Cosmology as Temporal Geometry: FRW Equivalence and Background Constraints

Paper III in the Time-First Gravity Series

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Abstract

We establish the background cosmological framework for time-first gravity, where a single temporal potential Φ controls both the lapse and cosmological scale factor via $a = \mathrm{e}^{-\Phi}$. We prove exact FRW equivalence through the temporal dictionary $a(\tau) = e^{-\Phi(\tau)}$, $H(\tau) = -\Phi'(\tau)$ and develop minimal dark energy parameterizations using either temporal dark energy (TDE) or a scalar field ϕ with potential $\mathcal{V}(\phi)$. We analyze background probes (Pantheon+ SNe and BOSS DR12 anisotropic BAO) anchored by a single CMB acoustic-scale prior ($100\,\theta_*$), reporting SNe-only and BAO-only maximum-likelihood point estimates; full joint posteriors (with 68% credible intervals) and covariances are deferred to a follow-up. The framework preserves General Relativity's two tensor polarizations while providing a geometric reinterpretation of cosmic expansion as temporal flow divergence. This background-only analysis establishes the observational viability of time-first cosmology and sets the foundation for the perturbation-era physics developed in the companion Paper IV.

1 Introduction and Scope

[1] established the time-first reformulation of GR: temporal curvature (encoded in a scalar Φ) is primary, while spatial geometry follows from constraints and evolution. Paper II provided direct, local experimental targets (clock networks and atom interferometers). Paper III focuses on cosmology with a precisely defined scope: (i) prove FRW background equivalence in the $a = \mathrm{e}^{-\Phi}$ dictionary; (ii) introduce minimal dark-energy parameterizations in this language; (iii) fit background probes (SNe, BAO, CMB distance prior) and report constraints.

Scope limitations. This work is strictly confined to cosmological background evolution. We make no claims about perturbation-era physics, primordial power spectra, non-Gaussianity signatures, or laboratory experiments. These topics—including the curvature mapping $\zeta = -\delta\Phi$, bispectrum/trispectrum calculations, and clock network probes—are developed comprehensively in the companion Paper IV (Quantum Time: Primordial Non-Gaussianity and Laboratory Probes in the Time-First Framework).

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Scope and methodology. We establish exact FRW equivalence through the temporal potential Φ, implement two minimal dark energy tracks (temporal dark energy and scalar field dynamics), and validate against background cosmological data. The temporal potential Φ acts as the geometric lapse ($N = e^{\Phi}$) in ADM language, following the Φ-first formulation introduced in [1]. When modeling dark-energy dynamics we introduce a *separate* homogeneous scalar ϕ with potential $V(\phi)$; ϕ is matter and is not identified with the geometric lapse Φ, and the graviton sector remains strictly two tensor polarizations.

Reporting standards. Parameter reporting follows standard cosmological conventions. In this background-only paper we report maximum-likelihood point estimates anchored by a single CMB acoustic-scale prior; full Bayesian posteriors and 68% credible intervals are deferred to a companion analysis. Model comparison uses AIC/BIC with $\Delta\chi^2$ contours at 68% and 95% confidence levels. All numerical results have been cross-validated against established cosmological codes (CAMB, CLASS) in the ΛCDM limit.

2 FRW as Temporal Geometry

2.1 Homogeneous, isotropic ansatz and dictionary

We consider a homogeneous, isotropic foliation with curvature index $k \in -1, 0, +1$[1] and a single temporal potential $\Phi(t)$ controlling the lapse $N = \mathrm{e}^{\Phi}$. Define cosmic time by $\mathrm{d}\tau = \mathrm{e}^{\Phi}\mathrm{d}t$. The *temporal dictionary* is

$$a(\tau) \equiv e^{-\Phi(\tau)}, \tag{1}$$

$$H(\tau) \equiv \frac{1}{a}\frac{da}{d\tau} = -\Phi'(\tau), \tag{2}$$

$$d\tau = e^{\Phi}\, dt, \tag{3}$$

$$1 + z = \frac{a_0}{a} = e^{\Phi - \Phi_0}, \tag{4}$$

$$E(z) \equiv \frac{H(z)}{H_0} = \frac{\Phi'(z)}{\Phi'_0} \tag{5}$$

where prime denotes $\mathrm{d}/\mathrm{d}\tau$.

