi R^2, \qquad dS_{\rm EB} = 2\pi R\, dR. \tag{3}$$

2 Formation of a timelike entropic barrier

Self-gravitating systems exhibit negative heat capacity ($C < 0$). Dissipation thereby drives the entropy to a maximum at a finite areal radius R_* before curvature invariants diverge. A thin, high-entropy spherical layer forms around R_*; we model it as a timelike thin shell with surface stress $\mathrm{diag}(-\sigma, p, p)$. The interior is taken as de Sitter with $H^2 = \Lambda_-/3$,

$$ds_-^2 = -(1 - H^2 r^2)\, dt_-^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega^2, \tag{4}$$

while the exterior is Schwarzschild (or Schwarzschild–de Sitter),

$$ds_+^2 = -f_+(r)\, dt_+^2 + f_+(r)^{-1} dr^2 + r^2 d\Omega^2, \qquad f_+(r) = 1 - \frac{2M}{r} - \frac{\Lambda_+ r^2}{3}. \tag{5}$$

Curvature bound inside: for de Sitter one has $K \equiv R_{abcd}R^{abcd} = \tfrac{8}{3}\Lambda_-^2 < \infty$.

3 Thin-shell dynamics (Israel junction conditions)

Let $R(\tau)$ denote the areal radius of the (timelike) shell. Israel's junction conditions imply

$$\sqrt{f_+(R)+\dot{R}^2}-\sqrt{f_-(R)+\dot{R}^2}=\kappa(R), \qquad \kappa(R)=4\pi\,\sigma(R)\,R, \tag{6}$$

with $f_-(r)=1-H^2r^2$. This can be cast as an effective potential equation $\dot{R}^2+V(R)=0$ with

$$V(R)=f_-(R)-\frac{\left(f_+(R)-f_-(R)-\kappa(R)^2\right)^2}{4\,\kappa(R)^2}. \tag{7}$$

Surface stress conservation gives

$$\frac{d}{d\tau}(\sigma A)+p\,\frac{dA}{d\tau}=0 \;\Rightarrow\; \frac{d\sigma}{dR}=-\frac{2(\sigma+p)}{R}, \qquad A=4\pi R^2. \tag{8}$$

We use a simple equation of state $p=w\,\sigma$ with constant w.

4 Outward-evolution threshold without accretion

Assume (i) no radial energy flux through the shell ($T^r{}_t|_{\text{shell}}=0$) and (ii) $dM\approx 0$. Balancing interior de Sitter acceleration with exterior attraction at $r=R$ yields a robust outward-growth condition

$$H^2R\gtrsim\frac{M}{R^2} \qquad\Longleftrightarrow\qquad R\gtrsim R_{\text{thr}}\equiv\left(\tfrac{3M}{\Lambda_-}\right)^{1/3}. \tag{9}$$

Define the dimensionless parameter $\chi\equiv\Lambda_-M^2$. Then

$$\frac{R_{\text{thr}}}{R_S}=\frac{1}{2}\left(\frac{3}{\chi}\right)^{1/3}, \qquad R_S=2M. \tag{10}$$

Proposition (sufficient outward evolution). Let $p=w\,\sigma$ with $w>-1/2$ (so the shell self-gravity weakens with R via (8)). If at some τ_0 the radius obeys $R(\tau_0)\geq R_{\text{thr}}$ of (9), then $\dot{R}(\tau_0)\geq 0$ and the shell evolves outward for a finite interval. (Sketch: with $w>-1/2$, $\kappa'(R)<0$; at threshold $\partial_R V<0$ so that $-V=\dot{R}^2$ increases with R.)

5 Wavelength cutoff and falsifiability (classical)

We drop any transparency assumption and formulate a *storage* criterion for electromagnetic (or scalar) modes in the interior bounded by the entropic barrier at areal radius $R(\tau)$. Let ω_{loc} be the local frequency measured in the shell frame and $k_{\text{loc}}=\omega_{\text{loc}}$ (units $c=1$). A minimal confinement condition for quasi-trapped modes is

$$k_{\text{loc}}R\;\geq\;\xi, \qquad \xi=\mathcal{O}(1), \tag{11}$$

meaning that at least one fundamental half-wave fits into the cavity set by the shell. The observed frequency is gravitationally redshifted,

$$\omega_\infty=\zeta_+(R)\,\omega_{\text{loc}}, \qquad \zeta_+(R)=\sqrt{f_+(R)}=\sqrt{1-\frac{2M}{R}-\frac{\Lambda_+R^2}{3}}. \tag{12}$$

Combining (11) and (12) yields a classical cutoff for trapped modes at infinity,

$$\boxed{\omega_{\infty,c}(R;M)\simeq\frac{\xi\,\zeta_+(R)}{R}}, \qquad f_c\equiv\frac{\omega_{\infty,c}}{2\pi}. \tag{13}$$

Modes with $\lambda_\infty\gtrsim 2\pi/\omega_{\infty,c}$ are not stored by the interior; shorter-wavelength modes can be trapped and are then observed redshifted.

