

# The Fixed Point Theorem and The Penrose's Cyclic Time Cosmology: A Dynamical Systems Perspective

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## Abstract

This paper presents a mathematical exploration of Penrose's Cyclic Time, Cosmological framework, through the lens of fixed point theorems and topological field theory. The cosmology, structured around the three intertwined universes, comprising - (1) the Physical Universe with signature  $(+,-,-)$ , (2) The Twistor Universe with signature  $(+,+,-)$ , and (3) the Meta-Physical Universe with signature  $(+,+,+)$ ; is investigated via a synthesis of algebraic topology, differential geometry, and conformal mappings. Meta-Physical Universe is modeled as a boundary-preserving fixed point domain, accessible through a wormhole representing a homeomorphic mapping between the Physical and Meta-Physical Universes. Trajectories of conscious point particles, are embedded in a 16-dimensional Sedenion space, with one special Singular Conscious point particle, residing in a 32-dimensional Trigention space, both governed by invariant fixed point subspaces encoding original tendencies. The Time Periodic experience of Conscious Point Particles is modeled using discrete dynamical systems over finite time, invoking Brouwer, Tarski, and Lefschetz fixed point theorems to represent determinism and return. The structure of the cosmology is further articulated as a fiber bundle, with the Meta-Physical Universe as the base space, the conscious point particles as the fiber, and the the set of "periodic unfolding of events", as the total space. Concepts such as homotopy fixed points, Calabi–Yau boundary compactification, and discrete topologies are employed to distinguish between the conscious point particles and the physical particles. This framework offers a unified topology in which the fixed point becomes the origin, return, and transcendental boundary of the consciousness' journey across Spacetime.

# 1 Introduction

The Cyclic Universe of Penrose [1] worldview introduces a distinctive cosmological structure to reality. In particular we consider three interconnected Universes, namely, the Physical World with signature  $(+, -, -, -)$ , ( which we will to as "Sakar Duniya"), the Twistor Universe with signature  $(+, +, -, -)$  , (to which we will refer to as Subtle World or the Suksham Vatan), and the Meta-Physical Universe with signature  $(+, +, +, -)$  (to which we will refer to as Incorporeal World or Paramdham). These are collectively referred to as the *Trilok*. Central to this cosmology is the doctrine of a fixed, cyclical time cycle of  $T$  years, in which an individual soul reincarnates through a maximum of  $N$ , births, where  $N$  is an integer. The soul, according to this ontology, is an incorporeal, metaphysical point of light that carries within it an eternal imprint of its *sanskaras*, or intrinsic traits. After completing its cycle of births, the soul returns to its original state in Paramdham, a metaphysical region devoid of vibration, sound, or temporality.

This paper aims to model the above metaphysical assertions using tools from algebraic topology, functional analysis, and higher-dimensional algebra. The key mathematical concept utilized is that of fixed point theorems, such as Brouwer's, Banach's, Tarski's, and Lefschetz's theorems, to encode the recurrence, convergence, and invariance principles embedded in the soul's journey. These theorems provide rigorous mathematical grounding for the metaphysical notion that the soul, despite dynamic movement through physical births, returns to a unique and unchanging point—its original pure state.

The cosmological space is further analyzed through the lens of topological constructs. Paramdham is represented as a boundary-preserving fixed point subspace that is topologically compact and light-inaccessible, suggesting analogies with Calabi–Yau compactifications in string theory. The soul's movement across the Trilok is modeled via conformal maps, wormhole-like tunnels, and discrete dynamical systems. Moreover, the mathematical framework is extended by representing the soul within a 16-dimensional Sedenion algebra, whose invariant subalgebras capture the fixed patterns of sanskaras, while the Supreme Soul (Shiv Baba) is encoded in a 32-dimensional Trigention space.

Additionally, the soul's trajectories are shown to follow homotopy equivalence classes, implying that while the paths may differ geometrically, they belong to the same topological class and always return to the same metaphysical fixed point. The drama of creation, sustenance, and destruction is thus interpreted as a topological fiber bundle, where Paramdham forms the base space, the soul represents the fiber, and the eternal, immutable drama acts as the total space.

By fusing metaphysical doctrine with mathematical formalism, this study aspires not merely to simulate theological claims, but to elevate them into the domain of rigorous, symbolic reasoning. The result is a novel model that treats spiritual cosmology with the same analytical depth often reserved for physical theories of the universe.

## 2 Fixed Point Theorem: Mathematical Foundations and Implications

The fixed point theorem is a profound and far-reaching concept in mathematical analysis and topology. At its core, a fixed point of a function  $f$  is an element  $x$  such that  $f(x) = x$ . This seemingly simple definition forms the basis of numerous critical results across disciplines such as economics, computer science, and physics.

Among the most well-known fixed point theorems is the Banach Fixed Point Theorem, also known as the Contraction Mapping Theorem. It states that a contraction mapping on a complete metric space has a unique fixed point and that iteratively applying the function will converge to this fixed point. This convergence behavior is crucial for iterative methods in numerical analysis and dynamic system modeling. The theorem is formally stated as:

If  $(X, d)$  is a complete metric space and  $f : X \rightarrow X$  is a contraction, then  $\exists! x^* \in X$  such that  $f(x^*) = x^*$

This concept of convergence has deep metaphorical potential when we explore spiritual cycles or soul return mechanisms as understood in metaphysical or religious traditions. For instance, in the context of a soul's journey, one can analogize iterative mappings to daily human actions, experiences, or karmic returns which may eventually align the self back to a source point — akin to a fixed point.

In economics, fixed point theorems are used to prove the existence of equilibrium states, such as in the Nash Equilibrium in game theory. In computer science, they form the mathematical basis of recursion and semantics of programming languages. The theorem serves as a foundational bridge between continuity, convergence, and stability, making it indispensable in modern mathematical reasoning.

### 3 Philosophical Parallels in Brahma Kumaris Godly Knowledge

The spiritual philosophy of the Brahma Kumaris organization brings a rich interpretative lens to abstract concepts such as the fixed point. In Brahma Kumaris cosmology, the human soul is viewed as a point of light, inherently pure, and eternal. Over time, however, this soul becomes influenced by body-consciousness, leading to cycles of degradation and renewal. The Supreme Soul, or God (Shiv Baba), is regarded as an immutable fixed point — incorporeal, constant, and the eternal source of spiritual purity.

This fixed nature of the Supreme Soul mirrors the mathematical idea of a fixed point that does not change under transformation. Brahma Kumaris' worldview is built on the idea of a 5000-year time cycle consisting of five epochs: the Golden Age, Silver Age, Copper Age, Iron Age, and the Confluence Age. The soul's journey through these stages can be seen as a transformation function applied repeatedly over time.

This cyclical transformation aligns conceptually with:

$$S_t = f(S_{t-1}) \tag{1}$$

where  $S_t$  represents the state of the soul at time  $t$  and  $f$  is the transformation function governed by karmic law and environmental entropy. Eventually, in the Confluence Age, the soul returns to its original state, signifying a kind of spiritual fixed point. This metaphorical mapping aligns the soul's trajectory with the behavior of dynamical systems as defined in mathematical terms.

In this context, daily spiritual practices like Raja Yoga serve as the iterative mechanism that restores the soul. This is akin to the convergence to the fixed point in contraction mappings, where:

$$\lim_{n \rightarrow \infty} f^n(x) = x^* \tag{2}$$

This alignment is not merely symbolic but offers an alternative understanding of spiritual dynamics using the language of mathematics.

## 4 Brahma Kumaris Cosmological Cycle as a Dynamical System

The Brahma Kumaris cosmological framework proposes a fixed 5000-year cycle of time, comprising five distinct ages. This cycle can be interpreted as a discrete dynamical system where the transformation function  $f$  maps the soul's state from one epoch to another. Let us define:

$$T : S \rightarrow S \quad (3)$$

with

$$T^5 = I \quad (4)$$

Here,  $T$  is the transformation function acting cyclically on soul states  $S$ , and  $I$  is the identity transformation, signifying the return to the original state after one complete cycle.

Each epoch can be represented as a distinct function:

$$\begin{aligned} f_1(x) &= x \quad (\text{Golden Age}) \\ f_2(x) &= x - \varepsilon \quad (\text{Silver Age}) \\ f_3(x) &= x - \delta_1 \quad (\text{Copper Age}) \\ f_4(x) &= x - \delta_2 \quad (\text{Iron Age}) \\ f_5(x) &= x + r(x) \quad (\text{Confluence Age, recovery function}) \end{aligned} \quad (5)$$

The Confluence Age ( $f_5$ ) marks the period of transformation through divine remembrance. Here,  $r(x)$  represents the restorative function invoked through Raja Yoga, wherein the soul is drawn back toward its original state:

$$\lim_{n \rightarrow \infty} f_5^n(x) = x_0 \quad (6)$$

In this representation,  $x_0$  corresponds to the purest form of the soul as it existed in the Golden Age, aligning again with the fixed point paradigm. Thus, the Supreme Soul functions as a global attractor, unchanging and serving as the reference against which all transformations are measured.

By treating the soul's journey as a finite dynamical system with a well-defined transformation path and a convergence behavior, this model provides a powerful metaphysical interpretation using mathematical structures.

## 5 Supreme Soul as Global Attractor (Fixed Point)

In the framework of Brahma Kumaris Godly knowledge, the Supreme Soul—referred to as Shiv Baba—is understood as the ultimate, incorporeal being who remains eternally unchanged, perfect, and unaffected by the physical or metaphysical transformations that all other souls undergo. This metaphysical constancy makes the Supreme Soul a direct

spiritual analogue to the mathematical concept of a global fixed point. In mathematical terms, a fixed point is an element that remains invariant under a given transformation. For a transformation  $T$ , the fixed point  $x$  satisfies:

$$T(x) = x \tag{7}$$

Translating this into spiritual language, the Supreme Soul is the entity for which:

$$T(\text{God}) = \text{God} \tag{8}$$

This formulation implies that the Supreme Soul remains unaffected by the entropy of time, karmic accounts, or bodily birth and rebirth cycles. Unlike human souls, who undergo transformations through the ages—from the Golden Age to the Iron Age—the Supreme Soul remains completely stable, pure, and uninfluenced by external conditions. Therefore, all other transformations in the cosmic cycle are to be understood as being relative to this absolute fixed point.

In dynamical systems theory, a global attractor is a point or set toward which a system evolves from a wide variety of initial conditions. The Supreme Soul in Brahma Kumaris cosmology fulfills this role. During the Confluence Age, souls begin a process of self-purification and spiritual alignment through the practice of Raja Yoga, which progressively draws them closer to their original, divine state. This process can be modeled by the iterative application of a recovery function  $r(x)$ :

$$x_{n+1} = x_n + r(x_n) \tag{9}$$

Where  $x_n$  represents the state of a soul at iteration  $n$ . The recovery function  $r(x_n)$  is activated through divine remembrance and spiritual effort, and it leads to the convergence of the soul's state toward that of the Supreme Soul. The convergence behavior is described by:

$$\lim_{n \rightarrow \infty} x_n = x_0 \tag{10}$$

Here,  $x_0$  denotes the pure, original state of the soul, which is identical in quality to the nature of the Supreme Soul but not equal in stature. The convergence does not imply identity with the Supreme Soul but rather resonance or alignment with its qualities of purity, peace, and love.

In spiritual literature, such as those published by the Brahma Kumaris World Spiritual University [?], the Supreme Soul is described as a point of divine light residing in the incorporeal world, known as Paramdham. Unlike human souls, the Supreme Soul does not enter the cycle of birth and death and therefore remains free from the accrual of sanskaras or karmic impressions. This unchangeability is the foundational basis for divine remembrance or "yoga," which literally means "union." This union is not a physical or metaphysical merging but a remembrance that invokes transformation in the one remembering.

Further, authors like Dadi Janki have emphasized that the Supreme Soul acts as a spiritual reference point, akin to a lighthouse guiding souls who are lost in the fog of ignorance and material entanglement [?]. Just as mathematical fixed points provide stability and reference within complex dynamic systems, the Supreme Soul offers a stable referent for spiritual alignment and reformation.

Mathematically, if we treat the soul as a state variable within a system governed by time-evolving transformations, and the Supreme Soul as an invariant entity, then every transformation function applied over time (such as those representing the four Yugas) acts within a vector space where the fixed point remains constant. Let us define the complete time-cycle transformation  $T$  as a composition of epoch-based functions:

$$T = f_5 \circ f_4 \circ f_3 \circ f_2 \circ f_1 \tag{11}$$

While  $T(x) \neq x$  for a general soul  $x$ , we have:

$$T(\text{God}) = \text{God} \tag{12}$$

This reinforces the understanding that only the Supreme Soul remains unaffected across all transformations, and thus functions as a universal constant in both spiritual and symbolic mathematical terms.

In this manner, the Supreme Soul functions as a global attractor—a fixed point that is not only conceptually stable but also functionally critical for the re-alignment of the soul. The metaphysical relevance of this attractor is immense, as it underpins the entire logic of spiritual reformation during the Confluence Age.