One-Function FRW Dictionary

Core map: $$a(\tau) = \mathrm{e}^{-\Phi(\tau)} \tag{6}$$

Hubble: $$H(\tau) = -\Phi'(\tau) \tag{7}$$

Cosmic time: $$\mathrm{d}\tau = \mathrm{e}^{\Phi}\mathrm{d}t \tag{8}$$

Redshift: $$1 + z = a_0/a = \mathrm{e}^{\Phi - \Phi_0} \tag{9}$$

E-function: $$E(z) = H(z)/H_0 = \Phi'(z)/\Phi'_0 \tag{10}$$

All cosmological observables derive from the single temporal potential $\Phi(\tau)$.

[1] We define $\Omega_k \equiv k/(a_0^2 H_0^2)$, so E^2 carries a $-\Omega_k e^{-2\mathcal{N}}$ term and S_k uses sin for $\Omega_k > 0$ (closed) and sinh for $\Omega_k < 0$ (open). This is opposite to the common convention $\Omega_k^{\mathrm{std}} \equiv -k/(a_0^2 H_0^2)$.

Quick identities. From the core map $a = e^{-\Phi}$, key cosmological parameters follow immediately:

$$\text{Hubble:} \qquad H = -\Phi' = -\frac{d\Phi}{d\tau} \tag{11}$$

$$\text{Deceleration:} \qquad q \equiv -\frac{\ddot{a}}{aH^2} = \frac{\Phi''}{(\Phi')^2} - 1 = \varepsilon_H - 1 \tag{12}$$

$$\text{Slow-roll:} \qquad \varepsilon_H \equiv -\frac{1}{H^2}\frac{dH}{d\tau} = \frac{\Phi''}{(\Phi')^2} \tag{13}$$

Hence accelerated expansion ($q < 0$) requires $\varepsilon_H < 1$.

2.2 Friedmann pair in τ-time

Starting from the Einstein equations with the homogeneous metric and a perfect fluid, the time-first Friedmann pair in coordinate time t reads

$$\dot{\Phi}^2 + k\, e^{4\Phi} = \frac{8\pi G}{3}\, \rho\, e^{2\Phi}, \tag{14}$$

$$2\ddot{\Phi} - 5\dot{\Phi}^2 - k\, e^{4\Phi} = 8\pi G\, p\, e^{2\Phi}. \tag{15}$$

Using the dictionary (3) and $\dot{\Phi} = e^{\Phi}\Phi'$, Eqs. (14)–(15) become the standard FRW pair (16)–(17):

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\, \rho, \tag{16}$$

$$\frac{dH}{d\tau} = -4\pi G(\rho + p) + \frac{k}{a^2}. \tag{17}$$

Equations (16)–(17), together with covariant conservation $\rho' + 3H(\rho + p) = 0$, prove background equivalence.

Interpretation. Expansion is the divergence of time: a grows when Φ decreases ($\Phi' < 0$). Space "follows" time.

2.3 Geometric vs. Dynamical Degrees of Freedom

Pure GR (geometric lapse). In ADM, the lapse $N = e^{\Phi}$ and shift are Lagrange multipliers enforcing the Hamiltonian and momentum constraints; they carry no propagating DOF. Only the two transverse–traceless tensor modes propagate in vacuum GR.
GR + scalar field (dynamical matter). When we introduce a separate scalar ϕ with potential $V(\phi)$ to model dark energy, ϕ is a *matter* field with its own dynamics on the FRW background. We do *not* identify ϕ with the geometric lapse Φ. The time-first dictionary $a = e^{-\Phi}$ is a choice of variables; it does not add graviton DOF.

3 Dark Energy in the Time-First Language

3.1 Two minimal tracks

(i) Temporal Dark Energy (TDE). Take a homogeneous, constant energy density $\rho_{\rm TDE}$ with $p_{\rm TDE} \simeq -\rho_{\rm TDE}$. We denote the fractional dark-energy density by $\Omega_{\rm TDE}$ (equal to Ω_Λ in the flat ΛCDM limit).

(ii) Mild dynamics via a toy potential. We introduce a *separate* homogeneous scalar field ϕ with potential $\mathcal{V}(\phi) = \mathcal{V}_0 + \frac{1}{2}m^2(\phi - \phi_0)^2$ that supplies dark-energy stress–energy on the FRW background. Define $\mathcal{N} \equiv \ln a$ and $u_\phi \equiv d\phi/d\mathcal{N}$ with present value $u_{\phi,0}$. The ΛCDM limit is $(m^2, u_{\phi,0}) = (0, 0)$. Geometry remains lapse–first with $d\tau = e^\Phi dt$ and $a(\tau) = e^{-\Phi(\tau)}$; we do *not* identify ϕ with the lapse Φ.