Dimensionless scaling. Setting $\Lambda_+ = 0$, let $R_S = 2M$ and $y \equiv R/R_S \geq 1$:

$$f_c\, R_S = \frac{\xi}{2\pi}\, \frac{\sqrt{1-1/y}}{y}. \tag{14}$$

This rises from zero at $y \to 1^+$, has a single maximum near $y \approx 1.5$, and decays $\propto 1/y$ for $y \gg 1$. Hence the absolute bound

$$f_c\, R_S \;\lesssim\; 0.061\, \xi \quad \text{(natural units)}. \tag{15}$$

6 Timeline (quantitative milestones)

Stage	**Radius / condition**	**Quantitative signature**
(0) Initial collapse	$R \gg R_S$; $C > 0 \to C < 0$	Temperature rises with contraction; onset of negative heat capacity.
(1) Entropy maximum	$R = R_*$	S reaches $S_{\max}$; thin dissipative layer forms.
(2) Causal barrier	$R \approx R_*$	Timelike shell forms; de Sitter interior ($\Lambda_- > 0$); trapped surface near shell.
(3) Post-transient	$dM \simeq 0$	Exterior mass fixed; $R_S = 2M$ constant; entropy $S_{\rm EB} = \pi R^2$.
(4) Threshold	$R \gtrsim R_{\rm thr}$	Condition (9) or (10).
(5) Expansion	$\dot{R} > 0$	$S_{\rm EB}$ grows via $dS_{\rm EB} = 2\pi R\, dR$; interior volume $\propto R^3$ increases.
(6) Outcome	—	No curvature singularity (de Sitter bound); no classical information paradox.

7 Limitations and outlook

We do not prove geodesic completeness; we provide a sufficient classical condition (9) that prevents further focusing beyond the shell. Quantum unitarity and correlations in any emitted radiation are beyond scope. A full stability analysis requires $V''(R_*) > 0$ from (7) with a specified w; we leave the algebraic details for a follow-up appendix.

A Classical emergence of an effective Λ_-

A.1 Constraints ⇒ constant-curvature interior

Assume: (i) a timelike thin shell at areal radius $R(\tau)$, (ii) no rest-mass flux across the shell ($T^r{}_t|_{\rm shell} = 0$), (iii) interior regularity and isotropy, (iv) quasi-stationarity near R_*. Under (iii)–(iv) the interior stress tensor must be proportional to the metric, $T^-_{ab} = -\rho_-\, g^-_{ab}$. Einstein's equation then gives $R^-_{ab} = 8\pi\rho_-\, g^-_{ab} + \Lambda_+ g^-_{ab}$, i.e. a maximally symmetric interior with constant curvature. Hence

$$H^2 = \Lambda_-/3, \qquad \Lambda_- \equiv \Lambda_+ + 8\pi\rho_-,$$

so an *effective* Λ_- emerges from a uniform interior energy density (vacuum-like $w = -1$) within GR.

A.2 Junction relation and a bound for Λ_-

At a static shell ($\dot{R} = 0$), Israel's condition is

$$\sqrt{1 - \frac{2M}{R} - \frac{\Lambda_+ R^2}{3}} - \sqrt{1 - \frac{\Lambda_- R^2}{3}} = 4\pi\sigma R. \tag{16}$$

Eq. (16) fixes Λ_- *implicitly* in terms of $(M, \Lambda_+, R, \sigma)$ (algebra omitted). Independently, the outward-evolution condition used in the main text,

$$H^2 R \gtrsim \frac{M}{R^2} \quad \Longleftrightarrow \quad \boxed{\Lambda_- \gtrsim \frac{3M}{R^3}}, \tag{17}$$

provides a *lower bound* set purely by exterior data (M, R). At the onset $(R = R_{\rm thr})$ the bound is saturated, so $\Lambda_- = 3M/R^3$.

B Classical origins of negative heat capacity ($C < 0$)

B.1 Virial (microcanonical) route

For a bound self-gravitating system: $2K + U = 0$, $K = \frac{3}{2} N k_B T$, hence $E = K + U = -K = -\frac{3}{2} N k_B T$ and

$$C \equiv \frac{dE}{dT} = -\tfrac{3}{2} N k_B < 0,$$

demonstrating generic negative heat capacity in the microcanonical ensemble.

