A Family of 24 Quadratic Polynomials Generating odd $\mathbb{N}$ Except the First Number of Twin Prime Pairs

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Abstract

We present a family of 24 quadratic polynomials in two variables that together generate the set of odd natural numbers $\mathbb{N}$, excluding the first number of every twin prime pair. The construction of these polynomials is based on a study of triangular numbers and results in overlapping outputs. While this note focuses on presenting the empirical result and the formulas, a complete proof has been developed by the author. The author welcomes collaboration with qualified researchers to assist in formalizing and preparing a full paper for peer-reviewed publication.

Introduction

Twin prime pairs are pairs of prime numbers of the form $(p, p+2)$. The study of prime gaps, and twin primes in particular, has been a rich field of inquiry in number theory. In this short note, we describe a family of 24 quadratic polynomials in two variables (x, y) that collectively generate all odd natural numbers except the **first** element in each twin prime pair.

The approach was inspired by analysis of triangular numbers and their interactions with quadratic forms.

The Polynomials

Let $(x, y) \in \mathbb{N}^2$. The 24 quadratic polynomials are as follows:

$$
\begin{aligned}
f_1(x,y) &= 32xy + 20x + 32y^2 + 60y + 23\\
f_2(x,y) &= 32xy + 36x + 32y^2 + 76y + 43\\
f_3(x,y) &= 16xy + 20x + 32y^2 + 92y + 63\\
f_4(x,y) &= 32xy + 20x + 32y^2 + 44y + 15\\
f_5(x,y) &= 16xy + 12x + 32y^2 + 60y + 27\\
f_6(x,y) &= 32xy + 36x + 32y^2 + 92y + 63\\
f_7(x,y) &= 16xy + 12x + 32y^2 + 52y + 19\\
f_8(x,y) &= 32xy + 28x + 32y^2 + 84y + 47\\
f_9(x,y) &= 32xy + 44x + 32y^2 + 100y + 75\\
f_{10}(x,y) &= 32xy + 28x + 32y^2 + 68y + 35\\
f_{11}(x,y) &= 16xy + 20x + 32y^2 + 84y + 55\\
f_{12}(x,y) &= 32xy + 44x + 32y^2 + 116y + 99\\
f_{13}(x,y) &= 32xy + 12x + 32y^2 + 52y + 13\\
f_{14}(x,y) &= 32xy + 28x + 32y^2 + 68y + 33\\
f_{15}(x,y) &= 16xy + 20x + 32y^2 + 84y + 53\\
f_{16}(x,y) &= 32xy + 12x + 32y^2 + 36y + 9\\
f_{17}(x,y) &= 32xy + 28x + 32y^2 + 84y + 49\\
f_{18}(x,y) &= 16xy + 12x + 32y^2 + 52y + 21\\
f_{19}(x,y) &= 32xy + 20x + 32y^2 + 44y + 13\\
f_{20}(x,y) &= 16xy + 12x + 32y^2 + 60y + 25\\
f_{21}(x,y) &= 32xy + 36x + 32y^2 + 92y + 61\\
f_{22}(x,y) &= 32xy + 20x + 32y^2 + 60y + 25\\
f_{23}(x,y) &= 16xy + 20x + 32y^2 + 92y + 65\\
f_{24}(x,y) &= 32xy + 36x + 32y^2 + 76y + 45
\end{aligned}
$$

Sample Outputs

At the limit of 100, those quadratic polynomials generates all odd natural numbers except those corresponding to the first members of the twin prime pairs, starting from 11. In other words, the sieve removes numbers which are the smaller prime in twin prime pairs, effectively filtering out numbers like 11, 17, 29,41,59,71 from the sequence of odd numbers.

Observations

- The construction was based on studying triangular numbers, which motivated the structure of each polynomial.
- Some outputs are duplicated across multiple polynomials.
- No polynomial in the list outputs the first number in any known twin prime pair.

Conclusion

This empirical discovery highlights an unexpected relationship between quadratic forms and the distribution of natural numbers, specifically in the context of twin primes as they act as a sieve for twin prime numbers. The polynomials were constructed through the study of triangular numbers and show consistent exclusion of the first elements of twin prime pairs. Although only the result is presented here, the author possesses a complete proof and invites collaboration with interested researchers to help write, formalize, and refine the full theoretical paper for peer-reviewed publication.