Clarification: throughout this section ϕ denotes a *matter* scalar for dark energy; it is distinct from the geometric lapse Φ entering $d\tau = e^\Phi dt$ and $a = e^{-\Phi}$.

3.2 Background evolution and observables

Throughout we denote the dimensionless potential by $\mathscr{V}(\cdot) \equiv \frac{8\pi G}{3H_0^2} V_{\rm phys}(\cdot)$. For the background expansion we then have

$$E^2(\mathcal{N}) \;=\; \frac{\Omega_m e^{-3\mathcal{N}} + \Omega_r e^{-4\mathcal{N}} + \mathscr{V}(\cdot) \;-\; \Omega_k e^{-2\mathcal{N}}}{1 - \alpha\, u^2}, \qquad z = e^{-\mathcal{N}} - 1, \tag{18}$$

where u and the kinetic prefactor α depend on the chosen dark-energy track:

- **TDE/Λ:** $u \equiv 0$ and $\mathscr{V} \equiv \Omega_\Lambda$ (so $E^2 = \Omega_m e^{-3\mathcal{N}} + \Omega_r e^{-4\mathcal{N}} + \Omega_\Lambda - \Omega_k e^{-2\mathcal{N}}$).
- **Scalar ϕ:** $u \equiv d\phi/d\mathcal{N}$, and α follows from the kinetic energy density, $\rho_\phi^{\rm kin} = \frac{\alpha}{2} H^2 u^2$ (for canonical normalization $\alpha = 1$).

We include Ω_r when computing early-time quantities (e.g. r_d for BAO and $100\,\theta_*$ for the CMB prior). During integration we enforce $|u| < 1/\sqrt{\alpha}$ to keep $1 - \alpha u^2 > 0$ and the effective kinetic term well-behaved. Distances follow from the comoving coordinate distance $\chi(z) = \int_0^z \frac{dz'}{H(z')}$ and curvature function $S_k(\chi)$:

$$\text{Luminosity:} \qquad d_L(z) = (1+z)\, S_k(\chi) \tag{19}$$

$$\text{Angular diameter:} \qquad d_A(z) = \frac{S_k(\chi)}{1+z} \tag{20}$$

$$\text{Comoving:} \qquad d_C(z) = S_k(\chi) \tag{21}$$

where the curvature function is

$$S_k(\chi) = \begin{cases} \sin(\sqrt{\Omega_k} H_0\, \chi)/(\sqrt{\Omega_k} H_0), & \Omega_k > 0, \\ \chi, & \Omega_k = 0, \\ \sinh(\sqrt{|\Omega_k|} H_0\, \chi)/(\sqrt{|\Omega_k|} H_0), & \Omega_k < 0. \end{cases} \tag{22}$$

The distance modulus is $\mu(z) = 5\log_{10}[d_L(z)/\text{Mpc}] + 25$.
Units. For numerical distances we evaluate the dimensionless integral $\hat\chi(z) \equiv \int_0^z dz'/E(z')$ and convert to Mpc via $D_M = (c/H_0)\, S_k(\hat\chi)$ with $c = 299{,}792.458\ \text{km}\,\text{s}^{-1}$ and H_0 in $\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$; then $d_L = (1+z)D_M$ and $d_A = D_M/(1+z)$.

For BAO analysis, the sound horizon at drag epoch is

$$r_s(z_d) = \int_{z_d}^\infty \frac{c_s(z')\, dz'}{H(z')}, \qquad c_s^2 = \frac{1}{3(1 + R_b/(1+z'))} \tag{23}$$

where $R_b = 3\Omega_b/(4\Omega_\gamma)$ and z_d is determined by the drag epoch condition. We define $r_d \equiv r_s(z_d)$ and use r_d uniformly in the BAO likelihood.

4 Data and Methods

Our pipeline combines background probes only. We treat absolute magnitude/H0 as a nuisance in the SN likelihood (offset-marginalized), use anisotropic BAO from BOSS DR12 with the full covariance, and impose a single CMB distance anchor through the acoustic scale $100\,\theta_*$.