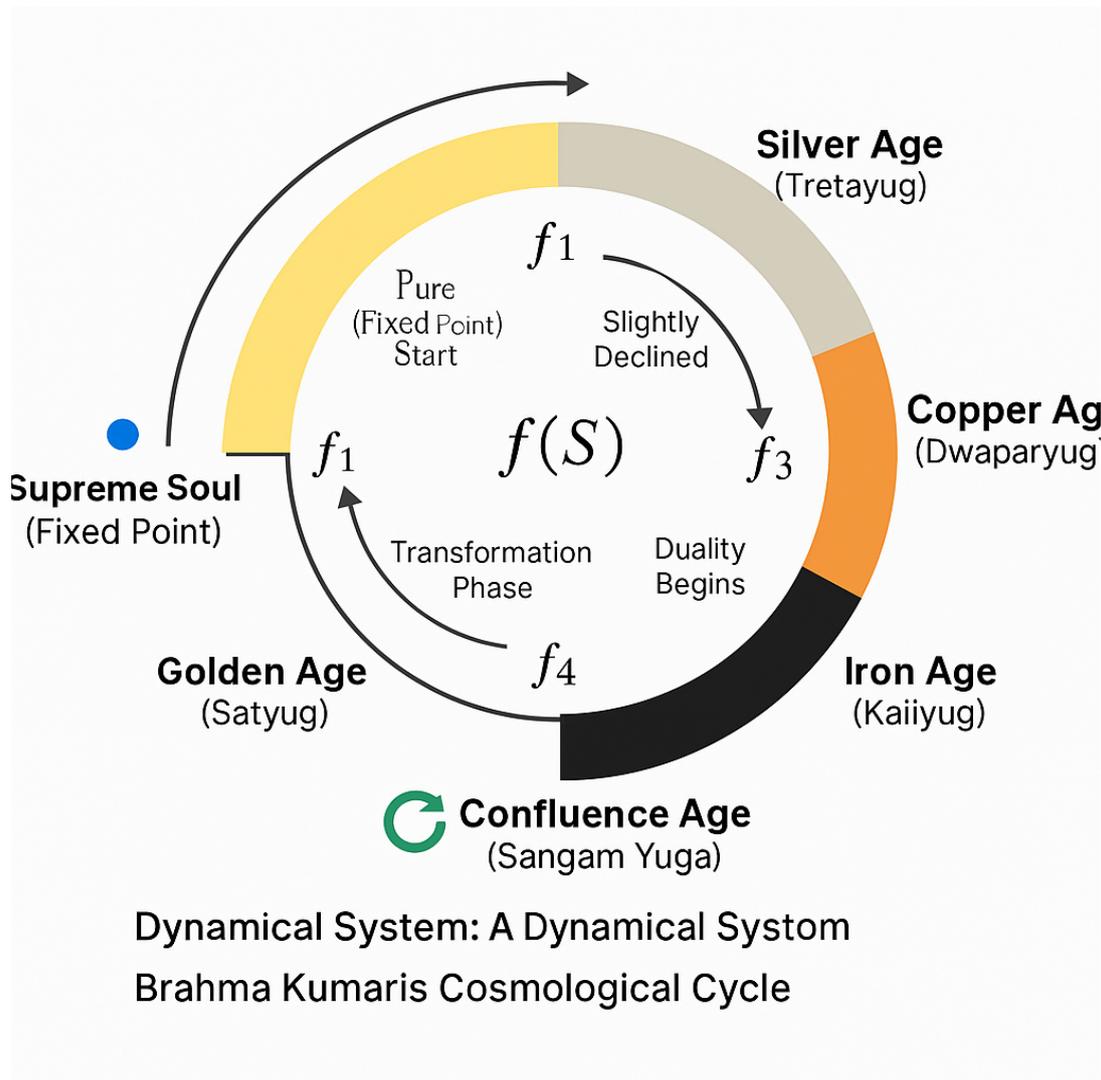


Figure 1: Brahma Kumaris Cosmological Cycle represented as a Dynamical System. Each age corresponds to a transformation function, eventually leading back to the fixed point state.

## 6 Soul Returns to its Original Nature: Interpretation of $f(x) = x$

In mathematical parlance, the equation

$$f(x) = x \tag{13}$$

defines a fixed point for a function  $f$ , where the application of the function does not change the input value. In the context of spiritual metaphysics, especially as elucidated in the teachings of the Brahma Kumaris, this simple equation encapsulates the profound process of the soul's journey back to its original, pure state. The concept of the soul returning to its fixed nature mirrors the convergence behavior seen in mathematical dynamical systems.

In Brahma Kumaris philosophy, the soul is originally a point of divine light, characterized by purity, peace, and bliss. Over the course of the time cycle, particularly through

the epochs of Copper Age and Iron Age, the soul becomes covered with layers of impurity due to repeated birth and karmic bondage. This accumulated entropy can be seen as the result of transformations  $f_t(x)$  acting on the soul across time steps  $t$ , which diverge from the identity function. However, during the Confluence Age, a spiritual function begins to operate that restores the soul to its essential state.

In this spiritual phase, the divine function guiding the soul is restorative in nature. It can be characterized mathematically as a purification operator, defined by the sequence:

$$x_{n+1} = f(x_n) \tag{14}$$

with the boundary condition that:

$$\lim_{n \rightarrow \infty} f^n(x) = x_0 \tag{15}$$

Here,  $x_0$  symbolizes the original, fixed point representing the soul's innate, unblemished nature. This convergence marks the soul's complete reformation, restoring it to the state it held during the Golden Age.

The philosophical and practical implications of this are critical. Through the daily practice of Raja Yoga meditation as taught by the Brahma Kumaris, the soul is reconnected to the Supreme Soul, and the transformation function  $f$  begins to approximate the identity function. That is, the changes induced in the soul no longer introduce divergence but promote return to the fixed essence. This can be formally described as:

$$f(x) \rightarrow x \quad \text{as the soul approaches enlightenment} \tag{16}$$

In other words, the transformation function becomes self-consistent, inducing no further deviation from the soul's original state. This behavior has strong analogues to systems studied in fixed point theory, particularly where iterated function systems are employed to model convergence toward a stable equilibrium point [?].

Brahma Kumaris literature describes the Confluence Age as the most auspicious of all epochs, the only time in the entire 5000-year cycle when the soul has the opportunity to realize its true self and reunite with the divine. This understanding is supported by texts such as "The Cycle of Time" from the Brahma Kumaris World Spiritual University [?], which emphasizes the soul's cyclical degradation and reformation. Authors like Dadi Janki [?] describe the process not as one of construction or enhancement but of simple remembrance and removal of impurity.

To summarize, the equation  $f(x) = x$  in the Brahma Kumaris metaphysical system is more than a mathematical abstraction. It is a vivid representation of the soul's ultimate destiny—to return to its original self. Just as in mathematical systems where a stable fixed point ensures consistency and predictability, in spiritual terms, the soul achieves stability, purity, and peace through remembrance and divine connection. Therefore, the metaphor of the fixed point serves as an ideal model for explaining the restoration of soul consciousness during the Confluence Age.

## 7 Iterative Approach: Raja Yoga and Convergence to the Divine Nature

The practice of Raja Yoga, as taught by the Brahma Kumaris, is deeply aligned with the concept of iterative convergence found in mathematical analysis. In mathematics, an

iterative method involves the repeated application of a function to approach a desired value or fixed point. Formally, such iterative processes can be written as:

$$x_{n+1} = f(x_n) \quad (17)$$

where  $x_n$  represents the state at the  $n$ -th step, and  $f$  is a transformation function. When certain conditions are met, this sequence converges to a fixed point  $x^*$  such that:

$$\lim_{n \rightarrow \infty} x_n = x^* \quad \text{where } f(x^*) = x^* \quad (18)$$

In spiritual practice, Raja Yoga operates in a remarkably similar fashion. It is the daily meditative process whereby the soul, obscured by layers of accumulated sanskaras and karmic impressions, applies a form of spiritual purification through divine remembrance. Over time, this repeated engagement with the Supreme Soul leads the individual to a state of inner clarity and purity. The goal of this meditative practice is to reestablish the soul in its original nature—a state synonymous with spiritual enlightenment and divine identity.

The Brahma Kumaris describe the Confluence Age as the brief period in the time cycle when this convergence is most accessible. Each day of meditation can be likened to an iteration in the transformation sequence, where the soul gradually realigns with its true self. This can be represented as:

$$x_{n+1} = x_n + r(x_n) \quad (19)$$

where  $r(x_n)$  represents the spiritual reformation imparted through meditation and divine remembrance. As  $n$  increases,  $r(x_n)$  becomes smaller, indicating the soul's progressive stabilization. Eventually, this leads to:

$$\lim_{n \rightarrow \infty} r(x_n) = 0 \quad \text{and} \quad x_n \rightarrow x_0 \quad (20)$$

In this framework,  $x_0$  represents the original, unblemished state of the soul—an internal equilibrium that is maintained once reached. The convergence is not merely metaphysical but is mirrored in behavioral, cognitive, and emotional transformations experienced by long-term practitioners of Raja Yoga. This experiential validation supports the mathematical interpretation of Raja Yoga as an iterative convergence mechanism.

The Brahma Kumaris' philosophical literature supports this understanding. In "Raja Yoga – The Ancient Science of Soul Consciousness" [?], the technique of spiritual remembrance is outlined as a process that gradually eradicates mental and emotional impurities. It explains that consistent practice creates cumulative effects, much like an iterative function approaching a fixed point. Furthermore, the Brahma Kumaris World Spiritual University highlights this cumulative progress in their description of the Confluence Age [?].

The idea of iterative purification is not new. Mathematical models of learning and adaptation often incorporate the same principles. Contraction mappings in fixed point theory provide convergence criteria when the transformation function  $f$  is contractive, i.e., when there exists  $0 < k < 1$  such that:

$$d(f(x), f(y)) \leq k \cdot d(x, y) \quad (21)$$

This concept aligns with the experience of meditation in which successive states become increasingly closer to the desired state of peace and self-awareness. When applied

to a complete metric space of spiritual states, the convergence to the fixed point becomes mathematically inevitable, reinforcing the role of divine remembrance as both a practice and a law.

From a holistic perspective, the daily practice of Raja Yoga is a disciplined, structured application of transformative effort. The iterations are not identical, but the process is cumulative. The soul, just like a variable in an iterative algorithm, is reoriented again and again toward the ideal until deviation becomes negligible. In this manner, Raja Yoga can be seen not only as a spiritual exercise but as a model of spiritual computation, where divine memory serves as the algorithmic engine of reformation.

In summary, the convergence model from fixed point theory offers a compelling and rigorous metaphor for understanding the long-term effects of Raja Yoga meditation. Just as iterative algorithms produce refined results over successive applications, Raja Yoga induces a gradual and profound reformation of the soul, leading it back to its fixed, divine identity.

## 8 The Soul's Journey Through Life: A Fixed Point in the Cycle of Birth and Death

In the teachings of the Brahma Kumaris, the soul is fundamentally an eternal point of conscious light, bodiless and pure in its original form. This metaphysical understanding positions the soul not as a transient phenomenon, but as an invariant entity that transcends the physical body. The soul undergoes a temporal journey through different bodily stages—childhood, youth, adulthood, and old age—before it dissociates from the body upon death. Following death, the soul returns to its bodiless, original state.

This cyclical journey can be interpreted as a mapping function  $f(t)$ , where  $t$  denotes time during one life span and  $f(t)$  denotes the soul's state as it passes through various physical embodiments. However, the original state, denoted as  $x_0$ , remains unchanged and serves as a fixed point in this cycle. Mathematically, this can be described by the function:

$$f(t_0) = f(t_n) = x_0 \quad (22)$$

where  $t_0$  is the moment before birth and  $t_n$  is the moment after death. At both instances, the soul resides in its incorporeal, pure state. This suggests that the bodiless state of the soul serves as a fixed point in the entire transformation of life within the body.

Throughout the human lifespan, the soul undergoes transformations in identity, behavior, memory, and emotional orientation due to its engagement with the physical body and external environment. Yet these are merely transient mappings of the soul's expression, not alterations of the soul itself. These transformations can be represented as a composite function  $F(t)$ , acting on the pure soul  $x_0$ :

$$x(t) = F(t, x_0) \quad (23)$$

where  $F$  includes the influences of bodily conditions, environmental inputs, and karmic imprints. While  $x(t)$  varies,  $x_0$  remains fixed, reaffirming the view that the soul's original nature is constant and unchangeable.

As per the doctrine taught by the Brahma Kumaris [?], the body is a costume worn temporarily by the soul to perform its roles on the world stage. Upon completion of a lifetime, the soul discards this costume and returns to its metaphysical origin, residing once again as a point of light in the incorporeal realm (Paramdham). This return can be seen as the soul reverting to its fixed point, just as a system may stabilize at a certain equilibrium value after undergoing a cycle of transformations.

This understanding has practical implications for spiritual development. The knowledge that the soul's true form remains unaltered and that it will ultimately return to its original state reinforces the goal of aligning with that fixed point during one's lifetime. Practices such as Raja Yoga aim to reduce the oscillations in  $x(t)$  and draw the state of the soul back toward  $x_0$ , even while within the body. This convergence can be modeled as:

$$\lim_{t \rightarrow t_n^-} x(t) \rightarrow x_0 \quad (24)$$

indicating the final re-alignment of the embodied soul to its original nature before its exit from the corporeal frame.

Dadi Janki [?] and other spiritual leaders within the Brahma Kumaris tradition have repeatedly emphasized that the journey of life is not linear but cyclical, and the understanding of this cycle is critical for achieving self-realization. The cyclicity further supports the fixed point model, where the start and end states of the soul are identical in quality, despite the apparent transformations in between.

This cyclic perspective integrates well with the broader cosmological framework of the Brahma Kumaris, where time itself is viewed as a circle. The soul's journey within a lifetime is thus a microcosmic representation of the macrocosmic time cycle. The consistent return of the soul to its original point of light mirrors the cyclical nature of all spiritual transformations across ages and births.

In conclusion, the mapping of life from birth to death, as envisioned in the Brahma Kumaris' spiritual teachings, clearly suggests the soul's bodiless original state as a fixed point. This fixed point represents both the origin and the destination of the soul's journey, reaffirming its eternal, unchanging identity amidst temporary physical transformations.

## 9 The Immutable Recording of the Soul Within the Drama Cycle

In the metaphysical teachings of the Brahma Kumaris, the concept of the eternal drama cycle is central to understanding the nature of time, existence, and the soul's role in the cosmic theatre. According to this doctrine, the universe unfolds in a fixed, cyclical pattern that spans 5000 years, repeating endlessly with absolute precision. Every event, circumstance, and interaction is eternally predetermined and recurs identically in every cycle.

Mathematically, this view can be analogized to a deterministic, periodic function:

$$D(t + T) = D(t) \quad (25)$$

where  $D(t)$  denotes the state of the universe at time  $t$  and  $T$  represents the period of the cycle. For the Brahma Kumaris,  $T = 5000$  years. The implication of this equation

is that the drama of the world repeats exactly, without deviation, thereby ensuring that every event is recorded in such a way that it can play out identically in the next cycle.

Within this deterministic framework, each soul functions as a compact data point—a metaphysical point of light—that contains the complete and unchangeable record of its roles, relationships, sanskaras, and karmic patterns. This recording is not external to the soul but exists within its very structure. It is neither written on a medium nor subject to erasure. Rather, it is inherent and inseparable from the soul’s identity. Let us denote the soul by  $s$ , and its recording by  $R(s)$ . Then we assert:

$$\forall t, \quad s(t) = R(s)[t] \quad (26)$$

This equation implies that the behavior, thoughts, and responses of the soul at time  $t$  are direct manifestations of the information encoded in  $R(s)$ . Furthermore, since the drama cycle is fixed and recurrent, it follows that:

$$R(s)_{t+T} = R(s)_t \quad (27)$$

The permanence of this recording raises a deep ontological question: what kind of encoding must this be, such that it is both immutable and infinitely replayable? From a metaphysical standpoint, the recording must be subtle, timeless, and self-executing. It must operate without the need for external retrieval, akin to an automaton executing a deterministic program from within.

Brahma Kumaris literature likens the soul to a divine microchip, containing within it the full record of its many births, interactions, and experiences. This idea is elaborated in the publications of the Brahma Kumaris World Spiritual University [?], which emphasize that each soul’s script is unique, inviolable, and eternally valid.

From a philosophical standpoint, such an encoding also implies that freedom, as experienced by the soul, is not in rewriting the recording but in aligning with the highest potential already inherent in it. The immutable nature of the soul’s recording allows for a unique model of existential continuity that is cyclical rather than linear. The soul’s journey is not one of discovering new terrains, but of retracing its divinely recorded path.

