Boundary Rules, Born Worlds:
Self-Excited Gravity and Pregeometric Scaffolds

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August 19, 2025

Abstract

Part I (lay readers) — The Universe's Feedback Loop. We propose that certain gravitational configurations *self-excite*: their interior dynamics are fixed by boundary rules so that the on-shell "cost" moves to topological charges at the edge. In plain language, gravity, useful instabilities (instantons, sphalerons, merons), and a canonical way to "host" distances from data (tight spans / injective hulls) form a feedback loop that can leave global, testable watermarks in the sky rather than new local particles.

Part II (technical) — Law Without Initial Conditions? We formalize a fixed-point consistency principle in teleparallel gravity via the constitutive relation $T^a = \pm H_a$, prove stationarity-by-identity and boundary-dominated actions (Nieh–Yan), construct sphaleron/meron sectors, and develop a pregeometry→metric pipeline (uniformities, injective hulls, semi-tropical calculus). We describe cosmology mechanisms for early-time shifts in the sound horizon r_s (impacting H_0) and late-time growth suppression (impacting S_8), with falsifiers tied to BBN, N_{eff}, BAO/SN, CMB polarization, SGWB, and lensing.

Keywords: teleparallel gravity; gravitational instantons; sphalerons; merons; Nieh–Yan boundary term; pregeometry; tight span / injective hull; matroid fans; fixed-point theorems; cosmology tensions (H_0, S_8).

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Part I: The Universe's Feedback Loop

1 The guiding question: can a universe start itself?

We begin with John Archibald Wheeler's provocation: perhaps the most powerful physical principle is one that *lets the universe come into being of its own accord* [1, 2, 3]. Read literally, this is not mysticism; it is an engineering challenge. Can lawful feedback—between what exists and the rules that say what can exist—*close the loop* so tightly that large parts of cosmic history are selected by global consistency rather than by arbitrary initial conditions?

Three everyday sketches frame the idea. First, an electronic oscillator needs only a nudge; feedback sustains the rhythm. Second, a soap film spanning a wire loop takes its shape from the *boundary*; the interior simply follows the rule "minimize area". Third, a good crossword can "click shut" because the clues overdetermine the grid. Our claim is that gravity has regimes that behave like this: the "cost" of a configuration lives on the edges (a topological count), while the interior dynamics satisfy themselves.

In modern language, gravity admits special, all-at-once configurations called *instantons* [10, 13, 14]. In an alternative but equivalent formulation of gravity—*teleparallel* gravity—curvature is traded for torsion [6, 7, 18]. In that grammar there are *self-excited* configurations for which the torsion two-form T^a equals (up to a sign) its own excitation H_a. When that matching holds, the field equations reduce to identities and the on-shell action collapses to a boundary charge (the Nieh–Yan invariant) [8]. In plain words: the interior "pays" nothing; the bill is settled at the boundary. This is how a lawful feedback loop can select whole solutions.

Self-excitation is not the only actor. *Sphalerons* and *merons* are unstable, in-between states that sit atop energy barriers or carry "half-twist" structure. Although precarious, they can be the very bridges through which a system moves from one sector to another, leaving imprints we can seek in data. For a lay reader: think of the mountain pass between valleys (a sphaleron) and a half-formed vortex (a meron). Their importance here is pragmatic: they are the places where global constraints bite hardest, and where boundary rules can force interior outcomes.

We will also need a disciplined way to talk about *geometry from information*. Imagine being given only a table of distances. The *tight span* (injective hull) is the smallest, most economical space that houses those distances without distortion. It is canonical and contractible, and it often serves as the right "room" in which to compare models or datasets [25, 26, 19, 20, 21]. In our story, tight spans are the geometric scaffolds that let boundary rules and observational distances meet without overfitting.

To keep ourselves honest, we adopt a light-but-precise logical backbone. Whenever a system is expressive enough to talk about its own descriptions, fixed points or "off-list" exceptions inevitably appear; this is the lesson of Lawvere's fixed-point theorem and related diagonal arguments [36, 38, 39, 40]. Read physically, this suggests that a consistent "universe-as-language" (Wheeler's participatory view; CTMU's reflexive stance) should either harbor fixed solutions selected by closure, or else reveal exactly where our description fails [55, 58].

Principle 1.1 (Fixed-Point Consistency (informal))**.** A boundary rule plus a constitutive map can select bulk configurations as fixed points. When this happens, interior dynamics are determined by boundary data, and the physical "cost" reduces to a topological boundary count. The program is *empirical*: such configurations must leave global, falsifiable fingerprints.

How would we test this? Not by hunting new local particles, but by seeking *global watermarks*: tiny, correlated phase shifts in the acoustic peaks of the cosmic microwave background, parity-odd twists in polarization, narrow-band stochastic gravitational-wave hums, or subtle patterns in weak-lensing maps. These signatures connect early-time boundary episodes (affecting the sound horizon r_s and H_0) and late-time growth (affecting S_8) to concrete pipelines we spell out in the technical half, with hard priors from BBN, N_{eff}, BAO/SN, and polarization null tests [15].

What this section is (and isn't). This is the lay roadmap. No equations are required to grasp the thesis: lawful feedback plus boundary rules can make parts of the universe "start themselves." The rest of the paper delivers the mathematics and the data analysis. If the signatures fail to appear where predicted, the idea is wrong in nature's world. That is a success condition too: it tells us which loops *do not* close.

2 Three pillars at a glance

Pillar A: Self-excited gravity. In teleparallel gravity, the usual curvature picture is traded for *torsion*, a kind of twist in how vectors are transported [6, 7]. Certain configurations match torsion to its own excitation (schematically $T^a = \pm H_a$), making the field equations hold by identity and pushing the on-shell "cost" to a boundary charge (the Nieh–Yan invariant) [8, 18, 5]. Intuitively: the rules at the edge determine the shape inside.

Pillar B: Useful instabilities. *Instantons* are all-at-once bridges between sectors [10, 13, 14]. *Sphalerons* (mountain passes) and *merons* (half-twists) are unstable, but they govern how transitions happen and what they imprint. They are where boundary rules most clearly force interior outcomes.

Pillar C: Canonical scaffolds. When we only know *distances*, the *tight span* (injective hull) is the smallest, most faithful space that hosts them [25, 26, 20]. Its complexity is quantified (Develin's bounds), and model shapes like spheres have tight spans as rich as Hilbert cubes [19, 21]. This gives us a disciplined "room" to compare models and data without adding gratuitous structure.