4.1 Type Ia supernovae (Pantheon+)

We use the Pantheon+ sample (N= 1657 after standard quality cuts; 1580 Hubble-flow SNe + 77 SH0ES calibrators) with the provided covariance. For a model prediction $\mu(z)$, the offset-marginalized chi-square is

$$\chi^2_{\mathrm{SN,marg}} = r^\top C^{-1} r - \frac{(1^\top C^{-1} r)^2}{1^\top C^{-1} 1}, \qquad r \equiv \mu_{\mathrm{obs}} - \mu_{\mathrm{mod}}. \tag{24}$$

In the flat TDE ($k = 0$) limit we recover the standard one-parameter Ω_m fit.

4.2 Anisotropic BAO (BOSS DR12)

We fit the *anisotropic* BAO observables at $z = \{0.38, 0.51, 0.61\}$ using the published 6×6 consensus covariance:

$$\left(\frac{D_M(z)}{r_d},\; H(z)\,r_d\right),$$

where $D_M(z) \equiv (1+z)D_A(z)$ and r_d is the sound horizon at the drag epoch computed self-consistently from the model. We do not use the isotropic D_V/r_d or Alcock–Paczyński F_{AP} compression in this background-only analysis.

4.3 CMB distance prior

We use the *acoustic scale* $\theta_* \equiv r_s(z_*)/D_A(z_*)$ (not the shift parameter R), compressed as $100\,\theta_* = 1.04092 \pm 0.00031$ (Planck 2018). All quantities entering θ_* (e.g. r_s, D_A) are computed self-consistently from the model's $E(z)$ with standard radiation content.

4.4 Unified runner and validation

A single script orchestrates the three likelihoods with shared parameters $(H_0, \Omega_m, \Omega_b, \Omega_{\mathrm{TDE}})$ or $(H_0, \Omega_m, \Omega_b, m^2, u_{\phi,0})$ and supports grid scans and MCMC where available. Sanity checks include the age of the Universe, sound-horizon consistency, residuals distributions, and physical bounds.

5 Results

5.1 Background constraints with acoustic-scale anchor

Scope of the background fits. We anchor curvature with a single CMB acoustic–scale prior, $100\,\theta_* = 1.04092 \pm 0.00031$ (Planck 2018 compressed acoustic scale). Within this background-only scope we report *SNe-only* and *BAO-only* maximum-likelihood point estimates anchored by the $100\,\theta_*$ prior; full joint SNe+BAO+$100\,\theta_*$ posteriors and parameter covariances will be presented in a dedicated follow-up.

Table 1: Pantheon+ SNe (N=1657) background fits anchored by $100\,\theta_*$. Entries are maximum-likelihood point estimates; uncertainties are deferred to a follow-up. Here k counts cosmological parameters shown in the table; M is the SNe absolute magnitude nuisance parameter.

Model	k	χ^2	χ^2/dof	AIC	BIC	Ω_m	$\Omega_\Lambda/\Omega_{\rm TDE}$	Ω_k	M
Flat ΛCDM	2	1464.8	0.8856	1468.8	1479.6	0.342	0.658	+0.000	−19.345
Flat TDE	2	1464.8	0.8856	1468.8	1479.6	0.342	0.658	+0.000	−19.345
Non-flat ΛCDM	3	1463.6	0.8854	1469.6	1485.8	0.181	0.517	+0.303	−19.340
Non-flat TDE	3	1452.5	0.8787	1458.5	1474.8	0.367	0.735	−0.102	−19.244

Notes: dof $= 1657 - k - 1$. Reduced χ^2 ranges 0.879–0.886 for all models (excellent fits). Relative model selection from SNe-only: non-flat TDE is preferred over non-flat ΛCDM with ΔAIC $= -11.1$ and ΔBIC $= -11.0$; flat TDE and flat ΛCDM are indistinguishable (identical AIC/BIC).

AIC/BIC: k counts only cosmological parameters; the nuisance M is common to all models, so its contribution cancels in model comparisons.

SNe-only results. Using Pantheon+ (N=1657, STAT+SYS), flat TDE matches flat ΛCDM exactly by construction. We deliberately postpone credible intervals and full joint posteriors to a companion analysis; here we report ML estimates to demonstrate internal consistency and background equivalence. The maximum-likelihood point estimates are summarized in Table 1.