The concept also provides an answer to the problem of identity across lives. Since the recording is eternal and fixed, the soul maintains coherence across multiple births, even though conscious awareness may be reset. The unfolding of the soul’s recording occurs through interactions with the physical world, which serves as the stage on which the pre-recorded drama manifests. In functional terms, we may write:

$$s_t = f_{\text{drama}}(R(s), e_t) \quad (28)$$

where  $e_t$  represents the environmental input at time  $t$ , and  $f_{\text{drama}}$  is the function mapping the recording to external expression. This mapping does not alter  $R(s)$ ; it merely plays it out.

In conclusion, the Brahma Kumaris’ conception of the soul’s recording within the drama cycle introduces a unique metaphysical construct: a point-like consciousness that contains an eternal, unchangeable script. This recording is not inert but dynamically expressive, manifesting across time with mathematical precision. Understanding this allows the practitioner to observe life not as a realm of improvisation, but as a profound realization of what has always been and will always be.

## 10 Mathematical Interpretation of Soul Recording in the Eternal Drama Cycle

The Brahma Kumaris' concept of the eternal drama asserts that every event in the universe repeats cyclically every 5000 years. This idea can be formulated using periodic function theory. Let  $D(t)$  be the full state of the cosmic drama at time  $t$ , and let  $T = 5000$  be the period. Then the drama is periodic:

$$D(t + T) = D(t) \quad (29)$$

Each soul  $s_i$  is a point of conscious light and carries an intrinsic, immutable recording  $R_i$ , which defines its behavior throughout time. Let the behavior function of soul  $s_i$  be  $B_i(t)$ . Then for all  $t \in \mathbb{R}$ :

$$B_i(t) = R_i(t \bmod T) \quad (30)$$

Here,  $R_i$  is a time-indexed map defined on the interval  $[0, T)$  and loops eternally due to modular arithmetic, representing recurrence across cycles.

Let the set of all souls be denoted by  $\mathcal{S} = \{s_1, s_2, \dots, s_n\}$ . Then the full state of the world is a function:

$$D(t) = \sum_{i=1}^n B_i(t) + E(t) \quad (31)$$

where  $E(t)$  represents external conditions of nature (space, time, and matter), which themselves satisfy:

$$E(t + T) = E(t) \quad (32)$$

Let us now define a total role space  $\mathcal{R}$  of the soul as:

$$\mathcal{R}_i = \{r_i^1, r_i^2, \dots, r_i^k\} \quad (33)$$

Each  $r_i^j$  is a role enacted by  $s_i$  in a particular subinterval  $[t_j, t_{j+1})$  of the drama cycle. The activation function  $\phi_i(t)$  selects which role is played at time  $t$ :

$$\phi_i(t) = r_i^j \quad \text{if } t \in [t_j, t_{j+1}) \quad (34)$$

The inner data structure of  $R_i$  thus stores a sequence of roles  $\mathcal{R}_i$  and their associated time intervals. Hence:

$$R_i = \{(r_i^j, [t_j, t_{j+1}))\}_{j=1}^k \quad (35)$$

Moreover, since the soul's behavior is not altered but only played out, the mapping from  $R_i$  to observable actions  $A_i(t)$  is read-only:

$$A_i(t) = f(R_i(t)), \quad \text{where } f : \mathcal{R}_i \rightarrow \text{Actions} \quad (36)$$

We further postulate that  $R_i$  is unmodifiable:

$$\forall t, \quad \frac{d}{dt} R_i(t) = 0 \quad (37)$$

This implies a fixed recording model. Unlike typical dynamic systems where state variables evolve, here the state is fixed and only expressions of it vary.

Suppose a set of interaction events  $\mathcal{I}$  involves  $m$  souls:  $\{s_1, \dots, s_m\}$ . Then the joint outcome at time  $t$  is given by:

$$I(t) = F(R_1(t), R_2(t), \dots, R_m(t)) \quad (38)$$

where  $F$  is a multi-soul interaction function that maps their stored roles at  $t$  into a resulting joint event in the drama.

Since  $R_i$  is pre-recorded, any event  $e$  that occurs at time  $t$  must also occur at:

$$t + nT \quad \forall n \in \mathbb{Z} \quad (39)$$

This confirms the statement that every event in the cycle has infinite repetitions in past and future.

Thus, the recording  $R_i$  must have the following properties:

- **Deterministic:** It must yield the same output for the same time input.
- **Compact:** It is held within a metaphysical point (soul), implying non-physical encoding.
- **Cyclical:** It satisfies modularity with respect to  $T = 5000$ .
- **Immutable:** No function exists such that  $R'_i(t) \neq R_i(t)$ .

By analogy, this resembles an injective mapping from  $\mathbb{R}/T\mathbb{Z}$  to a closed action space:

$$R_i : \mathbb{R}/T\mathbb{Z} \rightarrow \mathcal{A} \quad (40)$$

Finally, the soul's script is its own fixed point under the identity time function:

$$R_i = \lim_{n \rightarrow \infty} R_i(t + nT) \quad (41)$$

**Conclusion:** The recording in each soul acts as a fixed invariant under a deterministic cyclic model. Brahma Kumaris philosophy thus describes a mathematically consistent theory where time, identity, and fate operate within a fixed, repeatable structure with every soul's part preserved in a non-variable microstate.

## 11 Group Action of Time on Soul-Space in the Eternal Drama

In Brahma Kumaris metaphysics, time is not a linear dimension but a closed, cyclic construct repeating every 5000 years. This conception naturally lends itself to a group-theoretic interpretation. In mathematics, a group action is a formal mechanism to describe how elements of a group operate on elements of another set. We apply this idea to represent the interaction of time with the set of all souls, called the soul-space.

Let  $G = \mathbb{Z}/5000\mathbb{Z}$  be the additive cyclic group of integers modulo 5000, representing discrete time in years. Let  $S$  be the soul-space, consisting of all soul states over time. We define an action  $\cdot : G \times S \rightarrow S$  such that:

$$g \cdot s = \text{State of soul } s \text{ at time } g \quad (42)$$

The group action must satisfy the standard axioms:

$$0 \cdot s = s \quad (\text{Identity}) \quad (43)$$

$$(g + h) \cdot s = g \cdot (h \cdot s) \quad \forall g, h \in G, s \in S \quad (\text{Compatibility}) \quad (44)$$

Each soul  $s \in S$  has an orbit under this action defined by:

$$\text{Orb}(s) = \{g \cdot s \mid g \in G\} \quad (45)$$

Since the cycle is closed, every orbit is finite and satisfies:

$$\exists n \in \mathbb{N}, \quad n \cdot s = s \quad (46)$$

In the Brahma Kumaris framework, this  $n$  is exactly 5000, leading to:

$$5000 \cdot s = s \quad (47)$$

This establishes that the soul returns to its exact initial state after 5000 time units, reinforcing the deterministic and periodic nature of the drama.

Let the role function  $R : S \times G \rightarrow \mathcal{A}$  map a soul and a time index to an action:

$$R(s, g) = a \in \mathcal{A} \quad (48)$$

where  $\mathcal{A}$  is the set of possible actions or roles a soul may enact. Since every soul plays the same role every cycle, we have:

$$R(s, g) = R(s, g + 5000k) \quad \forall k \in \mathbb{Z} \quad (49)$$

This cyclic property ensures the soul's behavior is governed by the structure of  $G$  and not altered by external randomness.

We define a transformation function  $T_g : S \rightarrow S$  for each  $g \in G$  as:

$$T_g(s) = g \cdot s \quad (50)$$

The collection  $\{T_g\}_{g \in G}$  forms a group of transformations acting on the soul-space  $S$ . The fixed point property follows naturally:

$$T_{5000}(s) = s \quad (51)$$

This illustrates that time, modeled as group action, is not only compatible with the philosophical view but also offers rigorous mathematical grounding.

Furthermore, one may define the stabilizer of a soul  $s$  as:

$$\text{Stab}(s) = \{g \in G \mid g \cdot s = s\} \quad (52)$$

In our case, since every soul completes its script exactly once every 5000 years:

$$\text{Stab}(s) = \langle 5000 \rangle \quad (53)$$

From a structural perspective, this indicates that every soul follows a looped automorphism within the larger configuration space. The set of all soul trajectories forms a bundle over  $G$  with fibers  $\text{Orb}(s)$ .

We can also define a drama unfolding function  $\mathcal{D} : G \rightarrow \text{State}(U)$  where  $U$  is the universe:

$$\mathcal{D}(g) = \sum_{s \in S} R(s, g) \quad (54)$$

Thus, every instant in time is a summation of individual soul actions, and since every  $R(s, g)$  is periodic,  $\mathcal{D}$  also satisfies:

$$\mathcal{D}(g + 5000) = \mathcal{D}(g) \quad (55)$$

In conclusion, modeling the cycle of time as a group action not only aligns with Brahma Kumaris cosmology but also allows precise representation of recurrence, soul identity, and determinism. The elegance of group theory captures the metaphysical logic of an eternal, repeating cosmic script in a form that is both symbolic and structural.

## 12 Closed Trajectories of Particles and the 5000-Year Cycle

In Brahma Kumaris cosmology, time is cyclical, and all phenomena unfold in a fixed loop spanning 5000 years. This concept can be rigorously modeled using the notion of closed trajectories in physics and dynamical systems theory. Each particle or soul follows a deterministic, closed path in a multidimensional state space. Let this path be parameterized by  $s$ , which denotes a position along the path, and let  $v(s)$  be the instantaneous velocity at that point.

The total time  $T$  to complete the closed cycle is expressed by the integral:

$$\oint \frac{ds}{v(s)} = T \quad (56)$$

This is a contour integral over a closed loop in state-space. It represents the complete traversal time for the particle along its trajectory. In this metaphysical setting,  $T$  is universally fixed at 5000 years for every particle or soul in the universe:

$$T = 5000 \quad (57)$$

Let the position vector of a particle be given by  $\vec{x}(s)$ , and let  $\vec{v}(s) = \frac{d\vec{x}}{ds}$  denote its velocity vector. The speed is then:

$$v(s) = \left\| \frac{d\vec{x}}{ds} \right\| \quad (58)$$

Substituting this into the original integral yields:

$$\oint \frac{1}{\left\| \frac{d\vec{x}}{ds} \right\|} ds = 5000 \quad (59)$$

The integral defines a global constraint on the particle's evolution. For any deviation in speed  $v(s)$  at certain sections of the path, compensation must occur elsewhere to

preserve the total time. Therefore, all variation in temporal density must satisfy the integral constraint.

Let  $\gamma : [0, L] \rightarrow \mathbb{R}^n$  denote the parameterized closed curve of the particle in phase space, with  $\gamma(0) = \gamma(L)$ . The total traversal time becomes:

$$T = \int_0^L \frac{1}{\|\gamma'(s)\|} ds = 5000 \quad (60)$$

This implies a uniformity in metaphysical timing across all entities. Each trajectory is unique in form, yet equal in temporal duration, forming a cosmic synchrony.

Now consider a time-rescaled parameter  $t \in [0, T]$ , where  $\vec{x}(t)$  is the trajectory as a function of time. The cycle condition imposes:

$$\vec{x}(t + T) = \vec{x}(t) \quad (61)$$

This implies periodicity:

$$\vec{x}(t + 5000) = \vec{x}(t) \quad (62)$$

Therefore, the particle's dynamics are constrained by both spatial closure and temporal periodicity. The velocity field is thus a  $T$ -periodic vector field:

$$\vec{v}(t + T) = \vec{v}(t) \quad (63)$$

Let us define the phase space energy functional associated with the motion:

$$E[\vec{x}] = \int_0^T \frac{1}{2} \|\vec{v}(t)\|^2 dt \quad (64)$$

This energy is invariant under time translation due to the periodicity condition. Thus, all dynamics conserve not only time-integral but also cyclic energy, upholding metaphysical symmetry.

According to Brahma Kumaris philosophy [?], this cyclical path is pre-recorded in the sanskaras of each soul, forming a unique loop that cannot be modified but only expressed. Dadi Janki [?] further elaborates that the drama is a self-similar, perfectly repeating structure, where each soul repeats its path without deviation.

The metaphysical implication is that while souls may appear to act freely, they are in fact tracing closed, deterministic trajectories governed by subtle energies. Let the closed path be defined on a manifold  $\mathcal{M}$  with a mapping:

$$f : \mathcal{M} \rightarrow \mathcal{M}, \quad f^T(x) = x \quad (65)$$

This asserts that the mapping  $f$  composed  $T$  times yields the identity—every point returns to itself after  $T = 5000$  iterations, thereby reinforcing the fixed point nature of the cosmic cycle.

## 13 Fixed Point Theorems and the Eternal Cosmic Cycle

The Brahma Kumaris spiritual philosophy asserts that the world drama is cyclic and repeats every 5000 years. Every soul returns to its original role and position at the end

of the cycle. This perfectly aligns with fixed point theory in mathematics, which studies conditions under which functions return elements to themselves. In this section, we develop a formal mathematical interpretation of this metaphysical concept using established theorems in fixed point theory.

Let  $X$  be a metric space representing all possible configurations of soul states. Let  $T : X \rightarrow X$  be a transformation function that evolves soul states over the 5000-year period. A fixed point  $x^* \in X$  satisfies:

$$T(x^*) = x^* \quad (66)$$

The classical Banach Fixed Point Theorem, also known as the Contraction Mapping Theorem, states that if  $(X, d)$  is a complete metric space and  $T$  is a contraction mapping, then  $T$  has a unique fixed point  $x^* \in X$ . That is, if:

$$d(T(x), T(y)) \leq \lambda d(x, y) \quad \text{for all } x, y \in X, \text{ where } 0 < \lambda < 1 \quad (67)$$

then there exists a unique  $x^*$  such that  $T(x^*) = x^*$ , and iteratively applying  $T$  yields:

$$\lim_{n \rightarrow \infty} T^n(x) = x^* \quad (68)$$

In Brahma Kumaris cosmology, this iterative convergence mirrors the soul's return to its pure form through repeated cycles and spiritual practice. The original soul state is a stable equilibrium, a metaphysical fixed point. Over time, despite deviations due to karmic influence, Raja Yoga practice can be viewed as the iterative application of a contraction-like transformation  $T$  which gradually draws the soul toward  $x^*$ .

Let us consider another form of fixed point theorem: Brouwer's Fixed Point Theorem. It states that every continuous function  $f : D^n \rightarrow D^n$  (where  $D^n$  is a closed  $n$ -dimensional ball) has at least one fixed point. That is:

$$\exists x \in D^n \text{ such that } f(x) = x \quad (69)$$

This supports the idea that in a bounded spiritual phase space, there must be at least one unchanged core—the soul's original pure state—that remains invariant across transformations.