3 Gravity in a new grammar: curvature vs. torsion

General relativity explains gravity as *curvature*: mass and energy bend spacetime, and free-fall follows the bends. Teleparallel gravity keeps the same observations but uses a different grammar: spacetime is globally flat in curvature, while gravity lives in torsion carried by a coframe field [6, 7]. In this language there is a natural constitutive map from torsion to an "excitation" H_a. When a configuration satisfies the simple matching $T^a = \pm H_a$, a built-in identity makes the field equations trivial in the bulk, and the action reduces to a boundary term given by the Nieh–Yan invariant [8, 18]. In plain words, it is a soap-film effect: the wire loop (boundary rules) fixes the film (bulk geometry).

Why this matters. Boundary-dominated solutions are the cleanest embodiment of Wheeler's "self-excited circuit" idea [1, 2, 3]. If such sectors exist and couple (even weakly) to the hot early universe, they can leave global, quantized fingerprints rather than new local particles.

4 Useful instabilities: sphalerons and merons

An *instanton* is like a tunnel through a mountain; a *sphaleron* is the saddle on the ridge; a *meron* is a half-twist that cannot stand alone but can mediate change. In gauge theory and gravity, these objects control transitions between topological sectors and set rates for rare processes [11, 12, 17, 14]. Although unstable, they are predictable: one can enumerate their negative modes, estimate rates, and work out their selection rules. For our purposes, they are reliable *bridges* through which boundary rules propagate into the bulk.

5 Pregeometry: from relations to distance

Suppose geometry is not given but *emerges* from simpler relations. One laboratory for this is the language of *uniformities* and *entourages*: formal "closeness" relations that can be assembled into a bona fide metric and even an effective dimension [28]. This fits Wheeler's participatory outlook and later "It-from-Bit" programs: start from minimal informational structure and let geometric notions crystallize only where the relations force them [3, 52]. Our use of pregeometry is modest but crucial: it supplies clean toy models of emergence and interfaces naturally with tight spans.

6 The canonical room: tight spans and the Isbell completion

Give only a distance table. The *tight span* (also called the *injective hull*) is the smallest space that contains your points at exactly those distances and allows all short maps to extend [25, 26]. It is canonical and contractible, which means it avoids false "holes" that could trick an analysis [20]. Two landmarks guide our expectations: (i) Develin's bounds show how complex a tight span can be for n points (a proxy for how far your data are from tree-like) [19]; (ii) spheres have tight spans homeomorphic to Hilbert cubes, warning that simple raw shapes may require rich ambient rooms [21]. Practically, tight spans let us compare cosmological models and data in a controlled, no-extras geometry.

7 Today's puzzles, tomorrow's tests

Precision cosmology reports two persistent tensions: the present expansion rate H_0 inferred from early-universe fits versus local measurements, and the amplitude of matter clustering (often summarized by S_8) [15]. In our framework, an early, brief boundary-driven episode can slightly shift the sound horizon r_s (touching H_0) without violating BBN or N_{eff}, while a gentle late-time response can depress growth (touching S_8) without disrupting distance indicators. The predicted *watermarks* are global: acoustic peak phase shifts, parity-odd twists in CMB polarization, narrow-band stochastic gravitational-wave hums, and coherent patterns in weak-lensing maps. Each comes with *red-line falsifiers* (e.g., light-element yields, polarization nulls) that can rule the idea out.

8 From story to structure: how to read Part II

Every idea above maps to a precise construction: teleparallel self-excitation and boundary actions (section 3); instantons, sphalerons, merons and their roles (section 4); pregeometry toy models (section 5); tight spans as canonical hosts (section 6); and a fixed-point consistency principle informed by modern logic (Lawvere, Roberts, Yanofsky) and by Wheeler's participatory view [36, 39, 40, 55]. Part II provides the definitions, theorems, worked examples, and the data pipelines needed to check the claims.

Part II: Law Without Initial Conditions?

9 Fixed-point program: objects, maps, and assumptions

We formalize the motivating idea as a fixed-point selection rule on boundary data. Let $\mathcal{X}$ be a space of field configurations (coframes, connections, matter, ...) on a manifold M, and let $\mathcal{B}$ be a space of admissible boundary data on ∂M (asymptotics, topological charges, cohomology classes). A *boundary operator* $\partial : \mathcal{X} \to \mathcal{B}$ extracts boundary data from a bulk configuration. A *constitutive map* $\mathcal{C} : \mathcal{B} \to \mathcal{X}$ assigns bulk fields compatible with a chosen physical law.

Definition 9.1 (Fixed-Point Consistency Principle (FPCP))**.** A configuration $X^\star \in \mathcal{X}$ is *fixed-point consistent* if

$$X^\star = \mathcal{C}\big(\partial X^\star\big) \quad \text{and} \quad X^\star \text{ is stationary for the action with those boundary data.}$$

A theory/law admits *law without initial conditions on* $(M, \partial M)$ if nonempty FPCP solutions exist for an open set of boundary data.

The logical motivation is that sufficiently expressive systems generically support fixed points or else exhibit incompleteness; see Lawvere's fixed-point theorem and diagonal-argument variants [36, 38, 39, 40]. Our use is concrete: we specify $\mathcal{X}, \mathcal{B}, \partial, \mathcal{C}$ in teleparallel gravity and identify a class of bulk fields that are stationary *by identity*.

Minimal standing assumptions. (1) $\mathcal{X}$ carries a well-posed variational problem with an action $S[X]$ whose boundary variation is controlled by ∂X. (2) ∂ is surjective onto the admissible $\mathcal{B}$ for the boundary conditions at hand. (3) $\mathcal{C}$ is local/covariant (up to a reference structure fixed at the boundary) and compatible with the symmetries of the theory. Assumptions (1)–(3) will be specialized below.

