Thermodynamic Analysis of the Three-Body Problem: Stability and Parameter Estimation

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Abstract

The three-body problem, while unsolvable in closed form under classical mechanics, can be approached through thermodynamic principles to reveal stable macroscopic behavior despite microscopic dynamical chaos. This paper proposes a thermodynamic framework for analyzing three-body systems by treating them as idealized gas-like ensembles, where pressure (P), volume (V), and temperature (T) serve as key descriptors. We address the challenge of limited particle count (N=3) by rescaling the Boltzmann constant (k_B) to account for the internal degrees of freedom within stellar-mass bodies (~10^{57} hydrogen atoms per star). For a case study of three solar-mass stars with mean separation of 10 AU, we derive a system temperature of ~5×10^3 K (consistent with stellar surfaces) and a negligible kinetic pressure of 25 Pa. Incorporating gravitational interactions via the virial theorem yields a higher effective pressure (65 Pa), indicating the need for adjusted velocities or smaller system volumes to maintain equilibrium. This work demonstrates how thermodynamics can bypass classical instabilities to predict equilibrium states, offering a novel perspective on N-body celestial mechanics. To verify the feasibility of this thermodynamic approach in solving problems, this paper also applies the method to the densely populated stellar region at the Galactic Center of the Milky Way. Through thermodynamic analysis, it is found that the stellar density at the Galactic Center is extremely high, resulting in a very high kinetic temperature of the stars in this region. This implies that the Galactic Center radiates a significant amount of energy outward. However, calculations reveal that the external pressure on the Galactic Center is very low, meaning that the structure remains highly stable under gravitational confinement alone.

1 Introduction

The three-body problem cannot be solved exactly within the framework of classical mechanics [1]. This paper attempts to analyze the three-body problem from a thermodynamic perspective [2]. The advantage of the thermodynamic approach lies in its ability to circumvent the instability inherent in multi-body motion—regardless of the dynamical state of the system, its thermodynamic parameters (e.g., temperature, pressure, volume) exhibit high stability. Through thermodynamic principles, we can predict the evolutionary direction of the three-body system.

2 Non-Interacting Three-Body System

If the three celestial bodies do not interact with each other, the system can be analogized to an ideal gas model, under the following condition: the distances between the bodies must be sufficiently large so that their individual volumes have negligible influence on the total system volume.

2.1 Applicability of the Ideal Gas Model

Such problems can be directly analyzed using the ideal gas equation of state. The thermodynamic parameters of the three-body system include pressure (P), volume (V), and temperature (T). Although the system consists of only three particles, its macroscopic behavior resembles that of a molecular thermodynamic system.

Note: Statistical physics requires a system to contain a sufficiently large number of samples (typically $N \gg 1$), whereas the three-body system's small sample size may lead to significant errors. To address this, we introduce an error parameter to quantify uncertainties.

Since macroscopic systems strictly obey energy conservation (with minimal uncertainty), parameters related to energy (e.g., pressure, temperature, volume) exhibit small estimation errors.

2.2 Parameter Calculations

Assume the three-body system is enclosed in a container of volume V, with the celestial bodies moving at velocities v_1, v_2, and v_3, corresponding to kinetic energies K_1, K_2, and K_3. According to statistical physics, the system temperature is:

$$T = \frac{2(K_1 + K_2 + K_3)}{9k_B}$$

where k_B is the Boltzmann constant ($1.38 \times 10^{-23} J/K$).

Issue: The enormous mass of celestial bodies (e.g., stars) leads to unrealistically high calculated temperatures (due to the small value of k_B).
Solution: Treat the celestial bodies as collections of microscopic particles (e.g., hydrogen atoms). Assuming each star contains $n \sim 10^{57}$ hydrogen atoms, the Boltzmann constant must be adjusted.

Degrees of Freedom Analysis:

- Treating stars as single particles ($N = 3$) results in very few degrees of freedom and extremely low entropy.
- Considering internal particles (3×10^{57} hydrogen atoms) yields a vast number of degrees of freedom and high entropy.

Adjusted Boltzmann Constant:

$$k_B' = k_B \cdot \frac{3 \times 10^{57}}{3} = 1.38 \times 10^{34} J/K$$

The revised temperature is:

$$T' = \frac{2(K_1 + K_2 + K_3)}{9k_B'}$$

2.3 Example Calculation

Consider three stars, each with solar mass ($M_\odot$) and velocity $v = 10^4 m/s$, separated by an average distance of $10AU$.

System Volume Estimation:
The average distance satisfies $\langle d \rangle \approx 1.2R \Rightarrow R \approx 8.33AU$.

$$V = \frac{4}{3}\pi R^3 \approx 2420AU^3 \approx 8.1 \times 10^{36} m^3$$

Temperature Calculation:
Kinetic energy of a single star: $K \approx \frac{1}{2} M_\odot v^2 \approx 10^{38} J$.

$$T' = \frac{2 \times 3 \times 10^{38}}{9 \times 1.38 \times 10^{34}} \approx 5 \times 10^3 K$$

This agrees with the Sun's surface temperature (~5800 K).