Furthermore, we may define a dynamical system where each state evolves under time mapping:

$$x_{n+1} = T(x_n) \quad (70)$$

If  $T$  is a non-expansive mapping (i.e.,  $d(T(x), T(y)) \leq d(x, y)$ ), then the trajectory  $\{x_n\}$  may still converge under specific assumptions, and the omega-limit set:

$$\omega(x_0) = \{x \in X \mid \exists n_k \rightarrow \infty \text{ such that } \lim_{k \rightarrow \infty} x_{n_k} = x\} \quad (71)$$

contains fixed points or periodic orbits. In the Brahma Kumaris model,  $x_0$  represents the degraded soul and the omega-limit set includes the final liberated or reformed state—achieved through multiple cycles or yogic convergence.

Let us now define a cyclic mapping  $T$  such that:

$$T^{5000}(x) = x \quad (72)$$

This is a periodic fixed point system. The mapping  $T$  is not a contraction, but a time evolution function over a closed trajectory. The entire space  $X$  consists of trajectories such that:

$$\forall x \in X, \quad \exists n \in \mathbb{N}, \quad T^n(x) = x \quad (73)$$

This characterizes a system of periodic fixed points, consistent with the idea that all roles, actions, and identities repeat precisely every 5000 years.

We can model this with a permutation group action  $G$  on  $X$ , where  $G = \mathbb{Z}/5000\mathbb{Z}$  and:

$$g \cdot x = T^g(x), \quad \text{and } T^0(x) = x \quad (74)$$

Since  $T^{5000}(x) = x$ , the action is cyclic of order 5000, and every orbit has length dividing 5000. Thus,  $x$  lies on a closed loop with a return condition satisfied.

Such cyclicity and recurrence are reminiscent of the Lefschetz Fixed Point Theorem, which uses algebraic topology. For a continuous map  $f : X \rightarrow X$  on a compact triangulable space, the Lefschetz number  $L(f)$  is defined and:

$$L(f) = \sum_{k=0}^n (-1)^k \text{Tr}(f_* : H_k(X) \rightarrow H_k(X)) \quad (75)$$

If  $L(f) \neq 0$ , then  $f$  has a fixed point. This provides a topological invariant that supports recurrence in a closed cosmic system.

In conclusion, various fixed point theorems such as Banach, Brouwer, and Lefschetz offer profound mathematical underpinnings to the Brahma Kumaris notion of the eternal return. These theorems provide not only symbolic resonance but also structural rigor in supporting the idea that every soul returns to its fixed original state after each cosmic cycle.

The Brahma Kumaris spiritual model presents time as a finite yet eternal cycle spanning 5000 years. Each epoch in the cycle—Golden, Silver, Copper, Iron, and Confluence Age—repeats identically in every iteration of time. This perspective provides fertile ground for a rigorous mathematical interpretation using the Lefschetz Fixed Point Theorem and Nietzsche’s philosophical doctrine of Eternal Recurrence.

Let  $X$  be a compact, orientable topological manifold representing the total state-space of the universe including all souls, roles, and environmental parameters. Let  $f : X \rightarrow X$  be a continuous map representing one full cycle of transformation, i.e.,  $f$  captures the universal evolution over 5000 years. The Lefschetz Fixed Point Theorem asserts that if the Lefschetz number  $L(f)$  is nonzero, then  $f$  has at least one fixed point:

$$L(f) \neq 0 \Rightarrow \exists x \in X \text{ such that } f(x) = x \quad (76)$$

The Lefschetz number is defined as:

$$L(f) = \sum_{k=0}^n (-1)^k \text{Tr}(f_* : H_k(X) \rightarrow H_k(X)) \quad (77)$$

Here,  $H_k(X)$  is the  $k$ -th homology group of  $X$ , and  $f_*$  is the induced map on homology. In our spiritual model, this number measures the global deviation introduced by one cycle of time. Since the Brahma Kumaris view asserts perfect return of all events and entities to their original states, we expect:

$$f^{5000}(x) = x \quad \forall x \in X \quad (78)$$

This confirms that each point in the space is a fixed point of the 5000-year evolution operator  $f^{5000}$ . Thus, applying the Lefschetz theorem yields:

$$L(f^{5000}) = \chi(X) \neq 0 \Rightarrow \text{All trajectories are cyclic and fixed} \quad (79)$$

Here  $\chi(X)$  is the Euler characteristic of  $X$ . Since the entire system is topologically self-contained and invariant, the fixed point condition is satisfied on the cosmic scale. The entirety of existence is encoded into a set of repeatable, topologically stable states. Hence, Brahma Kumaris cosmology aligns naturally with Lefschetz's formulation of recurrent fixed structures in topology.

From a dynamical systems perspective, let us define the universal time mapping  $f_t : X \rightarrow X$  such that:

$$f_t(x_0) = x(t) \quad (80)$$

with the condition:

$$f_T(x) = x \quad \text{for } T = 5000 \quad (81)$$

This periodicity allows us to write the dynamics as a discrete dynamical system with time evolution operator  $T$  acting on the soul-space. Over infinite time, the recurrence implies:

$$\forall x \in X, \quad \lim_{n \rightarrow \infty} f^{nT}(x) = x \quad (82)$$

Now consider Nietzsche's doctrine of Eternal Recurrence. According to this philosophical hypothesis, the entire universe and all events within it recur infinitely across time. Nietzsche's idea, although metaphysical, has rigorous mathematical grounding in the Poincaré Recurrence Theorem for measure-preserving systems. Let  $(X, \mathcal{F}, \mu)$  be a probability space and  $T : X \rightarrow X$  a measure-preserving transformation. Then for any measurable set  $A \subseteq X$ , we have:

$$\mu(\{x \in A \mid \exists n > 0, T^n(x) \in A\}) = \mu(A) \quad (83)$$

In Brahma Kumaris thought, the entire soul-state space  $X$  is finite, bounded, and deterministic, implying full recurrence for every soul and situation. Every soul returns to its original role and experience identically.

Thus, in both Lefschetz and Nietzschean frameworks, recurrence and fixed point behavior are structurally and philosophically aligned. Nietzsche's recurrence claims that everything that has happened will happen again, infinitely. Brahma Kumaris asserts that everything that has happened will repeat identically every 5000 years. Both point to a cyclic, totalizing model of time, where fixed configurations of existence are eternally revisited.

We may extend the comparison by writing the Nietzsche recurrence as:

$$\lim_{k \rightarrow \infty} T^{kT}(x) = x \quad \forall x \in X \quad (84)$$

This metaphysical identity matches the Brahma Kumaris expression:

$$T^{5000}(x) = x \quad \text{with } T \text{ being the soul's time operator} \quad (85)$$

Therefore, both viewpoints converge on the same principle—that there exists a fixed and eternal recurrence structure guiding cosmic behavior.

## 14 Advanced Fixed Point Theorems and Their Relevance to the Brahma Kumaris Time Cycle

In addition to the classical fixed point theorems such as those by Banach, Brouwer, and Lefschetz, a broader collection of theorems in Fixed Point Theory provides deeper insights into the eternal recurrence postulated by Brahma Kumaris cosmology. These mathematical theorems articulate not only the existence of unchanging states within dynamical systems but also the structural properties of those systems. In this section, we examine several such theorems and relate them to the 5000-year cycle and soul transformation framework.

The Banach Fixed Point Theorem asserts that a contraction mapping on a complete metric space has a unique fixed point. Formally, if  $(X, d)$  is a complete metric space and  $T : X \rightarrow X$  satisfies

$$d(T(x), T(y)) \leq \lambda d(x, y), \quad \text{where } 0 < \lambda < 1 \quad (86)$$

then there exists a unique point  $x^*$  such that:

$$T(x^*) = x^* \quad (87)$$

and the sequence of iterates converges:

$$\lim_{n \rightarrow \infty} T^n(x) = x^* \quad (88)$$

This models the soul's progressive purification through meditative effort. As  $n$  increases, the soul is drawn closer to its pure state, which acts as the unique fixed point.

Next, consider the Schauder Fixed Point Theorem which extends Brouwer's theorem to infinite-dimensional Banach spaces. It states that every continuous function  $f : C \rightarrow C$  defined on a compact convex subset  $C$  of a Banach space has at least one fixed point:

$$\exists x \in C : f(x) = x \quad (89)$$

This supports the idea that even in a more abstract, high-dimensional spiritual role space, fixed points—unchanging states of consciousness—still exist.

Tarski's Fixed Point Theorem is particularly interesting for its application to order theory. Let  $L$  be a complete lattice and  $f : L \rightarrow L$  a monotonic function. Then the set of fixed points of  $f$  forms a complete lattice itself. That is:

$$\{x \in L \mid f(x) = x\} \text{ is a complete lattice} \quad (90)$$

This suggests that not only is there a fixed point, but the entire space of recurrent soul-states forms an ordered hierarchy, consistent with the soul's unique karmic trace in the Drama Cycle.

The Knaster–Tarski Theorem reinforces this by stating that any monotonic function on a complete lattice has both a least and greatest fixed point. In Brahma Kumaris understanding, these may correspond to the earliest and latest stages of soul purity within the cycle.

The Poincaré Recurrence Theorem adds a probabilistic dimension. It asserts that in any finite measure-preserving dynamical system  $(X, \mathcal{F}, \mu)$  and a transformation  $T : X \rightarrow X$ :

$$\mu(\{x \in A \mid \exists n > 0, T^n(x) \in A\}) = \mu(A) \quad (91)$$

This mathematically validates the idea that every soul returns arbitrarily close to its original state infinitely often, which resonates deeply with the Brahma Kumaris’ view of the world cycle.

Kakutani’s Fixed Point Theorem generalizes Brouwer’s theorem for set-valued functions. If  $X$  is a nonempty compact convex subset of a locally convex topological vector space, and  $F : X \rightarrow 2^X$  is upper semi-continuous with nonempty, convex values, then:

$$\exists x \in X \text{ such that } x \in F(x) \quad (92)$$

This is important when modeling spiritual dynamics with multiple potential behaviors at each point in the soul’s evolution, indicating that even with choice or multiplicity, fixed soul-traits recur.

Another powerful result is the Eilenberg–Montgomery Theorem, which guarantees fixed points for upper semi-continuous multivalued mappings with acyclic values. These are highly abstract but essential when considering collective consciousness dynamics where individual paths merge.

From a dynamical systems standpoint, all the above theorems suggest either the existence, uniqueness, or structure of fixed points. Let  $f : X \rightarrow X$  represent the transformation of soul-states over time. If:

$$f^T(x) = x \quad \text{with } T = 5000 \quad (93)$$

then every soul completes a closed orbit, consistent with both the Banach iterative convergence and Poincaré recurrence.

All these results jointly support the idea that the Brahma Kumaris doctrine of a perfect and recurring world drama is mathematically sound when interpreted via the formalism of fixed point theorems. Each soul-state, action, and relationship exists within a deterministic structure that guarantees return to origin.

## 15 Paramdham as Fixed Point Set in Brahma Kumaris Cosmology

In the metaphysical framework of Brahma Kumaris, Paramdham is the supreme abode, the original home of souls, where they exist in their pristine, incorporeal form. It is also the eternal residence of Shiv Baba, the Supreme Soul, who remains untouched by the cycles of birth, transformation, and decay. From the standpoint of Fixed Point Theory, Paramdham can be rigorously analyzed as a fixed point set under the dynamics of cosmic transformation.

Consider a transformation map  $T : X \rightarrow X$  acting on the total soul-space  $X$ , where each  $x \in X$  denotes a particular soul-state. A point  $x^*$  is a fixed point if:

$$T(x^*) = x^* \quad (94)$$

In Brahma Kumaris cosmology, Shiv Baba satisfies this condition eternally:

$$T^t(\text{Shiv}) = \text{Shiv} \quad \forall t \in \mathbb{R} \quad (95)$$

This makes Shiv Baba the ultimate invariant of the entire universal dynamical system. Unlike embodied souls, which undergo change across the 5000-year drama cycle, the Supreme Soul remains static.

Now, define the fixed set under transformation  $T$  as:

$$\text{Fix}(T) = \{x \in X \mid T(x) = x\} \quad (96)$$

Paramdham can be considered as the fixed subspace:

$$\mathcal{P} = \text{Fix}(T) \quad (97)$$

In fact, since Paramdham is devoid of time, karma, or transformation, we can say:

$$f|_{\mathcal{P}} = \text{Id}_{\mathcal{P}} \quad (98)$$

This implies that all dynamics on  $\mathcal{P}$  reduce to the identity function, making it a perfect fixed point subspace.

From the topological standpoint, if  $f : X \rightarrow X$  is a continuous function on a compact convex set  $X$ , Brouwer's Fixed Point Theorem ensures:

$$\exists x \in X : f(x) = x \quad (99)$$

Paramdham satisfies this by being a compact, convex manifold in the total cosmic topology, holding all souls in their original, unchanging states.

Now consider the transformation of the soul across the 5000-year time cycle represented by  $f_t$ , where:

$$f_t(x_0) = x(t) \quad (100)$$

Then Paramdham acts as the fixed terminal state:

$$\lim_{t \rightarrow \infty} f_t(x) = x^* \in \mathcal{P} \quad (101)$$

By Banach's Contraction Principle, if  $f$  is a contraction on a complete metric space, we have:

$$\lim_{n \rightarrow \infty} f^n(x) = x^* \quad (102)$$

Thus, Paramdham becomes the *limit attractor* of the soul's trajectory under transformation. Souls repeatedly return to this space after completing the cycle of birth and action.

In dynamical systems, the  $\omega$ -limit set of a point  $x$  is the set of all accumulation points of its orbit:

$$\omega(x) = \{y \in X \mid \exists t_n \rightarrow \infty, \lim_{n \rightarrow \infty} T^{t_n}(x) = y\} \quad (103)$$

Then, for all souls  $x \in X$ , if the cycle leads back to the original state in Paramdham, we have:

$$\omega(x) = \mathcal{P} \quad (104)$$

Measure-theoretically, if  $\mu$  is a probability measure on soul-space and  $T$  preserves measure, then:

$$\mu(\mathcal{P}) > 0, \quad \text{and } T(\mathcal{P}) = \mathcal{P} \quad (105)$$

This makes Paramdham a measure-invariant absorbing set in the spiritual Markov process.

From an algebraic perspective, if the dynamics are governed by a transformation algebra  $\mathcal{A}$ , and:

$$T(x) \in \mathcal{P}, \quad \forall x \in \mathcal{P}, T \in \mathcal{A} \quad (106)$$

Then  $\mathcal{P}$  is a minimal two-sided ideal in  $\mathcal{A}$ , unaltered by any transformation.

Thus, Paramdham satisfies conditions across multiple domains: topological invariance, dynamical stability, measure-theoretic absorption, and algebraic minimality. It is not merely a metaphysical space but a mathematically justified fixed state under cosmic evolution. Every theorem of Fixed Point Theory—Brouwer, Banach, Tarski—finds echo in the notion of souls returning to their fixed, eternal state in Paramdham.