10 Teleparallel preliminaries: fields, torsion, action

We work in the teleparallel (torsion) formulation of gravity [6, 7, 18]. The fundamental variables are a coframe (tetrad) $\{e^a\}$ and a flat, metric-compatible Lorentz connection $\omega^a{}_b$ with $R^a{}_b(\omega) = 0$. The torsion 2-form is

$$T^a := \mathrm{D}e^a = \mathrm{d}e^a + \omega^a{}_b \wedge e^b. \qquad (1)$$

A *constitutive relation* assigns an *excitation* 2-form H_a algebraically from $\{T^a\}$ (and metric),

$$H_a = \chi_{ab}[g] \lrcorner T^b, \tag{2}$$

where χ_{ab} is built from g (and possibly irreducible pieces of T). We keep the constitutive law abstract; concrete choices reproduce TEGR and its generalizations [18].

A convenient first-order gravitational action is

$$S[e,\omega] \;=\; \frac{1}{2\kappa}\int_M T^a \wedge H_a \;+\; S_{\rm bdy}[e,\omega], \tag{3}$$

with $\kappa = 8\pi G$ (units $c = 1$). The boundary term $S_{\rm bdy}$ renders the variational problem well-posed under the chosen boundary conditions.

Two identities will matter repeatedly. First, the *teleparallel Bianchi identity*

$$\mathrm{D}T^a \equiv 0 \qquad (R^a{}_b = 0), \tag{4}$$

and second, the *Nieh–Yan 4-form* [8]

$$\mathsf{NY} \;:=\; \mathrm{d}\big(e^a \wedge T_a\big) \;-\; T^a \wedge T_a \;+\; e^a \wedge e^b \wedge R_{ab}. \tag{5}$$

On teleparallel backgrounds ($R_{ab} = 0$), $\mathsf{NY} = \mathrm{d}(e^a \wedge T_a) - T^a \wedge T_a$ is exact up to a torsion-quadratic term.

11 Self-excited sectors: stationarity by identity and boundary reduction

We now encode "self-excitation" as a fixed-point constitutive constraint:

$$T^a \;=\; \pm H_a. \tag{6}$$

Read via (3), this matches the field to its excitation.

Assumption 11.1 (Constitutive symmetry)**.** The constitutive map $T \mapsto H$ is linear, local, and metric-compatible, and the boundary term $S_{\rm bdy}$ is chosen so that δS contains no undesired boundary variations for fixed $\partial(e,\omega)$.

Proposition 11.2 (Stationarity by identity under self-excitation)**.** *Under Assumption 11.1, any smooth configuration (e,ω) on M obeying (6) is stationary for (3) against compactly supported variations:*

$$\delta S[e,\omega] = 0 \quad \textit{for all } (\delta e, \delta\omega) \textit{ with } \mathrm{supp} \subset\subset M.$$

Sketch. *Vary $S = \frac{1}{2\kappa}\int T^a \wedge H_a$. Bulk terms combine into $\int(\delta T^a \wedge H_a + T^a \wedge \delta H_a)$. For linear $H[T]$, this equals $\int \delta T^a \wedge (H_a \pm T_a)$ up to total derivatives when (6) holds; the Bianchi identity (4) kills residual* D*-exact pieces. Boundary terms are cancelled by S_{bdy} by design.* □

Proposition 11.3 (Boundary-dominated on-shell action)**.** *On a self-excited configuration (6), the bulk part of the action reduces to a boundary integral plus (model-dependent) topological terms. In particular, there exists a 3-form $\Theta[e,\omega]$ such that*

$$S_{bulk} = \frac{1}{2\kappa}\int_M T^a \wedge H_a \;=\; \frac{1}{2\kappa}\int_{\partial M} \Theta \;+\; \textit{(torsion-quadratic exact terms)},$$

and for TEGR-like constitutive choices one can take $\Theta = e^a \wedge T_a$ so that $S_{bulk} = \frac{1}{2\kappa}\int_{\partial M} e^a \wedge T_a$ up to Nieh–Yan exact terms [8, 18].

Remarks. (1) Propositions 11.2 and 11.3 encode the lay claim that the "cost" lives on the edge: in the self-excited sector, the interior Euler–Lagrange content collapses to identities and the on-shell value reduces to a boundary form. (2) Regularity, asymptotics, and charge quantization require case-by-case analysis (examples in §11.1).

11.1 Examples and calibrations

Classical gravitational instantons provide calibration geometries (e.g. Eguchi–Hanson) [9, 14]. One exports coframes adapted to these metrics, chooses a teleparallel inertial connection with $R = 0$, and checks (6) for an appropriate $H[T]$. For nontrivial S^3 ends, the boundary integral $\int_{\partial M} e^a \wedge T_a$ can carry a nonzero topological charge; explicit families are discussed in [5].

12 Sphalerons and merons: unstable bridges and rates

Let $\mathcal{E}[X]$ denote the (Euclidean) action density functional on configuration space. A *sphaleron* is a static saddle $X_{\rm sp}$ with exactly one negative mode in the Hessian of $\mathcal{E}$. A *meron* is a (typically singular or regulated) half-instanton carrying fractional topological charge. In Yang–Mills, BPST instantons and merons control tunnelling and selection rules [11, 12]; gravitational analogues inherit similar roles [17, 14]. In a self-excited sector, sphalerons/merons act as *bridges* that transmit boundary selection into the bulk. Semiclassical rates scale as $\Gamma \sim A\,\mathrm{e}^{-\mathcal{E}[X_{\rm sp}]}$ with prefactor A set by fluctuation determinants; the single negative mode yields the characteristic "mountain-pass" instability.

13 Pregeometry to metric to injective hull

We model metric emergence over a finite set X via a uniformity base of entourages $\{U_i \subset X \times X\}$ closed under inversion, finite intersections, and entourage multiplication. A nested sequence $U_0 \supset U_1 \supset \cdots$ induces a pseudo-metric $d(x, y) := \inf\{\epsilon > 0 : \ (x, y) \in U_{\lfloor 1/\epsilon \rfloor}\}$ [28]. Given any finite metric (X, d), its *tight span* $T(X)$ is the injective hull: the smallest hyperconvex space containing an isometric copy of X, characterized equivalently by the Isbell completion [25, 26, 20]. Complexity is controlled by Develin's bounds on $\dim T(X)$ and by model shapes (e.g. spheres) whose tight spans are Hilbert cubes [19, 21]. These properties make $T(X)$ an ideal "canonical room" for comparing cosmology summaries or parameter-distance tables without adding spurious holes.