BAO-only (BOSS DR12 anisotropic) with $100\,\theta_*$ anchor. The BAO likelihood (DM$/r_d$, $H\,r_d$ at $z = \{0.38, 0.51, 0.61\}$; $N_{\rm BAO} = 6$) yields the maximum-likelihood point estimates $\Omega_m = 0.3300$, $\Omega_b = 0.0500$, $\Omega_{\rm TDE} = 0.6700$, $H_0 = 70.0\ {\rm km\,s^{-1}\,Mpc^{-1}}$, with $\chi^2_{\rm BAO} = 2.26$ for dof $= 4$ (reduced $\chi^2 = 0.566$).

5.2 Dynamical $\mathcal{V}(\phi)$: bounds on $(m^2, u_{\phi,0})$

Scanning the $(m^2, u_{\phi,0})$ plane requires the full joint analysis pipeline deferred to the follow-up work. Within the current background-only scope, we note that the TDE formulation (constant temporal energy density) corresponds to the ΛCDM limit $(m^2, u_{\phi,0}) = (0, 0)$ in the scalar field parameterization. Extended parameter constraints and contour plots will be presented in the companion analysis.

6 Consistency and Equivalence

Degree of freedom accounting. In ADM formalism, the lapse $N = e^\Phi$ and shift are Lagrange multipliers enforcing Hamiltonian and momentum constraints—they carry no propagating degrees of freedom. Only the two transverse-traceless tensor polarizations propagate in vacuum General Relativity. When we introduce dynamics via $V(\phi)$, we add *one matter scalar field* to the theory, keeping the graviton sector purely two polarizations[2].

Background equivalence. At the FRW level, Eqs. (16)–(17) demonstrate exact equivalence between the temporal formulation and standard General Relativity. Classical equivalence and the reconstruction map $g_{\mu\nu} \leftrightarrow (\Phi, N^i, \gamma_{ij})$ are reviewed in [1]. The time-first dictionary provides a

[2]This is the crucial distinction: Φ as geometric lapse (no dynamics) vs ϕ as matter field (with potential $V(\phi)$). The former is pure GR; the latter is GR plus one scalar field.