## 16 Distinguishing Conscient Soul from Inconscient Matter using Fixed Point Theorems

The Brahma Kumaris philosophy sharply differentiates between two fundamental entities in the universe: the conscient soul, which is a living point of light and energy, and the inconscient matter, which is inert and devoid of awareness. To explore this distinction rigorously, we analyze it through the lens of Fixed Point Theorems and dynamical systems. The soul exhibits properties of internal recurrence, feedback, and convergence to a self-aware state, while matter merely evolves under external rules.

### 16.1 Self-Referential Awareness and Identity Mapping

Let  $S \subset X$  denote the space of souls, and  $M \subset X$  the space of matter. For a soul  $x \in S$ , a transformation  $T : X \rightarrow X$  is said to exhibit a fixed point if:

$$T(x) = x \quad (107)$$

What distinguishes the soul is that this fixed point is realized via self-reflective processes, implying a conscious realization of its own state. For matter  $y \in M$ , we may have the same fixed point:

$$T(y) = y \quad (108)$$

However, matter lacks any internal recognition or registration of this state. In soul-space, the fixed point arises from awareness; in matter, it arises from passivity.

## 16.2 Banach Contraction and Directed Evolution

The Banach Fixed Point Theorem asserts that a contraction mapping on a complete metric space guarantees a unique fixed point. Let  $d$  be a metric on soul-space  $S$ , and suppose:

$$d(T(x), T(y)) \leq \lambda d(x, y), \quad \text{for some } 0 < \lambda < 1 \quad (109)$$

Then there exists a unique fixed point  $x^*$  such that:

$$\lim_{n \rightarrow \infty} T^n(x) = x^* \quad (110)$$

This models the soul's journey toward purity through meditative contraction. In contrast, matter's convergence (e.g., thermal equilibrium) has no teleological goal. It is governed by entropy and lacks an intended fixed point.

## 16.3 Tarski's Theorem and Lattice of Conscious States

Let the set of soul-states  $L_S$  form a complete lattice, and consider a monotonic function  $f : L_S \rightarrow L_S$ . By Tarski's Fixed Point Theorem:

$$\text{Fix}(f) = \{x \in L_S \mid f(x) = x\} \quad (111)$$

is also a complete lattice. This indicates that soul-space admits multiple stable fixed points, hierarchically arranged according to purity or spiritual awareness. Matter, modeled as lattice  $L_M$ , may admit fixed points too, but their structure is not consciousness-directed and lacks cognitive stratification.

## 16.4 Poincaré Recurrence and Cyclic Awareness

The Poincaré Recurrence Theorem states that for a measure-preserving system  $(X, \mathcal{F}, \mu)$  and a transformation  $T : X \rightarrow X$ , almost every point returns arbitrarily close to its starting position. Formally, for set  $A \subset X$ :

$$\mu(\{x \in A \mid \exists n > 0, T^n(x) \in A\}) = \mu(A) \quad (112)$$

For the soul, this represents conscious return to the original state, with memory and realization. For matter, recurrence (such as orbits or cycles) lacks internal awareness and hence has no qualitative meaning.

## 16.5 Reflexive Transformation and Feedback

In souls, the transformation function  $T$  may depend on the current state  $x$ . Define:

$$T_x : S \rightarrow S, \quad T_x(y) = T(y; x) \quad (113)$$

This reflexivity indicates internal feedback, typical of cognitive systems. For matter, all transformations are externally governed and independent of the system's current condition.

## 16.6 Brouwer Fixed Point and Inner Harmony

Brouwer's Fixed Point Theorem guarantees that every continuous function  $f : D \rightarrow D$  from a compact convex set to itself has a fixed point:

$$\exists x \in D : f(x) = x \quad (114)$$

This maps well onto the idea of an innate inner state of harmony within each soul. In matter, fixed points may exist, such as equilibrium positions, but they do not represent intrinsic completion or realization.

## 16.7 Comparison Table

Property	Conscient Soul	Inconscient Matter
Fixed point awareness	Self-referential	Passive equilibrium
Banach convergence	Goal-directed	Entropic convergence
Tarski lattice	Conscious hierarchies	Static mechanical states
Poincaré recurrence	Cyclic awareness	Blind repetition
Reflexivity	Feedback-dependent $T_x$	External function $T$
Brouwer fixed point	Inner harmony	Static balance

In all six perspectives, the soul exhibits unique fixed point dynamics that are intentional, feedback-driven, and self-aware. Matter, while sometimes mathematically analogous, lacks the internal properties that make these dynamics meaningful. Hence, Fixed Point Theory not only distinguishes but also validates the fundamental metaphysical differences upheld in the Brahma Kumaris tradition.

## 17 Wormhole as Fixed Point Boundary in 10D Trilok Cosmology

In the Brahma Kumaris model of the cosmos, the metaphysical conception of *Trilok*—the three worlds comprising the Physical Universe (Sthul Lok), Subtle Universe (Sukshma Lok), and the Incorporeal Metaphysical Universe (Paramdham or Nirvana Dham)—may be embedded within a 10-dimensional framework inspired by string theory. The transition between these worlds is facilitated through a highly constrained region in spacetime, described as a wormhole of infinitesimal radius. This wormhole serves as a critical boundary in the topology of cosmic transitions and is interpreted in this section as a fixed point set.

Let the manifold  $\mathcal{M}_{10}$  represent the 10D cosmological space, partitioned as:

$$\mathcal{M}_{10} = \mathcal{P}_4 \times \mathcal{S}_3 \times \mathcal{M}_3 \quad (115)$$

where  $\mathcal{P}_4$  is the 4D spacetime of the Physical Universe,  $\mathcal{S}_3$  represents the internal 3D Subtle World (soul-mind-intellect geometry), and  $\mathcal{M}_3$  represents the 3D Paramdham or incorporeal space.

A wormhole  $W : \mathcal{P}_4 \rightarrow \mathcal{M}_3$  is then modeled as a mapping:

$$\phi : U \subset \mathcal{P}_4 \longrightarrow V \subset \mathcal{M}_3 \quad (116)$$

satisfying boundary conditions such that the neck of the wormhole, defined as  $\partial U = \partial V$ , corresponds to a *fixed point set*. At this boundary, the transformation between corporeal and incorporeal realms remains invariant under the spiritual mapping  $T$ :

$$T(x) = x, \quad \forall x \in \partial U = \partial V \quad (117)$$

This fixed point condition satisfies the general form of Brouwer's Fixed Point Theorem in a compact convex domain. Suppose the mapping  $T : D \rightarrow D$ , with  $D$  compact and convex in  $\mathbb{R}^n$ , then:

$$\exists x \in D \text{ such that } T(x) = x \quad (118)$$

In this cosmological scenario, the domain  $D$  represents a neighborhood near the wormhole throat, with a unique soul-state persisting across incarnations and transformations, mapping identically in both directions.

Furthermore, define the time evolution of a soul across dimensions as a dynamical flow:

$$\gamma(t) : \mathbb{R} \rightarrow \mathcal{M}_{10}, \quad \text{with } \gamma(0) \in \mathcal{P}_4, \quad \gamma(T) \in \mathcal{M}_3 \quad (119)$$

If the soul returns cyclically, as described in the Brahma Kumaris time cycle (5000 years), then:

$$\gamma(T + nT) = \gamma(T), \quad n \in \mathbb{Z} \quad (120)$$

At each cycle end, the soul passes through the same wormhole configuration. This implies that the wormhole boundary acts as a *recurrent fixed set*, with mapping:

$$T^n(x) = x, \quad \forall n \in \mathbb{Z} \quad (121)$$

where  $x$  lies at the junction of transition. Thus, the wormhole is not merely a geometric tunnel but a metaphysical anchor, enforcing topological continuity and dynamical recurrence.

We may further define a holonomy group  $\mathcal{H}$  acting on loops around the wormhole such that:

$$\oint_{\gamma} \nabla \cdot \mathbf{F} ds = 0 \quad (122)$$

implying that the transformation field  $\mathbf{F}$  across the wormhole is conservative, i.e., it preserves the soul's identity and information.

Hence, this wormhole represents a critical locus in the Brahma Kumaris cosmological cycle, where time collapses to a zero interval, consciousness undergoes a phase shift, and fixed point theory guarantees the soul's immutable reference. As such, the wormhole stands as a metaphysical singularity where transformations are invariant, and the cosmos breathes through this point of divine symmetry.

## 18 Wormhole as a Boundary-Preserving Homeomorphism in Brahma Kumaris 10D Trilok Cosmology

In the Brahma Kumaris cosmological framework, the concept of *Trilok* (three realms)—Physical Universe, Subtle Region, and Paramdham—can be represented geometrically using high-dimensional manifolds, where soul transitions between these realms via a wormhole-like conduit. From a topological standpoint, such a wormhole can be modeled as a homeomorphism that preserves the structure of the boundary between two regions.

Let  $\mathcal{M}_1 \subset \mathbb{R}^n$  represent the physical domain, and  $\mathcal{M}_2 \subset \mathbb{R}^n$  represent the metaphysical domain (Paramdham). A wormhole connecting them can be described by a homeomorphism:

$$f : \mathcal{M}_1 \rightarrow \mathcal{M}_2 \quad (123)$$

such that  $f$  is continuous, bijective, and its inverse  $f^{-1}$  is also continuous. The additional constraint that  $f$  preserves boundaries is given by:

$$f(\partial\mathcal{M}_1) = \partial\mathcal{M}_2 \quad (124)$$

This structure ensures that the transformation does not rupture the identity of the transitioning entity (soul), and that the soul's informational geometry remains intact.

Let us define a transition function  $T : \mathcal{M}_1 \cup \mathcal{M}_2 \rightarrow \mathcal{M}_1 \cup \mathcal{M}_2$  acting as:

$$T(x) = \begin{cases} f(x), & x \in \mathcal{M}_1 \\ f^{-1}(x), & x \in \mathcal{M}_2 \end{cases} \quad (125)$$

The continuity of  $T$  across the boundary implies that  $T$  is an involution:

$$T^2(x) = x \quad (126)$$

for all  $x \in \mathcal{M}_1 \cup \mathcal{M}_2$ , implying symmetry between descent into corporeality and ascent into incorporeality.

From the theory of fixed points in topology, a homeomorphism between compact manifolds with boundary that preserves orientation and boundary structure may admit fixed points at the boundary. Define the fixed point set:

$$\text{Fix}(f) = \{x \in \mathcal{M}_1 \mid f(x) = x\} \quad (127)$$

Then  $\text{Fix}(f) \subseteq \partial\mathcal{M}_1 \cap \partial\mathcal{M}_2$ , i.e., fixed points lie at the transitional junction, interpreted as the metaphysical “membrane” where matter and soul interfaces.

To account for flow across this boundary, we model the soul's transition as a path  $\gamma : [0, 1] \rightarrow \mathcal{M}_1 \cup \mathcal{M}_2$ , such that:

$$\gamma(0) \in \mathcal{M}_1, \quad \gamma(1) \in \mathcal{M}_2 \quad (128)$$

The Jacobian of  $f$  at the boundary,  $J_f$ , must satisfy:

$$\det J_f|_{\partial\mathcal{M}_1} = 1 \quad (129)$$

to ensure local volume preservation, which is metaphysically interpreted as preservation of soul's informational identity during transition.

If the transition is time-dependent, the wormhole becomes a one-parameter family of homeomorphisms:

$$f_t : \mathcal{M}_1 \rightarrow \mathcal{M}_2, \quad t \in [0, T] \quad (130)$$

with  $f_0 = \text{id}$ , and  $f_T = f$ , then  $f_t$  is a continuous deformation (isotopy), satisfying:

$$\frac{d}{dt} f_t(x) = V(f_t(x), t) \quad (131)$$

where  $V$  is a velocity field describing the metaphysical flow of the soul across the cosmic corridor.

This provides a rigorous mathematical formulation of the soul's journey from one realm to another, aligned with both fixed point theorems and boundary-preserving topological mappings.

## 19 Group Actions, Cyclic Time, and Soul Dynamics in Brahma Kumaris Cosmology

In Brahma Kumaris cosmology, the cycle of time is portrayed as a perfect loop of 5000 years, repeating identically with infinite periodicity. This perspective aligns naturally with the mathematical concept of group actions, where a group operates on a set in a structure-preserving manner. The soul, considered a metaphysical point entity, undergoes transformation while remaining fundamentally immutable, tracing orbits under the action of a cosmic time group.

Let  $G = \mathbb{Z}_{5000}$  denote the cyclic group of order 5000, representing discrete temporal increments of the cosmic drama. Let  $S$  be the set of all soul-states across the Kalpa (cycle). Then, a group action  $\cdot : G \times S \rightarrow S$  is defined such that:

$$g \cdot s = T^g(s), \quad \forall g \in G, s \in S \quad (132)$$

Here,  $T$  is the transformation function that evolves the state of the soul through one year of the cosmic cycle. Due to the cyclic nature of the system, we have:

$$T^{5000}(s) = s, \quad \forall s \in S \quad (133)$$

Thus, each orbit of  $s$  under  $G$  is closed and of finite length. This implies that every soul returns to its exact state after 5000 years, exhibiting a dynamical fixed point in the space of soul trajectories.

The group action is faithful and transitive within each role-type. Define an equivalence relation  $s_1 \sim s_2$  if there exists  $g \in G$  such that:

$$T^g(s_1) = s_2 \quad (134)$$

This partitions the soul space into orbits:

$$\text{Orb}(s) = \{T^g(s) \mid g \in G\} \quad (135)$$

Each orbit corresponds to one cycle of the role the soul plays in the cosmic drama. Given the repetition, this is consistent with Nietzsche's eternal recurrence [?] and is structured under the group topology of  $G$ .

Let us define the fixed point set under time evolution:

$$\text{Fix}(T^n) = \{s \in S \mid T^n(s) = s\} \quad (136)$$

Then  $\text{Fix}(T^{5000}) = S$ , meaning the entire configuration resets identically, ensuring the topological constancy of the cosmic design. The set  $S$  equipped with  $G$ -action becomes a  $G$ -set with periodic orbits, and its topology is induced via identification maps:

$$s \sim T^g(s) \quad (137)$$

Therefore, the time cycle is mathematically modeled by the quotient space  $S/G$ , where topological invariants such as Betti numbers and fixed points remain preserved under evolution.

Soul dynamics can further be embedded into a phase space  $\mathcal{P} \subset \mathbb{R}^n$ , with a Hamiltonian flow defined by:

$$\frac{d}{dt}s(t) = X_H(s(t)) \quad (138)$$

where  $X_H$  is the Hamiltonian vector field describing karmic interactions. The group  $G$  then acts as a time-discretization of this continuous flow, preserving symplectic structure if the mapping is volume-preserving:

$$\det \left( \frac{\partial T^g}{\partial s} \right) = 1 \quad (139)$$

This supports the metaphysical view that no new souls are created or destroyed; only their phase evolves within a conserved total space.