14 Outlook to data: what the fixed-point sector predicts

Early-time, brief self-excited episodes can shift the sound horizon r_s while preserving BBN and $N_{\rm eff}$, producing correlated acoustic-peak phase shifts; late-time, mild effective responses suppress growth and shift S_8 without spoiling distance ladders. Global observables include parity-odd CMB polarization, narrow-band stochastic gravitational-wave backgrounds, and lensing-reconstruction residuals. We implement these as add-on modules in standard Boltzmann codes with priors from BBN/ $N_{\rm eff}$ and BAO/SN, following the testing philosophy of [15]. Each mechanism carries explicit falsifiers; null results at forecast sensitivities would rule out the corresponding parameter regions.

15 Pipelines and reproducibility

Our empirical strategy is conservative: add the minimal fixed-point sector required by section 11 to standard cosmological pipelines, expose its parameters to data, and publish all artifacts for replication.

15.1 Inference setup

We implement background and perturbation-level modifications as effective sources compatible with (3)–(6). Early-time episodes alter the pre-recombination sound speed history so that the inferred sound horizon r_s can shift within BBN/$N_{\rm eff}$ bounds; late-time responses modify growth while preserving distance indicators. We provide patches for Boltzmann codes (CLASS/CAMB), with switchable modules:

(a) **Early-time module** (ETM): short-duration, boundary-driven episode parameterized by $(z_\star, \Delta z, \varepsilon_{r_s})$; internal priors enforce compatibility with BBN/$N_{\rm eff}$.

(b) **Late-time module** (LTM): gentle torsion-flavored effective stress with parameters $(\beta_{\rm grow}, z_{\rm trans})$; internal priors tied to BAO/SN distances.

(c) **Polarization/GW module** (PGM): parity-odd birefringence angle $\alpha(\nu)$ and a narrow-band SGWB template characterized by (ν_0, $\Delta\nu$, $\Omega_{\rm GW}$).

All new parameters have bounded, physically motivated priors; nested sampling or HMC explores the posterior in each data combination.

15.2 Data sets and hard priors

Our baseline follows the testing philosophy of [15], with "red-line" constraints that automatically veto unphysical regions.

Table 1: Baseline data and hard priors (illustrative).

Channel	**Data / Prior**
BBN / $N_{\rm eff}$	Light-element yields; conservative $N_{\rm eff}$ prior (hard cut).
Distances	BAO + SNe (Pantheon-like); SH0ES or TRGB optional as external.
CMB (T/E/B)	Planck/ACT baseline; CMB-S4/LiteBIRD forecasts for birefringence.
Lensing	CMB lensing recon.; galaxy weak lensing (DES/Rubin/Euclid/Roman).
Growth	RSD $f\sigma_8(z)$ from DESI/BOSS-like compilations.
SGWB	PTAs (nHz), ground-based (Hz–kHz), LISA (mHz) for narrow-band templates.

15.3 Public artifacts

We will release: (i) code patches (ETM/LTM/PGM); (ii) configuration files spelling out priors, sampler settings, and data vectors; (iii) mock-data generators for end-to-end checks; (iv) a small "`reproduce.sh`" script that runs the main tables and plots on commodity hardware. All figures will carry machine-readable overlays (CSV of binned spectra, posteriors).

16 No-go theorems, obstructions, and a decision tree

The fixed-point program fails cleanly under any of the following:

(A) Boundary ⇒ bulk mismatch. If (6) cannot be satisfied with the chosen constitutive map while keeping $R^a{}_b = 0$ and the required asymptotics, the sector is empty. This manifests as the absence of stationary-by-identity solutions (Proposition 11.2) for admissible boundary data.

(B) On-shell action not boundary-dominated. If $S_{\rm bulk}$ fails to reduce (up to exact forms) to a boundary integral consistent with Nieh–Yan structure [8, 18], the mechanism does not realize "boundary-only cost."

(C) Topology/regularity obstructions. Singularities or non-quantized charges at S^3 infinity veto the configuration; classical calibrations (e.g. Eguchi–Hanson) guide allowed ends [9, 14].

(D) Logical consistency. If the assumptions required for fixed-point selection contradict the expressive resources of the theory (Lawvere/Roberts/Yanofsky [36, 39, 40]), the closure claim must be weakened (e.g. from global to sectorial).

(E) Cosmology red lines. Any improvement in H_0 or S_8 that violates BBN/$N_{\rm eff}$, BAO/SN distance consistency, CMB polarization nulls, or SGWB bounds is rejected [15].

Decision tree. Start with boundary data $(\partial M, \mathsf{NY})$; construct candidate (e, ω) with $R = 0$; test (6); if stationarity holds, compute $\int_{\partial M} e^a \wedge T_a$ and regularity; only then expose parameters to cosmology modules. If any red line in section 15.2 is tripped, prune that branch.

17 Claims–evidence–methods matrix (core items)

Table 2: Matrix for three core claims (abbrev. CE = confidence estimate).

Claim	Evidence (target)	Method / falsifier	CE
Self-excited sector exists with boundary-only cost	Explicit teleparallel constructions with $T^a = \pm H_a$; on-shell $S_{\rm bulk}$ is boundary Nieh–Yan	Construct (e, ω) with $R = 0$; verify (6), Propositions 11.2 and 11.3; falsify if reduction fails	Med: depends on constitutive choice [8, 18, 5].
Early-time episode shifts r_s within BBN/$N_{\rm eff}$	Correlated acoustic-peak phase shifts in TT/TE/EE with unchanged light elements	ETM module in Boltzmann codes; falsify if BBN/$N_{\rm eff}$ or phases disagree [15]	Low–Med: data-driven.
Tight spans as canonical hosts reduce modeling bias	Stable embeddings; Develin bounds; spheres $\to$ Hilbert-cube behavior	Compute $T(X)$; check dimension bounds; falsify if embeddings distort distances [19, 20, 21]	High (math theorem-level).

18 Conclusions and next steps

We have translated "self-excited universe" from metaphor to mechanism: a fixed-point sector in teleparallel gravity where interior dynamics collapse to identities and the on-shell action reduces to a boundary charge. Unstable connectors (sphalerons/merons) provide controlled bridges between sectors; pregeometry and injective hulls supply canonical scaffolding for comparing models and data. The upshot is empirical: look for global watermarks (CMB phase shifts and parity, narrow-band SGWB, lensing residuals) under hard priors. Either the signatures appear where they must, or the loop fails to close in nature's world. In both outcomes, our understanding advances.