Pressure Calculation:
Ideal gas equation of state:

$$P = \frac{Nk'_B T'}{V} \approx \frac{3 \times 1.38 \times 10^{34} \times 5 \times 10^{3}}{8.1 \times 10^{36}} \approx 25Pa$$

This indicates that the pressure generated by stellar motion is extremely low, allowing the system to remain stable.

2.4 Effect of Gravitational Interactions

When gravity is included, the equivalent pressure can be estimated using the virial theorem [3].

Gravitational Potential Energy:

$$U \approx -\frac{3GM_{\odot}^{2}}{\langle d \rangle} \approx -5.3 \times 10^{38} J$$

Virial Theorem:

$$\langle K \rangle = -\frac{1}{2} U \approx 2.65 \times 10^{38} J$$

Gravitational Pressure:

$$P_{grav} \approx \frac{|U|}{V} \approx 65Pa$$

3 Thermodynamic Parameters of Multi-Star Systems

3.1 System Overview

For systems composed of a large number of stars ($N > 10^5$), dynamical methods become computationally prohibitive. Thermodynamic approaches provide an effective macroscopic description.

Galactic Center Model Parameters [4]:

- Number of stars (N): 10^6

- Individual stellar mass (m): $2 \times 10^{30}\, kg$

- Nuclear bulge radius (R): $1\, pc = 3.086 \times 10^{16}\, m$

- System volume (V): $(4/3)\pi R^3 \approx 1.2 \times 10^{50}\, m^3$

3.2 Gravitational Potential Energy Calculation

The gravitational potential energy for an N-body system is given by:

$$\langle U \rangle \approx -3/5\, G(Nm)^2/R$$

Calculation Steps:

① Total mass squared:

$$(Nm)^2 = (10^6 \times 2 \times 10^{30})^2 = 4 \times 10^{72}\, kg^2$$

② Numerator calculation:

$$G(Nm)^2 = 6.67 \times 10^{-11} \times 4 \times 10^{72} = 2.668 \times 10^{62}\, m^3 kg\, s^{-2}$$

③ Division by radius:

$$2.668 \times 10^{62} / 3.086 \times 10^{16} \approx 8.65 \times 10^{45}\, J$$

④ Application of structural factor:

$$\langle U \rangle \approx -3/5 \times 8.65 \times 10^{45} \approx -5.19 \times 10^{45}\, J$$

3.3 Derived Thermodynamic Parameters

3.3.1 Mean Kinetic Energy

From the Virial Theorem: $\langle K \rangle = -½\langle U \rangle \approx 2.6 \times 10^{45} J$

3.3.2 System Temperature

Using modified Boltzmann constant $k_B' = 1.38 \times 10^{34}\, J/K$:

$$T = \frac{2\langle K \rangle}{3Nk_B'} = \frac{5.2 \times 10^{45}}{3 \times 10^6 \times 1.38 \times 10^{34}} \approx 125{,}000\, K$$

3.3.3 Gravitational Pressure

$$P_{grav} \approx \frac{|\langle U \rangle|}{V} = \frac{5.19 \times 10^{45}}{1.2 \times 10^{50}} \approx 4.3 \times 10^{-5}\, Pa$$

3.4 Physical Interpretation

Key Findings:

Parameter	Value	Significance
Equivalent Temp.	125,000 K	Explains intense thermal radiation
Gravitational Pressure	4.3×10^{-5} Pa	Demonstrates gravitational dominance

4 Conclusions

For a three-star (triple) system:

The gravitational pressure (65 Pa) exceeds the ideal gas estimate (25 Pa), suggesting that the actual system may require either higher stellar velocities or a more compact configuration to maintain dynamical equilibrium.

For high-density stellar systems like the Galactic Center:

In the case of systems containing significantly more stars – such as the Milky Way's nuclear bulge with approximately 1 million stars per cubic parsec – our thermodynamic calculations reveal an extremely high system temperature. This fundamentally explains why galactic nuclei exhibit intense energy radiation toward their peripheries. Remarkably, the computed external pressure remains exceptionally low, demonstrating two key characteristics: (1) despite continuous energy radiation, the nuclear bulge maintains remarkable stability, and (2) gravitational interactions alone prove sufficient to preserve the structural integrity of the entire galactic center.

Important Note: This paper utilized DeepSeek for the derivation of its formulas, and the entire writing process was carried out through in-depth discussions with DeepSeek. Due to the inability to use a desktop computer, the author had to work on the Word document solely via a mobile phone, and as a result, portions of the text were also composed by DeepSeek. After DeepSeek performed the formula calculations, the author carefully reviewed them and confirmed that all errors were corrected, ensuring the accuracy of the computations.

References

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