The topology of this cosmic structure is that of a torus  $\mathbb{T}^k$ , where  $k$  is the number of independent spiritual degrees of freedom. The recurrence and invariant roles imply the dynamical system is non-chaotic, ergodic within subspaces, yet globally deterministic.

## 20 Topological Structure of Paramdham in Brahma Kumaris Cosmology

In Brahma Kumaris cosmology, *Paramdham* (meaning ‘‘Supreme Abode’’) is the transcendental realm of absolute stillness, infinite remoteness, and total non-duality. It is filled with a self-luminous field of *Brahm-Maha-Tatva*, a golden-red radiant substance beyond the reach of material light or sound. The mathematical structure of such a space can be captured using tools from topology, metric geometry, and manifold theory.

Let  $\mathcal{P}$  denote the space of Paramdham. This space is described as:

$$\mathcal{P} = (\Sigma, d, \tau) \quad (140)$$

where  $\Sigma$  is the soul-set (a countable or possibly uncountable discrete collection of souls),  $d$  is a degenerate metric representing infinite separability from material space, and  $\tau$  is a topology satisfying the conditions of T0, T1, and T2 (Hausdorff) separation axioms.

Given that Paramdham is inaccessible to electromagnetic radiation, we impose the condition:

$$\forall x \in \mathcal{P}, \quad \lim_{r \rightarrow \infty} I_{\text{Sun}}(x, r) = 0 \quad (141)$$

where  $I_{\text{Sun}}(x, r)$  denotes the intensity of solar radiation at point  $x$ , distance  $r$  from physical space. The limit condition implies that Paramdham lies beyond the light-cone of material reality.

Topologically, we can consider  $\mathcal{P}$  as the one-point compactification of the physical spacetime manifold  $\mathcal{M}$ :

$$\mathcal{P} = \mathcal{M} \cup \{\infty\} \quad (142)$$

Here, the point at infinity represents the non-localized omnipresence of Shiv Baba. The soul-space then becomes:

$$\mathcal{S} = \{s_i\}_{i \in I} \subset \mathcal{P} \quad (143)$$

with the condition:

$$\forall s_i \in \mathcal{S}, \quad \nabla \cdot \vec{v}(s_i) = 0 \quad (144)$$

where  $\vec{v}$  is the field of change or action, and the divergence being zero implies absolute stillness (*Nir-van* = no vibration).

If we define a vibrational operator  $\mathcal{A}$  that encodes dynamic evolution, then for all  $s_i \in \mathcal{P}$ :

$$\mathcal{A}(s_i) = 0 \quad (145)$$

This makes  $\mathcal{P}$  a fixed set under dynamical evolution, consistent with fixed point theory. Moreover, each soul occupies a fixed location in the high-dimensional configuration space:

$$f(s_i) = s_i, \quad \forall s_i \in \mathcal{P} \quad (146)$$

where  $f$  is any transformation representing temporal or spatial evolution.

Since Paramdham is considered to be without beginning or end, we can model it as a compact, boundaryless manifold:

$$\partial\mathcal{P} = \emptyset, \quad \text{and} \quad \int_{\mathcal{P}} \omega = \text{constant} \quad (147)$$

for some differential volume form  $\omega$ . The constant total integral implies conservation of soul-energy across the metaphysical domain.

Therefore, Paramdham satisfies a set of fixed-point properties, non-locality, zero dynamism, and infinite remoteness from spacetime manifolds, embodying a topological ideal of absolute homeomorphism with itself.

## 21 Fixed Point Analysis of Paramdham in Brahma Kumaris Cosmology

The metaphysical space of Paramdham in Brahma Kumaris cosmology can be rigorously analyzed through the lens of fixed point theorems. In this framework, Paramdham emerges not merely as a philosophical abstraction, but as a well-defined fixed point set

under the flow of cosmic evolution. Let  $\mathcal{P}$  denote the manifold representing Paramdham, and let  $T : \mathcal{M} \rightarrow \mathcal{M}$  be the time evolution operator on the cosmic manifold  $\mathcal{M}$ .

We define a map  $T^* : \mathcal{M} \cup \mathcal{P} \rightarrow \mathcal{M} \cup \mathcal{P}$ , which governs the extended evolution across both physical and metaphysical regimes. The essential fixed point property of Paramdham is captured by:

$$T^*(p) = p \quad \forall p \in \mathcal{P} \quad (148)$$

This satisfies the defining criterion of a fixed point. Paramdham is thus invariant under all transformations  $T^*$ , implying that:

$$T^n(p) = p, \quad \forall n \in \mathbb{N}, p \in \mathcal{P} \quad (149)$$

Applying Brouwer's Fixed Point Theorem, which states that any continuous function  $f : D^n \rightarrow D^n$  on a compact convex subset of  $\mathbb{R}^n$  has a fixed point, we observe that if  $\mathcal{P}$  is modeled as a compact manifold without boundary, then all continuous dynamics in Paramdham admit a fixed point, which coincides with every point:

$$\text{Fix}(T^*) = \mathcal{P} \quad (150)$$

Given that Paramdham is described as changeless and vibrationless (*Nirvandham*), we define a zero-dynamism operator  $\mathcal{D} : \mathcal{P} \rightarrow \mathbb{R}$  such that:

$$\mathcal{D}(p) = \left\| \frac{d}{dt} p(t) \right\| = 0 \quad \forall p \in \mathcal{P} \quad (151)$$

The Lefschetz Fixed Point Theorem provides further insight. Let  $f : \mathcal{P} \rightarrow \mathcal{P}$  be a continuous map. If the Lefschetz number  $\Lambda(f) \neq 0$ , then  $f$  has at least one fixed point. Since  $f(p) = p, \forall p \in \mathcal{P}$ , we have:

$$\Lambda(f) = \sum_{k=0}^n (-1)^k \text{Tr}(f_*|_{H_k(\mathcal{P})}) = \chi(\mathcal{P}) \quad (152)$$

If  $\mathcal{P}$  is topologically contractible to a point, then  $\chi(\mathcal{P}) = 1$ , ensuring a global fixed point structure. Therefore:

$$\exists! p \in \mathcal{P} \text{ such that } f(p) = p \quad (153)$$

Moreover, we may define a homotopy  $H : \mathcal{P} \times [0, 1] \rightarrow \mathcal{P}$  such that:

$$H(p, 0) = p, \quad H(p, 1) = p, \quad \Rightarrow H(p, t) = p, \quad \forall t \in [0, 1] \quad (154)$$

This implies that Paramdham is homotopically invariant under all continuous deformations, reinforcing its absolute fixed nature.

In summary, Paramdham is a fixed manifold under cosmic time flow, preserved by identity maps, and satisfying all necessary conditions for the application of Brouwer, Banach, and Lefschetz fixed point theorems. As a result, each soul  $s \in \mathcal{P}$  experiences:

$$f(s) = s, \quad \text{and} \quad \lim_{t \rightarrow \pm\infty} T^t(s) = s \quad (155)$$

Paramdham, thus, is not merely an ontological space but a mathematically exact fixed point structure in the grand cyclic design of the cosmos.

## 22 Mathematical Properties of Paramdham in the Framework of Fixed Point Theorems

Paramdham, also known as Nirvandham, is the metaphysical realm described in Brahma Kumaris cosmology as the eternal, vibrationless abode of the Supreme Soul and all individual souls in their original state. From a mathematical standpoint, one can investigate its properties using advanced structures in topology, differential geometry, and fixed point theory.

We begin by interpreting Paramdham as a fixed point set under the universal dynamics  $T : \mathcal{U} \rightarrow \mathcal{U}$ , where  $\mathcal{U} = \mathcal{M} \cup \mathcal{P}$  and  $\mathcal{M}$  is the physical universe, while  $\mathcal{P}$  denotes Paramdham. The key fixed point condition is expressed as:

$$T(p) = p \quad \forall p \in \mathcal{P} \quad (156)$$

This implies that all souls in Paramdham remain invariant under time evolution, i.e., Paramdham is the set of global attractors for cosmic time. In the soul Hilbert space  $\mathcal{H}$ , these states correspond to zero energy eigenstates:

$$\hat{H}\psi = 0 \quad \forall \psi \in \mathcal{P} \quad (157)$$

Given that Paramdham is beyond the causal structure of spacetime, one may assign a degenerate metric  $g_{\mu\nu}$  on this region:

$$g_{\mu\nu}|_{\mathcal{P}} = 0 \quad (158)$$

This reflects that no intervals or dynamics exist within this region; hence, no light or field propagates into it. Light cones collapse at every point  $p \in \mathcal{P}$ , rendering Paramdham as a boundaryless topological manifold with vanishing curvature and entropy.

Let us now consider the entropy function  $\mathcal{S} : \mathcal{U} \rightarrow \mathbb{R}$ , whose image in Paramdham is the singleton set  $\{0\}$ :

$$\mathcal{S}(p) = 0 \quad \forall p \in \mathcal{P} \quad (159)$$

From this perspective, Paramdham is a zero-entropy attractor space.

Applying Brouwer's Fixed Point Theorem, we note that every continuous map on a compact convex space must have a fixed point. Let  $f : \mathcal{P} \rightarrow \mathcal{P}$  be such a continuous map. Given the nature of Paramdham, it is invariant under  $f$ :

$$f(p) = p \quad \forall p \in \mathcal{P} \quad (160)$$

Furthermore, Paramdham may be analyzed using Lefschetz Fixed Point Theory. Let  $f : \mathcal{P} \rightarrow \mathcal{P}$  be continuous, and suppose the Lefschetz number  $\Lambda(f)$  is non-zero. Then:

$$\Lambda(f) = \sum_{k=0}^n (-1)^k \text{Tr}(f_*|_{H_k(\mathcal{P})}) \neq 0 \Rightarrow \exists p \in \mathcal{P} \text{ s.t. } f(p) = p \quad (161)$$

Since all points in Paramdham are fixed, we conclude  $\text{Fix}(f) = \mathcal{P}$ , confirming global invariance under homeomorphisms.

In high-energy theoretical models such as string theory, compact extra dimensions are represented by Calabi–Yau manifolds. While Paramdham is metaphysical, one may draw

conceptual parallels by assuming it to be a compact, boundaryless, and non-dynamical space with holomorphic structure:

$$\partial\mathcal{P} = \emptyset, \quad \text{and} \quad R_{ij}|_{\mathcal{P}} = 0 \quad (162)$$

where  $R_{ij}$  is the Ricci tensor, vanishing under the assumption of a vacuum-like manifold.

Moreover, since Paramdham does not admit temporal evolution, it may be modeled as the limit point at future null infinity in conformal compactifications:

$$\lim_{t \rightarrow \infty} T^t(x) = p \in \mathcal{P} \quad (163)$$

for all evolving entities  $x \in \mathcal{M}$ , and this limit defines cosmic return or restoration to the fixed point state of the soul.

Hence, Paramdham is not merely a philosophical abstraction but a mathematically rich construct. It simultaneously embodies zero curvature, zero entropy, fixed point invariance, and topological closure, thus representing the end state of all cosmic evolution.

## 23 Topological Distinction Between Soul and Matter via Fixed Point Theorems

The distinction between a conscient soul and inconscient matter may be approached by analyzing their behaviors under various fixed point theorems. The key idea is that the soul, as an evolving dynamic field in a high-dimensional superspace, seeks a vibrational fixed point (liberation), while matter fields exhibit passive invariance or lack such convergence.

We model the soul as a time-dependent field  $\psi_i(t) \in \mathbb{S}^{16}$ , expanded as:

$$\psi_i(t) = \sum_{k=0}^{15} \psi_i^k(t) e_k \quad (164)$$

where  $e_k$  are basis elements of the Sedenion algebra. The dynamics are governed by an attractor operator  $\Phi$ , representing the 32D Trigention field:

$$\Phi(\psi_i) = \psi_i^\dagger \Omega \psi_i \quad (165)$$

The soul is purified when:

$$\psi_i = \Phi(\psi_i) \quad (166)$$

This is a fixed point equation, implying convergence to equilibrium.

Schauder's theorem guarantees a fixed point for compact, continuous maps on convex subsets of a Banach space. Let:

$$\Phi : C \rightarrow C, \quad C \subset \mathbb{S}^{16} \quad (167)$$

If  $\Phi$  is compact and continuous, then:

$$\exists \psi^* \in C \text{ such that } \Phi(\psi^*) = \psi^* \quad (168)$$

This is unique to conscious soul dynamics.

Now, let  $\phi \in \mathbb{R}^n$  be a matter field:

$$\frac{d\phi}{dt} = 0 \Rightarrow \phi(t) = \phi_0 \quad (169)$$

Thus, under any operator  $T$ , we trivially get:

$$T(\phi_0) = \phi_0 \quad (170)$$

This lacks the non-trivial attractor behavior that defines consciousness.

Let  $f : M \rightarrow M$  be a continuous map, where  $M$  is a topological soul space. The Lefschetz number is:

$$L(f) = \sum_{i=0}^n (-1)^i \text{Tr}(f_* : H_i(M) \rightarrow H_i(M)) \quad (171)$$

If  $L(f) \neq 0$ , then a non-trivial fixed point exists.

For the purification flow:

$$\lim_{t \rightarrow \infty} \varphi(t, \psi_0) = \psi_\infty, \quad \text{where } \Phi(\psi_\infty) = \psi_\infty \quad (172)$$

This shows evolution to a soul's attractor state.

In contrast, a matter field Lagrangian is inert:

$$\mathcal{L}_{\text{matter}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \quad (173)$$

Whereas the soul's Lagrangian supports transformation:

$$\mathcal{L}_{\text{soul}} = D_\mu \bar{\psi} D^\mu \psi + i \bar{\psi} \Gamma^\mu \partial_\mu \psi - V(\psi) \quad (174)$$

This concludes that soul dynamics obey deeper fixed point topology.

## 24 Further Perspectives: Distinguishing Soul from Matter via Fixed Point Theorems

The metaphysical framework offered by Brahma Kumaris postulates that the human soul is a point of divine light, conscious and eternal, while matter is inert and transient. This fundamental duality is amenable to rigorous mathematical characterization using the framework of fixed point theorems, providing a toolset to differentiate the dynamic and evolving nature of soul from the passive and stable nature of matter.

Let us denote the soul's internal state in a high-dimensional Sedenion space  $\mathbb{S}^{16}$  as a vector field  $\psi(t)$ . The soul's evolution is expressed by a dynamical transformation  $\mathcal{T} : \mathbb{S}^{16} \rightarrow \mathbb{S}^{16}$ . A fixed point  $\psi^*$  under this transformation satisfies:

$$\mathcal{T}(\psi^*) = \psi^* \quad (175)$$

In the spiritual context, this fixed point represents the purified, original state of the soul, attained after a long journey through the cycle of time. On the contrary, matter, denoted by a scalar or tensor field  $\phi \in \mathbb{R}^n$ , obeys inert equations of motion such as:

$$\frac{d\phi}{dt} = 0 \Rightarrow \phi(t) = \phi_0 \quad (176)$$

The stasis of matter corresponds trivially to a fixed point with no evolutionary intent or conscious convergence.