Data and code availability

Reproducible artifacts (source code patches for CLASS/CAMB, configuration files, mock generators, and CSV overlays for figures) will be posted in a public repository upon submission. A minimal `reproduce.sh` script will regenerate the main tables and plots on commodity hardware.

Appendices

A Notation and conventions

We work on an oriented four-manifold M with boundary ∂M. Greek indices $(\mu, \nu, \ldots)$ label spacetime components; Latin indices $(a, b, \ldots)$ label orthonormal coframe components. Differential forms are denoted by boldface wedge $\wedge$; inner product $\langle\cdot,\cdot\rangle$ is metric-induced; Hodge dual $\star$ is defined by g but is not used centrally. Signature is $(-,+,+,+)$ in Lorentzian sections; we use $c = 1$ and $\kappa = 8\pi G$. A teleparallel (flat, metric-compatible) connection $\omega^a{}_b$ satisfies $R^a{}_b(\omega) = 0$; torsion is $T^a = \mathrm{D}e^a$. The excitation H_a is an algebraic 2-form built from T^a and g (constitutive law). The Nieh–Yan 4-form is

$$\mathsf{NY} = \mathrm{d}\big(e^a \wedge T_a\big) - T^a \wedge T_a + e^a \wedge e^b \wedge R_{ab}. \tag{7}$$

On teleparallel backgrounds ($R_{ab} = 0$), $\mathsf{NY} = \mathrm{d}(e^a \wedge T_a) - T^a \wedge T_a$.

B Worked template: a self-excited teleparallel configuration

This template lists checks needed to verify a self-excited solution on a given M with specified asymptotics.

(1) **Coframe and inertial connection.** Choose e^a adapted to the desired metric (e.g. an instanton seed) and select an inertial Lorentz connection $\omega^a{}_b$ with $R(\omega) = 0$ and the required falloffs at ∂M.

(2) **Compute torsion.** Evaluate $T^a = \mathrm{D}e^a$ and its irreducible pieces (vector, axial, purely tensorial) if needed.

(3) **Constitutive map.** Pick $H_a = \chi_{ab}[g]\lrcorner T^b$ (TEGR-like choice suffices). Verify locality, linearity, and metric-compatibility (Assumption 11.1).

(4) **Self-excitation.** Solve $T^a = \pm H_a$ for free parameters/functions. This may fix moduli and impose boundary relations.

(5) **Stationarity by identity.** Check the variation of $S = \frac{1}{2\kappa}\int T^a \wedge H_a + S_{\mathrm{bdy}}$ under compactly supported $(\delta e, \delta\omega)$; confirm $\delta S = 0$ using $\mathrm{D}T^a \equiv 0$ and the constitutive symmetry (Proposition 11.2).

(6) **Boundary reduction.** Evaluate S_{bulk} and show $S_{\mathrm{bulk}} = \frac{1}{2\kappa}\int_{\partial M}\Theta$ up to exact torsion terms; for TEGR-like laws one may take $\Theta = e^a \wedge T_a$ (Proposition 11.3).

(7) **Charge and regularity.** Compute $\int_{\partial M} e^a \wedge T_a$ on S^3 ends; check quantization/regularity constraints appropriate to the topology.

C Pregeometry toy model and induced metric

Let $X = \{A, B, C, D\}$ with entourages $U_0 = X \times X \supset U_1 \supset U_2 \supset \cdots$ defined by

$$U_1 = \{(x,x)\} \cup \{(A,B),(B,A),(C,D),(D,C)\}, \qquad U_2 = U_1 \cup \{(A,C),(C,A),(B,D),(D,B)\}.$$

Define a pseudo-metric by $d(x,y) := \inf\{2^{-n} : (x,y) \in U_n\}$. Then $d(A,B) = d(C,D) = 2^{-1}$, $d(A,C) = d(B,D) = 2^{-2}$, and $d(A,D) = d(B,C) = 2^{-2}$ by entourage composition. This constructs distances from minimal "closeness" data; refining entourages refines the metric.

D Tight span cheat sheet and a 4-point example

For a finite metric (X, d), the tight span $T(X)$ can be realized as the set of functions $f : X \to \mathbb{R}_{\geq 0}$ such that

$$f(x) + f(y) \geq d(x, y) \quad \forall x, y \in X, \qquad \text{and} \qquad \min_{x \in X} f(x) = 0,$$

with metric $d_T(f, g) = \sup_{x \in X} |f(x) - g(x)|$. Embed X via $x \mapsto d(x, \cdot)$. For the toy metric in appendix C with $d(A, B) = \frac{1}{2}$, $d(A, C) = \frac{1}{4}$, etc., $T(X)$ is a 2-dimensional rectilinear cell complex (tree-like if triangle equalities are tight, thicker otherwise). Develin's bounds give $\lfloor n/3 \rfloor \leq \dim T(X) \leq \lfloor n/2 \rfloor$ for generic n-point metrics; for spheres, $T(S^n)$ is homeomorphic to a Hilbert cube.

E Fixed-point theorems in one page

Principle E.1 (Lawvere-style fixed point, schematic)**.** Let $\mathcal{C}$ be a cartesian closed category. If there exists a "weakly point-surjective" arrow $E \to Y^E$, then every endomorphism $t : Y \to Y$ has a fixed point. Diagonalization (self-application) supplies the construction.

Variants: substructural settings weaken the hypotheses (Roberts); universal diagonal templates capture self-reference and incompleteness schemes (Yanofsky). Physical reading: a sufficiently expressive, reflexive description of a universe tends to admit fixed solutions or else formal obstructions, guiding what a "law without initial conditions" can coherently mean.

F Cosmology module parameters and priors (compact)

Early-time module (ETM): $(z_\star, \Delta z, \varepsilon_{r_s})$ with priors $z_\star \in [10^3, 10^5]$, $\Delta z / z_\star \in [10^{-2}, 10^{-1}]$, $\varepsilon_{r_s} \in [-0.05, 0.05]$; enforce BBN/$N_{\text{eff}}$ compatibility. Late-time module (LTM): $(\beta_{\text{grow}}, z_{\text{trans}})$ with $\beta_{\text{grow}} \in [-0.2, 0.2]$, $z_{\text{trans}} \in [0, 2]$; distances fixed to BAO/SN. Polarization/GW (PGM): birefringence angle $\alpha(\nu)$ as a low-order expansion about ν_0; narrow-band SGWB at $(\nu_0, \Delta\nu)$ with amplitude prior set by PTA/LIGO/LISA bounds. All runs publish posterior samples and prior files.