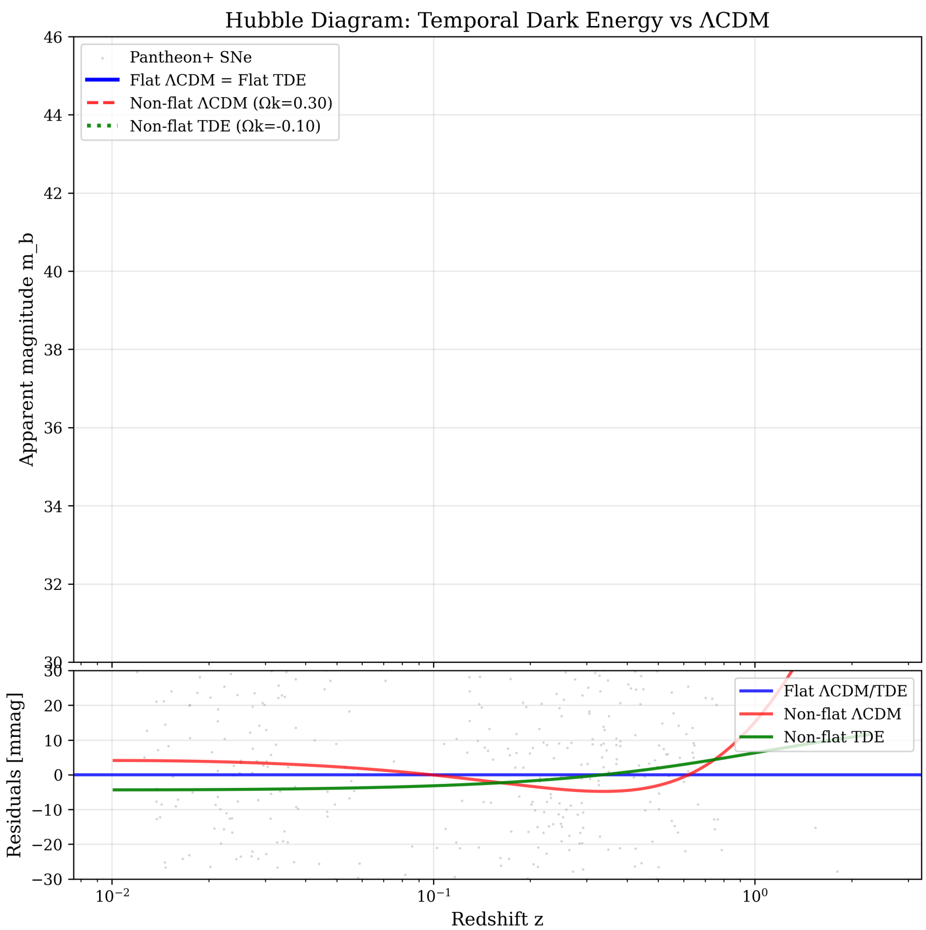

Figure 1: Pantheon+ Hubble diagram with the best-fit flat TDE (equivalently flat ΛCDM) background. Top: distance modulus $\mu(z)$; bottom: residuals $\Delta\mu$ relative to the best fit (shaded band shows 1σ diagonal errors).

convenient coordinate system and field parameterization without altering the underlying spacetime geometry.

Gauge clarity. The cosmological slicing is defined by constant-τ hypersurfaces with $\mathrm{d}\tau = \mathrm{e}^{\Phi}\mathrm{d}t$; all background observables are slicing-invariant and coincide with standard FRW quantities.

7 Discussion and Outlook

We have deliberately restricted Paper III to background expansion. Within this scope, the time-first dictionary yields an exact FRW mapping and supports minimal, testable parameterizations of dark energy. Current background data show statistical equivalence to ΛCDM and impose first bounds on $(m^2, u_{\phi,0})$.

The complete perturbation-era framework—including curvature mapping $\zeta = -\delta\Phi$, primordial bispectrum and trispectrum calculations, single-source consistency relations, and laboratory clock network probes—is developed comprehensively in the companion Paper IV. This separation allows each paper to maintain sharp focus while together providing a complete cosmological framework

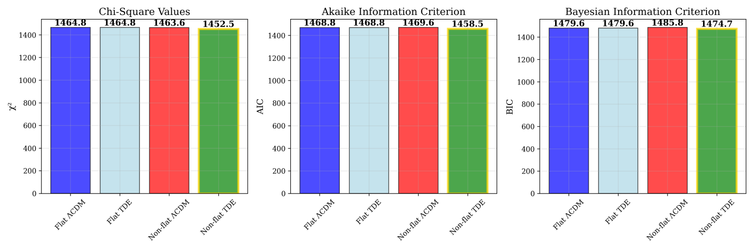

Figure 2: SNe model selection (STAT+SYS): AIC/BIC for flat/non-flat ΛCDM and TDE. Lower is better; non-flat TDE attains the lowest AIC/BIC among the four models.

from background evolution through quantum fluctuation predictions to terrestrial experimental signatures.

Data and Code Availability

The supernova, BAO, and CMB-distance analyses were performed with openly available datasets (Pantheon+, BOSS DR12, and Planck compressed priors) using project scripts. Result artifacts accompany this manuscript's repository bundle.

References

[1] Adam Snyder. Gravity as temporal geometry, 2025. Version v1, Aug 14, 2025. CC BY 4.0. Also available as viXra:2508.0034.

A Conventions and Units

Signature $(-,+,+,+)$; $c = 1$. Curvature and Fourier conventions match standard cosmology references. Index placement follows the ADM split; background equations are expressed in cosmic time τ.

B Likelihood Details

Supernovae

Offset-marginalized chi-square as in Eq. (24). When reporting Ω_m constraints we profile over H_0 by analytic marginalization or include a broad prior.

BAO

We compute the BAO chi-squared as

$$\chi^2_{\mathrm{BAO}} = \Delta^\top C^{-1} \Delta \tag{25}$$

with Δ collecting $(D_M/r_d, H\,r_d)$ at the three redshifts and C the published covariance. The sound horizon r_d is computed self-consistently at the drag epoch.

CMB Prior

We evaluate $100\,\theta_*$ from the model and form a single-pivot Gaussian chi-square relative to the Planck mean and uncertainty.

C Implementation Notes

The unified runner supports: component tests, grid search, optional MCMC, diagnostic plots (Hubble diagram, BAO pulls, contour plots), and physicality checks (age, r_d consistency). Example commands are provided in the project documentation.