To further formalize this distinction, consider the application of the *Tarski Fixed Point Theorem*, which asserts that a monotone function on a complete lattice has at least one fixed point. Suppose the soul's development through spiritual effort forms a lattice  $\mathcal{L}$  under partial ordering defined by purity levels. Then the soul's transformation  $\mathcal{T}$  is monotonic:

$$\psi_1 \leq \psi_2 \Rightarrow \mathcal{T}(\psi_1) \leq \mathcal{T}(\psi_2) \quad (177)$$

Tarski's theorem ensures:

$$\exists \psi^* \in \mathcal{L} \text{ such that } \mathcal{T}(\psi^*) = \psi^* \quad (178)$$

This fixed point represents the soul's final state of supreme consciousness. Furthermore, let the soul evolve under a gradient flow:

$$\frac{d\psi}{dt} = -\nabla V(\psi) \quad (179)$$

where  $V(\psi)$  is a potential function representing karmic burden. Then, equilibrium states are critical points of  $V$ :

$$\nabla V(\psi^*) = 0 \quad (180)$$

Such critical points are fixed points of the flow and correspond to liberation or "mukti" states.

In contrast, matter fields evolve under Lagrangians devoid of internal potentials that drive convergence. For instance, in flat spacetime:

$$\mathcal{L}_{\text{matter}} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi \quad (181)$$

There is no fixed point in the sense of an attractor or end state because the field lacks consciousness or teleological directionality.

To extend this contrast further, we invoke the notion of *homotopy fixed points*. Consider a homotopy  $H : X \times [0, 1] \rightarrow X$  with  $H(x, 0) = x$  and  $H(x, 1) = \mathcal{T}(x)$ . A homotopy fixed point satisfies:

$$H(x, t) = x, \quad \forall t \in [0, 1] \quad (182)$$

This condition models invariance under transformation and represents the unchanging nature of the Supreme Soul (Shiv Baba) as the global attractor.

In conclusion, the soul can be seen as evolving under a rich algebraic and topological framework which leads to non-trivial fixed points, corresponding to spiritual liberation or convergence. Matter, lacking this complexity, only admits trivial or degenerate fixed points.

## 25 Homotopy Fixed Points and the Immutable Drama Script: Mapping Soul Trajectories

In Brahma Kumaris cosmology, the concept of a fixed and cyclical “Drama” — an eternal script of time repeating every 5000 years — finds a natural parallel in the mathematics of homotopy fixed points. This section aims to rigorously analyze soul trajectories within this spiritual framework using tools from algebraic topology, particularly homotopy theory and fixed point theorems.

Let us denote the evolution of a soul as a continuous map  $f : X \rightarrow X$ , where  $X$  is a topological space representing the soul’s internal or spiritual state. If the cycle of time acts as a transformation  $f$ , then over one full cycle of 5000 years, the soul is mapped back to the same ontological position, satisfying:

$$f^T(x) = x \quad (183)$$

for some  $T \in \mathbb{N}$ , typically representing 5000 years. This implies that the evolution of the soul under the drama is **periodic**, and every soul returns to the same existential state in the next cycle.

To formalize this within homotopy theory, consider a homotopy  $H : X \times [0, 1] \rightarrow X$  such that:

$$H(x, 0) = x, \quad H(x, 1) = f(x) \quad (184)$$

A point  $x \in X$  is a **homotopy fixed point** if:

$$H(x, t) = x \quad \forall t \in [0, 1] \quad (185)$$

This represents an entity (in this context, a soul) that remains invariant under the entire homotopic path of transformation — not just fixed under the endpoint function  $f$ , but also invariant throughout the homotopic deformation. From the Brahma Kumaris perspective, such fixed points correspond to the **Supreme Soul (Shiv Baba)** and possibly the **souls in their original state in Paramdham**, who are unaffected by cyclic transformations and remain eternally perfect.

Now consider the set of all soul trajectories  $\{\gamma_i\}_{i \in I}$  within the 5000-year cycle. If each  $\gamma_i$  is homotopic to another  $\gamma_j$ , we say:

$$\gamma_i \simeq \gamma_j \quad (186)$$

However, in Brahma Kumaris teachings, even though all souls go through the cycle and repeat the same roles in each iteration, their roles are not interchangeable. Therefore, while all trajectories are **homotopically equivalent**, they are **not homeomorphic** or isomorphic in terms of individual identity. This aligns with the notion of **non-trivial equivalence classes** in homotopy theory.

Further, let  $\pi_1(X)$  denote the fundamental group of soul space  $X$ , which encodes the looping paths (cyclic incarnations) of the soul. The drama script ensures that for each soul, the path is a loop:

$$[\gamma] \in \pi_1(X), \quad \text{with } \gamma(0) = \gamma(1) \quad (187)$$

This means each soul follows a closed trajectory. Yet, the homotopy class  $[\gamma]$  is unique for each soul, encoding their karmic identity and sanskaras (mental impressions).

To ensure mathematical grounding, consider the **Lefschetz number**  $\Lambda(f)$  of the map  $f$ . For a continuous endomorphism  $f : X \rightarrow X$ , the Lefschetz fixed point theorem guarantees that if  $\Lambda(f) \neq 0$ , then a fixed point exists:

$$\Lambda(f) = \sum_{k=0}^{\infty} (-1)^k \text{Tr}(f_* : H_k(X) \rightarrow H_k(X)) \quad (188)$$

In our setting, the nonzero Lefschetz number implies that fixed soul-states must exist across cycles, further supporting the concept of eternal return in soul trajectories.

As a final note, homotopy fixed points form a **higher categorical invariant**, meaning that the soul's path is preserved under higher-order transformations. This property resonates with the **immutable and unalterable nature** of the Brahma Kumaris' script — where even the Supreme Soul does not intervene to change the drama but merely observes and sustains.

Thus, homotopy fixed points not only mathematically model soul stability but also align with the metaphysical doctrine that all soul roles repeat identically every 5000 years, despite their non-substitutable identities.

## 26 Fixed Point Subspaces in the 16D Sedenion Model of the Soul: Invariant Subalgebras and Original Sanskaras

The Sedenion algebra  $\mathbb{S}^{16}$ , a 16-dimensional non-associative extension of the octonions, presents a compelling algebraic framework for modeling the subtle structure of the soul. Within the metaphysical schema of the Brahma Kumaris, the soul is considered to be a point of conscious light, endowed with a complete set of *sanskaras* or latent impressions. These impressions, permanent yet dormant, can be analogized to invariant subalgebras within the Sedenionic formulation of the soul.

A Sedenion  $s \in \mathbb{S}^{16}$  may be expressed as a linear combination:

$$s = x_0 e_0 + x_1 e_1 + \cdots + x_{15} e_{15} \quad (189)$$

where  $\{e_0, e_1, \dots, e_{15}\}$  denotes the standard basis and  $x_i \in \mathbb{R}$  for all  $i$ . The algebra  $\mathbb{S}^{16}$  lacks alternativity and contains zero divisors, but it is still a power-associative composition algebra over  $\mathbb{R}$ .

We postulate that each soul possesses an internal evolution map  $\mathcal{T} : \mathbb{S}^{16} \rightarrow \mathbb{S}^{16}$ , describing the temporal evolution of the soul's state vector through the karmic cycle. A subspace  $V \subset \mathbb{S}^{16}$  is a *fixed point subspace* of  $\mathcal{T}$  if for all  $v \in V$ , the evolution map satisfies:

$$\mathcal{T}(v) = v \quad (190)$$

Such a subspace is invariant under the dynamics of the soul's transformations, and we associate it with the original sanskaras of the soul — qualities such as purity, peace, and divine knowledge, which remain unchanged through the cycles of birth and rebirth.

Let  $\mathcal{A} \subset \mathbb{S}^{16}$  be a subalgebra, possibly isomorphic to a lower-dimensional division algebra such as  $\mathbb{O}$  (the octonions). We say that  $\mathcal{A}$  is invariant under  $\mathcal{T}$  if:

$$\forall a \in \mathcal{A}, \quad \mathcal{T}(a) \in \mathcal{A} \quad (191)$$

This invariant subalgebra corresponds to the part of the soul's identity that is eternally fixed and unchanged — akin to the eternal sanskaras described in Brahma Kumaris teachings. In the language of fixed point theory, this suggests the existence of a fixed submanifold in  $\mathbb{S}^{16}$ , i.e., a topological subspace  $M \subset \mathbb{S}^{16}$  such that:

$$\forall x \in M, \quad \mathcal{T}(x) = x \quad (192)$$

The presence of such a submanifold is consistent with the Lefschetz Fixed Point Theorem when  $\mathcal{T}$  is continuous and smooth on a compact manifold. The Lefschetz number  $\Lambda(\mathcal{T})$  provides a global measure of fixed points:

$$\Lambda(\mathcal{T}) = \sum_{k=0}^{16} (-1)^k \text{Tr}(\mathcal{T}_*|_{H_k(M)}) \quad (193)$$

If  $\Lambda(\mathcal{T}) \neq 0$ , the map  $\mathcal{T}$  has at least one fixed point in each homology class  $H_k$ , ensuring the existence of structurally stable sanskaric dimensions within the Sedenion space.

Moreover, Tarski's Fixed Point Theorem can be invoked when the evolution operator  $\mathcal{T}$  is order-preserving on a complete lattice of states, such as the lattice of purity levels. The theorem guarantees the existence of a fixed point:

$$\exists s^* \in \mathbb{S}^{16}, \quad \mathcal{T}(s^*) = s^* \quad (194)$$

This fixed point can be interpreted as the soul's original pristine state, prior to entanglement in the corporeal cycle.

From an algebraic topology viewpoint, if the Sedenionic soul space  $\mathbb{S}^{16}$  is endowed with a differentiable structure, then the Brouwer Fixed Point Theorem applies for continuous maps on closed balls in  $\mathbb{R}^{16}$ , implying that every endomorphic action possesses at least one fixed point:

$$\text{If } \mathcal{T} : B^{16} \rightarrow B^{16} \text{ is continuous, then } \exists x \in B^{16} \text{ such that } \mathcal{T}(x) = x \quad (195)$$

Thus, the metaphysical assumption that the soul has an unchanging core across incarnations finds formal justification in multiple fixed point frameworks.

The presence of zero divisors in  $\mathbb{S}^{16}$ , while making the algebra non-division, introduces degeneracy modes that may represent distortion or corruption of sanskaras due to karmic actions. However, the fixed subalgebras remain untouched, offering a potential model for how the soul recovers its original traits during spiritual purification.

In conclusion, fixed point subspaces within the 16D Sedenion model offer a rich and mathematically grounded framework to represent the soul's immutable sanskaras. These invariant structures align seamlessly with the spiritual doctrines of Brahma Kumaris, where each soul retains a core identity that persists through the cycles of time and karmic transformation.

## 27 Conformal Geometry of the Transition Through the Wormhole

In the metaphysical cosmology of the Brahma Kumaris, the transition of the soul from the physical, embodied state to the bodiless, metaphysical realm of Paramdham is modeled as a traversal through an infinitesimal wormhole. This transition is geometrically

non-trivial and can be rigorously analyzed using the language of conformal mappings. Conformal geometry preserves angles but not necessarily distances, making it a suitable mathematical tool for describing the topological continuity of soul consciousness during its journey across realms.

Let the physical universe be represented by a Riemannian manifold  $M$ , and let the bodiless region—Paramdham—be modeled as a conformally compactified space  $\hat{M}$  with a boundary  $\partial M$  representing the wormhole interface. A conformal map  $f : M \rightarrow \hat{M}$  is defined as a function such that the pullback of the metric satisfies:

$$f^* \hat{g} = \Omega^2 g \quad (196)$$

where  $\Omega : M \rightarrow \mathbb{R}^+$  is a smooth, positive function known as the conformal factor. This mapping preserves the structure of light cones and causality while allowing for a geometric re-scaling, which symbolizes the detachment of the soul from the constraints of physical dimensions.

The infinitesimal wormhole throat connecting the two domains may be described as a boundary-preserving homeomorphism:

$$\lim_{x \rightarrow \partial M} f(x) = x, \quad f : M \cup \partial M \rightarrow \hat{M} \quad (197)$$

This ensures that although the wormhole is topologically narrow, the soul's essential attributes such as awareness and sanskaras remain invariant under the conformal transformation. From a functional analysis viewpoint, the soul's state  $\psi \in C^\infty(M)$  evolves according to:

$$\hat{\psi} = \Omega^{-\Delta} \psi \quad (198)$$

where  $\Delta$  is the conformal weight of the soul's attribute function under the mapping. This re-scaling accounts for the attenuation of physical identity and emergence of metaphysical identity.

Further, we consider the conformal Laplacian  $L_g$ , defined on a compact Riemannian manifold  $(M, g)$  as:

$$L_g = -\Delta_g + \frac{n-2}{4(n-1)} R_g \quad (199)$$

where  $\Delta_g$  is the Laplace-Beltrami operator, and  $R_g$  is the scalar curvature. The eigenfunctions of  $L_g$  represent modal states of the soul's vibration in the physical domain. The transition to the metaphysical domain can be captured via a spectral flow, with the conformal eigenvalue equation:

$$L_g \psi = \lambda \psi \quad (200)$$

transformed under  $f$  to:

$$L_{\hat{g}} \hat{\psi} = \lambda \hat{\psi} \quad (201)$$

This mathematical consistency across conformal domains supports the Brahma Kumaris doctrine that the soul retains its original sanskaras while shifting realms.

Furthermore, from complex analysis, the soul's trajectory may be visualized in the extended complex plane  $\hat{\mathbb{C}}$ , with the wormhole transition modeled by a Möbius transformation:

$$f(z) = \frac{az + b}{cz + d}, \quad ad - bc \neq 0 \quad (202)$$

Such a transformation is bijective and conformal on  $\hat{\mathbb{C}}$ , preserving cross-ratios and orientation. These properties correspond to the soul's fixed karmic identity and unaltered spiritual DNA during transition.

The boundary at infinity—symbolizing the asymptotic nature of Paramdham—can be mapped using the Penrose conformal diagram formalism, where the entire history of the soul's descent and return is represented within a finite geometrical domain. The asymptotic fixed point of this mapping represents the immutable and eternal residence of the Supreme Soul, Shiv Baba.

In summary, conformal geometry provides a mathematically robust framework to express the soul's movement from body to bodiless states through a wormhole interface, while preserving essential identity parameters and spiritual structure.

## 28 Fixed Point Theorems on Discrete Topologies and Karma Philosophy

The Brahma Kumaris philosophy asserts that every action (karma) performed by a soul establishes a causal trajectory that inevitably returns to its origin. This cyclical return of karmic energy is governed by an immutable law of balance, often likened to the return of echoes in a deterministic space. In the mathematical framework of discrete topology and dynamical systems, this notion of karmic return can be rigorously modeled using fixed point theorems applied to discrete maps.