G Myths & misreadings: a short FAQ

Is this metaphysics? No. The claim is operational: certain sectors of gravity appear as fixed points of a boundary-to-bulk law; they must leave testable, global signatures or be rejected.

Does "self-excited" mean observers create reality? No. "Participatory" here means descriptions are reflexive enough to admit fixed points or reveal inconsistencies [3, 55]. Our tests are observer-independent.

Is this a replacement for GR or ΛCDM? No. It is a minimal *sector* compatible with GR's observations, added to standard pipelines and constrained by hard priors [15].

Is boundary-only action a hand-wave? No. In teleparallel form with $R = 0$ and suitable constitutive laws, the on-shell bulk term reduces to a boundary form related to Nieh–Yan [8, 18].

H Complexity and invariants: matroid fans and Hodge-type checks

We summarize invariants used to track discrete-to-continuum transitions. A (finite) matroid M captures independence; its *Bergman fan* Σ_M is the cone complex built from flags of flats and encodes tropicalized combinatorics, while the *conormal fan* $\Sigma_{M,M^\perp}$ ties M to its dual and supports a Chow ring with Hodge-type inequalities (log-concavity) [29]. These fans serve as deformation-stable fingerprints: if a pregeometry flows to a metric model, the associated $(\Sigma_M, \Sigma_{M,M^\perp})$ should satisfy the same convexity inequalities before and after embedding into the tight span/Isbell completion [20, 19, 21]. Failure is a red flag for overfitting or inconsistent emergence.

I Cosmological mechanisms and tests: compact equations

We write a flat FRW background with scale factor a and Hubble rate $H(a)$. The comoving sound horizon is

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}\,\mathrm{d}z, \qquad c_s(z) = \frac{1}{\sqrt{3\,(1+R_b(z))}}, \quad R_b(z) \equiv \frac{3\rho_b}{4\rho_\gamma}. \tag{8}$$

An early, brief boundary-driven episode modifies $H \to H(1+\varepsilon_H)$ and/or $c_s \to c_s(1+\varepsilon_c)$ over $z \in [z_* - \Delta z,\; z_* + \Delta z]$, giving a controlled δr_s subject to BBN/N_{eff} priors [15]. Late-time effective response appears as a growth modifier $\mu(a)$ in

$$\delta'' + \left(2 + \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a}\right)\delta' - \frac{3}{2}\,\Omega_m(a)\,\mu(a)\,\delta = 0, \tag{9}$$

with primes $\mathrm{d}/\mathrm{d}\ln a$. We model $\mu(a) = 1 + \beta_{\mathrm{grow}}\,\Theta(a - a_{\mathrm{trans}})$ (Heaviside). Polarization rotation is $\alpha(\nu) = \alpha_0 + \alpha_1 \ln(\nu/\nu_0)$, constrained by TB/EB null tests; a narrow-band SGWB is

$$\Omega_{\mathrm{GW}}(f) = \Omega_0\,\exp\left[-\frac{(f-f_0)^2}{2\,\sigma_f^2}\right], \tag{10}$$

compared against PTA/LIGO/LISA bounds. All parameters are sampled in CLASS/CAMB patches with the priors of table 1.

J Superspace and participatory context

Wheeler's superspace is the space $\mathcal{S}$ of 3-geometry equivalence classes; canonical quantum gravity seeks a wavefunctional Ψ[geometry] constrained by a Wheeler–DeWitt-type equation [50, 56]. Our fixed-point sector is compatible with this picture: boundary rules define admissible classes in $\mathcal{S}$, and the constitutive map selects bulk fields whose on-shell action is boundary-dominated. Philosophically, "participatory" is read as reflexive model selection subject to empirical falsification, aligning with Wheeler's intent and later expositions [51, 52, 55].

K Proof sketches and constitutive examples

Stationarity-by-identity (sketch). With $S = \frac{1}{2\kappa}\int T^a \wedge H_a$ and linear, local $H[T]$, vary S; use $\mathrm{D}T^a \equiv 0$ (teleparallel Bianchi) and $T^a = \pm H_a$ to collapse bulk terms to exact forms (killed by compact support) and cancel residual boundary pieces by S_{bdy}; cf. Proposition 11.2.

Boundary reduction (sketch). For TEGR-like laws, $T^a \wedge H_a$ differs from $\mathrm{d}(e^a \wedge T_a)$ by torsion-quadratic exact terms; with $R = 0$, Nieh–Yan reduces the on-shell bulk action to $\int_{\partial M} e^a \wedge T_a$; cf. Proposition 11.3 and [8, 18].

Constitutive catalog (examples). TEGR-like: $H_a = \sum_I c_I$ (irreducible part $T^{(I)}$) with constants c_I chosen so that GR phenomenology is recovered [18]. In many such choices, $T^a = \pm H_a$ can be satisfied on instanton-seeded coframes with inertial connections; details belong in appendix B.

Glossary (lay readers)

Teleparallel gravity An equivalent formulation of gravity where curvature vanishes and gravity is encoded in torsion carried by a coframe.

Instanton An all-at-once field configuration that connects sectors in the quantum path integral; often minimizes an action in Euclidean time.

Sphaleron An unstable saddle-point configuration (a "mountain pass") controlling transition rates between sectors.

Meron A "half-instanton" carrying fractional topological charge; typically unstable or needing regulation.

Nieh–Yan invariant A boundary-related 4-form built from coframe and torsion; on teleparallel backgrounds it becomes an exact form.

Self-excited configuration A solution where torsion equals its own excitation ($T^a = \pm H_a$), making bulk equations hold by identity and pushing cost to the boundary.

Tight span / injective hull The smallest, contractible metric space that houses a given distance table without distortion; equals the Isbell completion.

Bergman / conormal fan Combinatorial–tropical blueprints (from matroid theory) used as invariants through deformations.

Superspace The space of all 3-geometries up to diffeomorphisms; a stage for canonical quantum gravity.

Acronym guide

BBN Big Bang Nucleosynthesis.

BAO Baryon Acoustic Oscillations.

CMB Cosmic Microwave Background.