B.2 Schwarzschild benchmark

With $T_H = 1/(4\pi R_S)$ and $M = R_S/2$ one finds

$$C_{\rm BH} = \frac{dM}{dT_H} = -2\pi R_S^2 < 0,$$

a purely GR benchmark for $C < 0$.

B.3 Thin-shell energetics (Tolman law)

Tolman redshift gives $T_\infty = \zeta_+(R)\, T_{\rm loc}$ with $\zeta_+(R) = \sqrt{1 - 2M/R - \Lambda_+ R^2/3}$. The shell energy $E_{\rm shell} = 4\pi R^2 \sigma(R)$ obeys $\frac{d\sigma}{dR} = -\frac{2(\sigma+p)}{R}$. For $p = w\sigma$ with $w > -1/2$ one gets $\sigma \propto R^{-2(1+w)}$ and thus (at fixed M) $E_{\rm shell} \propto R^{-2w}$. Increasing T_∞ (shrinking R) deepens the binding energy in magnitude, amounting to an *effective* negative specific heat, consistent with B.1–B.2.

C Overcritical regime as constrained entropy extremum (classical)

We formalize the "overcritical" stage without invoking negative absolute temperatures. The coarse-grained entropy is the area of the entropic boundary,

$$S_{\rm EB}(R) = \pi R^2, \qquad dS_{\rm EB} = 2\pi R\, dR.$$

Once accretion stops, the exterior ADM mass M is fixed. Admissible states $\{R, \sigma(R), \Lambda_-(R)\}$ obey: (i) static Israel junction ($\dot{R} = 0$)

$$F(R, \sigma, \Lambda_-; M) \equiv \sqrt{1 - \frac{2M}{R} - \frac{\Lambda_+ R^2}{3}} - \sqrt{1 - \frac{\Lambda_- R^2}{3}} - 4\pi\sigma R = 0, \tag{18}$$

(ii) surface-stress conservation

$$\frac{d\sigma}{dR} = -\frac{2(\sigma + p)}{R}, \qquad p = w\,\sigma,\ w > -1/2. \tag{19}$$

We extremize $\mathcal{S} = S_{\rm EB}(R) + \mu\, F(R,\sigma,\Lambda_-;M)$. Stationarity implies

$$2\pi R + \mu\,\partial_R F = 0, \quad \mu\,\partial_\sigma F = 0, \quad \mu\,\partial_{\Lambda_-} F = 0.$$

With $\partial_\sigma F = -4\pi R \neq 0$ we have $\mu \neq 0$ and $\partial_{\Lambda_-} F = 0$. Using $F = 0$ and (19) to eliminate $\sigma'(R)$, differentiation of $F(R,\sigma(R),\Lambda_-(R);M) = 0$ w.r.t. R yields (algebra omitted) the leading extremum condition

$$H^2 R \;\simeq\; \frac{M}{R^2}\,[1+\mathcal{O}(w)] \quad \Longleftrightarrow \quad \boxed{\Lambda_- \simeq \frac{3M}{R^3}\,[1+\mathcal{O}(w)]}. \tag{20}$$

D Classical confinement and optional semi-classical leakage

D.1 Null geodesics and confinement (classical)

The de Sitter interior has $r_{\rm dS} = H^{-1}$. For $R < r_{\rm dS}$, inward rays redshift strongly; outward rays emitted near the shell are reduced by $\zeta_+(R)$. Modes with local wavelength short enough to fit the cavity ($k_{\rm loc} R \geq \xi$, $\xi = \mathcal{O}(1)$) can form quasi-trapped standing patterns. Observationally this yields the low-frequency cutoff used in the main text: $\omega_{\infty,c}(R;M) \simeq \xi\,\zeta_+(R)/R$.

D.2 Regge–Wheeler WKB (optional, semi-classical)

Scalar perturbations satisfy $\partial_t^2\Psi - \partial_{r_*}^2\Psi + V_\ell(r)\Psi = 0$ with $V_\ell = f(r)\,\ell(\ell+1)/r^2$ (f as above). A thin shell adds a contact term $g(\omega)\,\delta(r-R)$. The standard WKB estimate is

$$\mathcal{T}_\ell(\omega) \approx \Big[1 + \exp\Big(2\int_{r_1}^{r_2} dr_*\,\sqrt{V_\ell(r)-\omega^2}\Big)\Big]^{-1},$$

with turning points $V_\ell(r_{1,2}) = \omega^2$. This is provided for future phenomenology and is *not* used to support the core classical claims.

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