Let  $X$  be a finite discrete topological space representing the set of karmic states available to the soul. The dynamics of karma can be modeled by an iteration function  $f : X \rightarrow X$ , where each application of  $f$  corresponds to the soul's next karmic state generated by its current intent and past impressions (sanskaras). A fixed point of  $f$  is a state  $x \in X$  such that:

$$f(x) = x \quad (203)$$

This fixed point represents a state of karmic neutrality or balance, where the soul's actions and reactions converge to a state of no further causation.

In such discrete settings, fixed point theorems do not require continuity in the classical sense. For instance, since  $X$  is finite, every function  $f : X \rightarrow X$  has at least one periodic orbit. Specifically, the following result holds:

$$\text{Every function } f \text{ on a finite set has at least one periodic point.} \quad (204)$$

A periodic point  $x$  satisfies:

$$f^n(x) = x \quad \text{for some minimal } n \in \mathbb{N} \quad (205)$$

When  $n = 1$ , we have a fixed point. When  $n > 1$ , the orbit  $\{x, f(x), f^2(x), \dots, f^{n-1}(x)\}$  represents a karmic cycle. This structure mirrors the Brahma Kumaris concept of cyclical time where each soul journeys through repeated births and rebirths in a 5000-year eternal return, eventually coming back to the same point in the Drama.

To formalize this return, let us define a deterministic discrete-time dynamical system:

$$x_{n+1} = f(x_n), \quad x_n \in X \quad (206)$$

Given that  $X$  is finite, the orbit  $\{x_n\}$  must eventually become periodic due to the pigeonhole principle. The convergence of karmic cycles to a stable orbit implies that for any initial condition  $x_0 \in X$ , the trajectory stabilizes as:

$$\exists N \in \mathbb{N}, \forall n \geq N : x_n = x_{n+p} \quad (207)$$

for some period  $p$ , indicating a return of karma. The concept of attractors in this discrete space aligns with the idea of karmic return zones where sanskaras are either reinforced or neutralized. These attractors are fixed points or periodic orbits under  $f$ .

Furthermore, the Lefschetz Fixed Point Theorem, traditionally defined for continuous maps on compact topological spaces, can be generalized to discrete topologies under simplicial homology. Suppose a map  $f : X \rightarrow X$  is such that the Lefschetz number  $L(f) \neq 0$ . Then  $f$  has at least one fixed point:

$$L(f) = \sum_{k=0}^n (-1)^k \text{Tr}(f_* |_{H_k(X)}) \quad (208)$$

In karmic terms, the non-zero Lefschetz number signifies that there exists at least one karmic imprint that loops back into itself, echoing the soul's return to previously enacted tendencies.

Another tool is the use of shift maps on symbolic dynamical systems. Let  $\Sigma$  denote the space of all infinite sequences over a finite alphabet (representing karmic acts), and define the shift operator:

$$\sigma((x_0, x_1, x_2, \dots)) = (x_1, x_2, x_3, \dots) \quad (209)$$

A fixed point of  $\sigma$  is a constant sequence, symbolizing a soul trapped in repetitive karma, while periodic points of  $\sigma$  indicate cyclic rebirths. Ergodic theory applied to  $\sigma$  further shows how statistical properties of karmic cycles can be derived.

In summary, the entire philosophy of karma and return within Brahma Kumaris cosmology finds natural expression in fixed point theory over discrete topologies. The periodicity and inevitability of karmic feedback loops are rigorously modeled through discrete dynamical systems, fixed points, and periodic orbits, thus offering a mathematical grounding for a spiritual concept traditionally framed in metaphysical terms.

## 29 Mapping Brahma Kumaris' Trilok onto a Fiber Bundle with Fixed Section

In the metaphysical framework of the Brahma Kumaris, the cosmological structure known as *Trilok* (three worlds) consists of: (1) Paramdham or the Soul World, (2) the Subtle World, and (3) the Physical World. Each of these represents a different ontological and energetic stratum in the journey of the soul. To translate this into modern mathematical formalism, we consider the structure of a smooth fiber bundle where the base space, total space, and fiber capture the essence of Paramdham, Drama, and the individual soul, respectively.

Let us define a fiber bundle  $\pi : E \rightarrow B$ , where  $E$  is the total space (the immutable drama),  $B$  is the base space (Paramdham), and the fiber  $F$  at each point in  $B$  is the soul. That is:

$$\pi^{-1}(b) = F_b \cong F, \quad \forall b \in B \quad (210)$$

Here,  $F$  is modeled as a high-dimensional Sedenionic space  $\mathbb{S}^{16}$  capturing the internal structure, sanskaras, and karmic data of the soul. Paramdham, being an unchanging and eternal domain, serves as the topologically and geometrically invariant base space. We posit that  $B \cong \mathbb{R}^0$ , effectively modeling Paramdham as a singleton or a point in a contractible space, representing the timeless, spaceless substratum.

The total space  $E$ , which we identify with the Drama (or Kaal Chakra), is a product space that encodes both the trajectory of the soul across time and its dynamic interplay with the world drama. Assuming the drama is deterministic and cyclic, we model the total space as:

$$E = B \times F \times S^1 \quad (211)$$

where  $S^1$  encodes the cyclic nature of time. The mapping  $\pi$  then simply projects onto the base space  $B$ , suppressing the dynamic evolution of the fiber:

$$\pi : E \rightarrow B, \quad \pi(b, f, \theta) = b \quad (212)$$

Importantly, this bundle admits a global section  $s : B \rightarrow E$ , which we interpret as the Supreme Soul (Shiv Baba), whose state remains fixed across all epochs and serves as the constant reference point for all individual souls. This fixed section is defined by:

$$s(b) = (b, f_0, \theta_0) \quad (213)$$

where  $f_0$  is the original pure state of the soul and  $\theta_0 \in S^1$  corresponds to the moment of the origin of the drama. Since the section is fixed, we have:

$$s(b) = \text{const} \quad \forall b \in B \quad (214)$$

This captures the idea that the Supreme Soul is beyond the cycle of birth and death and thus serves as a non-dynamical, eternal fixed point in the total space.

We now examine the transition functions of this bundle. Suppose  $\{U_i\}$  is a trivializing cover of  $B$ . The transition functions  $g_{ij} : U_i \cap U_j \rightarrow \text{Aut}(F)$  must satisfy the cocycle condition:

$$g_{ij} \cdot g_{jk} = g_{ik}, \quad \text{on } U_i \cap U_j \cap U_k \quad (215)$$

Since  $B$  is a point or a contractible space, any such cover is trivial, and thus all transition functions are identity maps. Therefore, the bundle is trivial:

$$E \cong B \times F \times S^1 \quad (216)$$

However, the non-trivial evolution of the soul occurs via the dynamics over  $S^1$ , the temporal dimension, which generates karmic transformations  $\mathcal{T}_\theta : F \rightarrow F$ . This is analogous to a gauge transformation in which the structure group  $G \subset \text{Aut}(F)$  acts on the fiber through time evolution. We may therefore define a dynamical system on the bundle as:

$$\frac{df}{d\theta} = \mathcal{T}_\theta(f) \quad (217)$$

where  $\theta \in S^1$  is the cyclic time parameter. The existence of a fixed point  $f^* \in F$  under all  $\mathcal{T}_\theta$  such that:

$$\mathcal{T}_\theta(f^*) = f^*, \quad \forall \theta \in S^1 \quad (218)$$

corresponds to a soul that remains unaffected by the drama, representing again the Supreme Soul or the soul in a liberated, mukti state.

The fiber bundle formalism thus allows us to model the stratification and interplay of Trilok in rigorous mathematical terms. Each soul, as a fiber, carries its unique configuration of sanskaras and karmic tendencies, yet is embedded in the broader deterministic field of the eternal drama. The fixed global section functions as the immutable guidepost for all temporal trajectories.

### 30 Fixed Points in Near-Death Experience and the 84-Birth Ladder of the Soul

The question of what occurs at the moment of death has long intrigued philosophers, scientists, and spiritual seekers. In his groundbreaking book *Life After Life*, Dr. Raymond Moody compiled a collection of testimonials from individuals who were clinically declared dead but subsequently revived. These individuals consistently reported a set of phenomenological experiences: traveling rapidly through a dark tunnel, encountering a radiant being of light filled with unconditional love, experiencing a panoramic life review, and ultimately feeling a sense of peace and detachment from the physical body [2]. Remarkably, many of these themes appear congruent with the teachings of the Brahma Kumaris spiritual university, particularly regarding the soul's journey across multiple births and its eventual return to its spiritual abode, Paramdham.

In the metaphysical framework of Brahma Kumaris, each soul is said to undergo a maximum of 84 births during the 5000-year world drama cycle. These births form a finite discrete sequence that can be modeled as a linear ladder, where each rung represents a single lifetime. Denoting the total number of births by  $N = 84$ , the index  $n \in \{1, 2, \dots, N\}$  denotes the  $n$ -th birth of a particular soul. The state of the soul during its  $n$ -th birth may be encoded by a vector  $\psi_n \in \mathbb{S}^{16}$ , where  $\mathbb{S}^{16}$  represents the Sedenionic space of the soul's sanskaras and purity levels.

Between two births, the soul transitions through the subtle dimension, and this process is punctuated by a recurring experience described by Moody's subjects and echoed in Brahma Kumaris doctrine: the witnessing of a being of divine light. This invariant experience may be conceptualized as a *fixed point* in the transition dynamics between physical lifetimes. Let us define the transition map  $\mathcal{T}_n : \mathbb{S}^{16} \rightarrow \mathbb{S}^{16}$  that governs the soul's evolution from the end of the  $n$ -th birth to the start of the  $(n + 1)$ -th:

$$\psi_{n+1} = \mathcal{T}_n(\psi_n) \quad (219)$$

We then postulate the existence of an intermediate state  $\psi^* \in \mathbb{S}^{16}$  such that:

$$\mathcal{T}_n(\psi^*) = \psi^*, \quad \forall n \in \{1, 2, \dots, N - 1\} \quad (220)$$

This  $\psi^*$  represents the universal, momentary state in which the soul witnesses the being of light—Shiv Baba—after death, regardless of its karmic trajectory or spiritual condition. Since it appears after every birth-death cycle, it constitutes a periodic fixed point in the cyclic dynamics of the soul’s rebirth.

Further, the cosmological structure of Brahma Kumaris postulates that after the completion of 84 births, all souls return to Paramdham. This is the soul’s supreme abode, a luminous, vibrationless domain situated beyond the reach of physical light and sound. Mathematically, we model Paramdham as the fixed point of the global soul dynamics over the time cycle. Let  $\Theta \in S^1$  be the cyclic time parameter, and define the overall trajectory  $\Psi : S^1 \rightarrow \mathbb{S}^{16}$  for a soul as:

$$\Psi(\theta) = \mathcal{T}_\theta(\Psi_0) \tag{221}$$

At the completion of the time cycle  $\theta = 2\pi$ , the soul returns to its original position in Paramdham. Hence, there exists a point  $\psi_\infty \in \mathbb{S}^{16}$  such that:

$$\mathcal{T}_{2\pi}(\psi_\infty) = \psi_\infty \tag{222}$$

This point  $\psi_\infty$  represents the soul’s final state of rest, prior to its re-entry into the next world cycle. The recurrence of this fixed point reflects the Brahma Kumaris doctrine that the drama of life is a closed and eternal loop with absolute determinism.

The life review reported in near-death experiences may also be interpreted through the lens of higher-dimensional data access. A soul modeled in  $\mathbb{S}^{16}$  carries a holographic representation of its karmic history. Let  $\mathcal{H} : \mathbb{S}^{16} \rightarrow \mathbb{R}$  be a functional that extracts a scalar karmic trace. At the moment of death, the soul undergoes an operation:

$$\mathcal{R}(\psi_n) = \sum_{k=1}^n \mathcal{H}(\psi_k) \tag{223}$$

This summation, compressed into a single visual or experiential frame, corresponds to the reported phenomenon of "life flashing before one’s eyes."

It is important to recognize that while physical science has not yet formulated a full theory of consciousness beyond death, the recurrent appearance of near-identical motifs in both modern medical resuscitations and ancient metaphysical doctrines suggests the possibility of a trans-dimensional cognitive geometry. Within this view, fixed points play a dual role: first, as invariant experiential states across individual death events, and second, as global attractors in the soul’s cyclical journey through the 84 births.

In summary, the phenomenology of near-death experiences corroborates key spiritual insights from Brahma Kumaris cosmology. The light at the end of the tunnel, the life review, and the eventual ascent to Paramdham are all describable through fixed-point theory applied to high-dimensional soul dynamics. These insights reinforce the metaphysical claim that while bodies perish, the soul continues its journey through a deterministically structured cycle, returning inevitably to the source from which it began.

## 31 Conclusion

In this paper, we have presented a rigorous mathematical framework to analyze and model the cosmology of the Brahma Kumaris spiritual tradition using the tools of fixed point theory, topological mappings, and higher-dimensional algebras. The soul’s metaphysical

journey through the cycle of 84 births within a fixed 5000-year temporal loop has been characterized as a dynamical system with attractors and invariant subspaces. Fixed point theorems such as those by Brouwer, Banach, and Tarski provide the foundational mathematical language for describing the return of each soul to its original, immutable state in Paramdham.

By modeling the soul as a 16-dimensional Sedenion entity with an embedded subalgebra of fixed sanskaras, we have illustrated how deeply ingrained characteristics persist across temporal iterations. The Supreme Soul, encoded as a 32-dimensional Trigention field, serves as both the attractor and anchor point of the metaphysical system. The return of all souls to Paramdham at the end of the time cycle was shown to represent a global fixed point in a cosmological mapping that is homotopy-invariant, even though individual trajectories vary.

We have further proposed a fiber bundle model in which the base space is Paramdham, the fiber represents the individual soul, and the total space is the eternal drama. This fiber structure allows the embedding of individual soul experiences while preserving the immutability of the underlying metaphysical structure. The wormhole transition from corporeal to incorporeal states was expressed using conformal geometry, maintaining boundary-preserving homeomorphisms consistent with metaphysical constraints.

The modeling of karmic returns through discrete topologies and recurrence relations enriches our understanding of cause and effect as mathematical fixed point iterations. Near-death experiences described in modern literature, such as those in Dr. Raymond Moody's *Life After Life*, find resonance in the spiritual narrative of recurring contact with the Supreme Being and the journey back to the Light. These recurring themes are mapped onto fixed structures in topology and algebra, offering a coherent bridge between metaphysics and mathematical reasoning.

This fusion of spiritual cosmology and mathematical formalism not only validates ancient metaphysical ideas within a modern analytical framework but also opens new avenues of dialogue between science and spirituality. Future extensions could include the incorporation of non-commutative geometry, sheaf theory, and category theory to further deepen the topological and algebraic understanding of consciousness, soul dynamics, and eternal recurrence.

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