ETM Early-Time Module (our pre-recombination episode parameterization).

LTM Late-Time Module (our growth-response parameterization).

PGM Polarization/Gravitational-Wave Module.

PTA Pulsar Timing Array.

SGWB Stochastic Gravitational-Wave Background.

TEGR Teleparallel Equivalent of General Relativity.

WDW Wheeler–DeWitt (constraint equation/context).

Symbol index

Symbol	Meaning
M, ∂M	Spacetime manifold and its boundary.
e^a	Orthonormal coframe (tetrad one-forms).
$\omega^a{}_b$	Flat, metric-compatible Lorentz connection ($R^a{}_b = 0$).
$T^a = \mathrm{D}e^a$	Torsion two-form.
H_a	Excitation two-form defined by the constitutive map $H[T]$.
D	Exterior covariant derivative with respect to $\omega^a{}_b$.
NY	Nieh–Yan 4-form $\mathrm{d}(e^a \wedge T_a) - T^a \wedge T_a$.
S	Gravitational action $\frac{1}{2\kappa}\int T^a \wedge H_a + S_{\mathrm{bdy}}$.
κ	$8\pi G$ (units $c = 1$).
$\mathcal{X}, \mathcal{B}$	Spaces of bulk fields and boundary data.
∂	Boundary operator $\mathcal{X} \to \mathcal{B}$.
$\mathcal{C}$	Constitutive map $\mathcal{B} \to \mathcal{X}$.
r_s	Sound horizon at recombination, Eq. (8).
$\mu(a)$	Growth-modifier in Eq. (9).
$\Omega_{\mathrm{GW}}(f)$	SGWB energy density spectrum, Eq. (10).
H_0, S_8	Hubble constant today; clustering amplitude summary.
$T(X)$	Tight span (injective hull) of a finite metric space (X, d).
Σ_M	Bergman fan of matroid M; $\Sigma_{M,M^\perp}$ conormal fan.

References

[1] J. A. Wheeler (1989), *Information, Physics, Quantum: The Search for Links*, in W. H. Zurek (ed.), *Complexity, Entropy, and the Physics of Information*, Addison–Wesley.

[2] J. A. Wheeler (1994), *At Home in the Universe*, American Institute of Physics.

[3] J. A. Wheeler and W. H. Zurek (eds.) (1983), *Quantum Theory and Measurement*, Princeton University Press.

[4] P. C. W. Davies (1992), *The Mind of God*, Simon & Schuster.

[5] M. Krssak (2025), Self-Excited Gravitational Instantons, arXiv:2408.01140 [gr-qc].

[6] R. Aldrovandi and J. G. Pereira (2013), *Teleparallel Gravity: An Introduction*, Springer.

[7] F. W. Hehl and Y. N. Obukhov (2007), Elie Cartan's torsion in geometry and in field theory, *Ann. Fond. Louis de Broglie* **32**, 157–194.

[8] H. T. Nieh and M. L. Yan (1982), An identity in Riemann–Cartan geometry, *J. Math. Phys.* **23**, 373–374.

[9] T. Eguchi, P. B. Gilkey, and A. J. Hanson (1980), Gravitation, gauge theories and differential geometry, *Physics Reports* **66**, 213–393.

[10] G. W. Gibbons and S. W. Hawking (1977), Action integrals and partition functions in quantum gravity, *Phys. Rev. D* **15**, 2752–2756.

[11] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Y. S. Tyupkin (1975), Pseudoparticle solutions of the Yang–Mills equations, *Phys. Lett. B* **59**, 85–87.

[12] S. Coleman (1977), The Uses of Instantons, in A. Zichichi (ed.), *The Whys of Subnuclear Physics*, Plenum.

[13] J. B. Hartle and S. W. Hawking (1983), Wave function of the Universe, *Phys. Rev. D* **28**, 2960–2975.

[14] M. Dunajski (2025), Gravitational Instantons, old and new, arXiv:2501.00688 [hep-th].

[15] CosmoVerse Collaboration (2025), The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics, arXiv:2504.01669 [astro-ph].

[16] M. Blagojevic and F. W. Hehl (eds.) (2013), *Gauge Theories of Gravitation: A Reader with Commentaries*, Imperial College Press.

[17] G. W. Gibbons and S. W. Hawking (1979), Classification of gravitational instanton symmetries, *Commun. Math. Phys.* **66**, 291–310.

[18] M. Krssak, R. J. Van den Hoogen, J. G. Pereira, C. G. Boehmer, and A. A. Coley (2019), Teleparallel theories of gravity: Illuminating a fully invariant approach, *Class. Quantum Grav.* **36**, 183001.

[19] M. Develin (2004), Dimensions of tight spans, arXiv:math/0407317.

[20] S. Willerton (2013), Tight spans, Isbell completions and semi-tropical modules, arXiv:1302.4370.

[21] S. Lim, F. Memoli, Z. Wan, Q. Wang, and L. Zhou (2021), Some results about the Tight Span of spheres, arXiv:2112.12646.

[22] M. Joswig et al., Tight Spans (resource page). Available at: https://page.math.tu-berlin.de/˜joswig/tightspans/index.html

[23] nLab entry: span. Available at: https://ncatlab.org/nlab/show/span

[24] nLab entry: injective envelope. Available at: https://ncatlab.org/nlab/show/injective+envelope

[25] N. Aronszajn and P. Panitchpakdi (1956), Extension of uniformly continuous transformations and hyperconvex metric spaces, *Pacific J. Math.* **6**, 405–439.

[26] J. R. Isbell (1964), Six theorems about injective metric spaces, *Proc. Amer. Math. Soc.* (details by edition).

[27] A. Gefter (2024), John Wheeler Saw the Tear in Reality, *Quanta Magazine*, 25 Sept 2024. Available at: https://www.quantamagazine.org/john-wheeler-saw-the-tear-in-reality-20240925/

[28] W. M. Stuckey (2002), On a Pregeometric Origin for Spacetime Dimensionality and Metric Structure, arXiv:gr-qc/0208057.

[29] F. Ardila (2021), The geometry of geometries: matroid theory, old and new, arXiv:2111.08726.

[30] Wikipedia: Pregeometry (physics). Available at: https://en.wikipedia.org/wiki/Pregeometry_(physics)

[31] nLab entry: matroid. Available at: https://ncatlab.org/nlab/show/matroid

[32] nLab entry: geometry (for structured (infinity,1)-toposes). Available at: https://ncatlab.org/nlab/show/geometry+(for+structured+(infinity,1)-toposes)

[33] nLab entry: geometry of physics – prequantum geometry. Available at: https://ncatlab.org/nlab/show/geometry+of+physics+–+prequantum+geometry

[34] nLab entry: geometric stability theory. Available at: https://ncatlab.org/nlab/show/geometric+stability+theory

[35] nLab entry: supergeometry. Available at: https://ncatlab.org/nlab/show/supergeometry

[36] F. W. Lawvere (1969; reprinted 2006), Diagonal arguments and cartesian closed categories, *Reprints in Theory and Applications of Categories* **15**, 1–13. PDF: http://tac.mta.ca/tac/reprints/articles/15/tr15.pdf

[37] nLab entry: Lawvere's fixed point theorem. Available at: https://ncatlab.org/nlab/show/Lawvere%27s+fixed+point+theorem

[38] A. Bauer (2007), On a proof of Cantor's theorem. Available at: https://math.andrej.com/2007/04/08/on-a-proof-of-cantors-theorem/

[39] D. M. Roberts (2023), Substructural fixed-point theorems and the diagonal argument: theme and variations, arXiv:2110.00239 (v3).

[40] N. S. Yanofsky (2003), A Universal Approach to Self-Referential Paradoxes, Incompleteness and Fixed Points, arXiv:math/0305282.

[41] X. D. Arsiwalla, H. Elshatlawy, and D. Rickles (2023), Pregeometry, Formal Language and Constructivist Foundations of Physics, arXiv:2311.03973.

[42] D. Chester, X. D. Arsiwalla, and L. H. Kauffman (2025), Preons, Braid Topology, and Representations of Fundamental Particles, arXiv:2501.03260.

[43] H. Elshatlawy, D. Rickles, X. D. Arsiwalla, and A. Blum (2025), Towards a Generalized Theory of Observers, arXiv:2504.16225 (v2).

[44] X. D. Arsiwalla (2024), Qualia and the Formal Structure of Meaning, arXiv:2405.01148.

[45] D. Rickles, H. Elshatlawy, and X. D. Arsiwalla (2023), Ruliology: Linking Computation, Observers and Physical Law, arXiv:2308.16068 (v2).

[46] A. L. Furtado Neto (2025), It from Bit: A Concrete Attempt, Preprints.org, doi:10.20944/preprints202502.0830.v1.

[47] B. d'Espagnat (2011), Quantum Physics and Reality, arXiv:1101.4545.

[48] G. Jaroszkiewicz (2004), Selected notes on quantum information and causality, arXiv:0408037.

[49] A. Colling and M. Dunajski (2025), Quasi-Einstein structures and Hitchin's equations, arXiv:2504.18475 [math].

[50] nLab (accessed Aug. 2025), Wheeler superspace. Available at: https://ncatlab.org/nlab/show/Wheeler+superspace

[51] W. H. Zurek, A. van der Merwe, and W. A. Miller (eds.) (1988), *Between Quantum and Cosmos: Studies and Essays in Honor of John Archibald Wheeler*, Princeton University Press.

[52] J. D. Barrow, P. C. W. Davies, and C. L. Harper, Jr. (eds.) (2004), *Science and Ultimate Reality: Quantum Theory, Cosmology, and Complexity*, Cambridge University Press.

[53] H. C. Ransford (2019), *In Search of Ultimate Reality: Inside the Cosmologist's Abyss*, ibidem Press.

[54] H. Weyl (2009), *Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics* (P. Pesic, ed.), Princeton University Press.

[55] A. V. Nesteruk (2013), A Participatory Universe of J. A. Wheeler as an Intentional Correlate of Embodied Subjects and an Example of Purposiveness in Physics, arXiv:1304.2277 [physics]. Available at: https://arxiv.org/abs/1304.2277

[56] D. Giulini (2009), The Superspace of Geometrodynamics, arXiv:0902.3923 [gr-qc]. Available at: https://arxiv.org/abs/0902.3923

[57] C. M. Langan (1989), The Resolution of Newcomb's Paradox, *Noesis* **44**. Available at: https://megasociety.org/noesis/44/newcomb

[58] C. M. Langan (n.d.), *The Theory of Theories* (CTMU). Archived Jan 19, 2018. Available at: https://web.archive.org/web/20180119060743/http://www.megafoundation.org/CTMU/Articles/Theory.htm

[59] C. M. Langan (2002), The Cognitive-Theoretic Model of the Universe: A New Kind of Reality Theory, *Progress in Complexity, Information, and Design* **1**(2–1.3), 1–56. Available at: https://www.infolab.ho.ua/Langan_CTMU_092902(1).pdf

[60] C. M. Langan (2004), Cheating the Millennium: The Mounting Explanatory Debts of Scientific Naturalism, in W. A. Dembski (ed.), *Uncommon Dissent: Intellectuals Who Find Darwinism Unconvincing*, ISI Books.

[61] C. M. Langan (2017), An Introduction to Mathematical Metaphysics, *Cosmos and History: The Journal of Natural and Social Philosophy* **13**(2), 313–330. Available at: https://cosmosandhistory.org/index.php/journal/article/view/618/1040

[62] C. M. Langan (2018), Metareligion as the Human Singularity, *Cosmos and History: The Journal of Natural and Social Philosophy* **14**(1), 321–332. Available at: https://cosmosandhistory.org/index.php/journal/article/view/694

[63] C. M. Langan (2018), The Metaformal System: Completing the Theory of Language, *Cosmos and History: The Journal of Natural and Social Philosophy* **14**(2), 207–227. Available at: https://cosmosandhistory.org/index.php/journal/article/view/740/1214

[64] C. M. Langan (2019), Introduction to Quantum Metamechanics (QMM), *Cosmos and History: The Journal of Natural and Social Philosophy* **15**(1), 265–300. Available at: https://cosmosandhistory.org/index.php/journal/article/view/788

[65] C. M. Langan (2020), The Reality Self-Simulation Principle: Reality is a Self-Simulation, *Cosmos and History: The Journal of Natural and Social Philosophy* **16**(1), 466–486. Available at: https://cosmosandhistory.org/index.php/journal/article/view/867