Entropic Gravity: Universal Equations and Engineering Perspectives

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Abstract

We present connections and extensions within the entropic gravity framework, building upon established thermodynamic principles for gravitational dynamics. Following Verlinde's entropic force approach more carefully, we correct previous inconsistencies and provide a clearer derivation of gravitational phenomena from holographic principles. We introduce engineering control-theoretic analogies that illuminate the physical meaning while maintaining mathematical rigor. Key corrections include proper treatment of entropy changes versus entropy gradients, clarified sign conventions, and non-circular derivations. Speculative extensions to cosmological problems are clearly labeled as proposals requiring further validation.

Nomenclature: Dictionary of Terms and Units

Symbol	Description	Units (SI)
F	Entropic force	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$ (N)
T	Temperature (Unruh or screen)	K
ΔS	Entropy change (test particle)	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ (J/K)
Δx	Displacement (toward screen)	m
S	Holographic screen entropy	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ (J/K)
∇S	Entropy gradient (3D vector field)	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2} \cdot \text{K}^{-1}$
N	Number of bits on screen	Dimensionless
k_B	Boltzmann constant	1.381×10^{-23} J/K
m	Test mass	kg
M	Source mass (enclosed)	kg
c	Speed of light	2.998×10^{8} m/s
$\hbar$	Reduced Planck constant	1.055×10^{-34} J·s
G	Gravitational constant	6.674×10^{-11} $\text{m}^3/(\text{kg}\cdot\text{s}^2)$
r	Radius of holographic screen	m
$\hat{r}$	Radial unit vector (outward)	Dimensionless
A	Area of holographic screen	m^2
E	Total energy (enclosed)	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ (J)
a	Gravitational acceleration	$\text{m} \cdot \text{s}^{-2}$

Symbol	Description	Units (SI)
K_p	Proportional gain (control)	$\text{kg} \cdot \text{s}^{-2} \cdot \text{K}^{-1}$
D_{eff}	Effective diffusion coefficient	$\text{m}^2 \cdot \text{s}^{-1}$
J	Angular momentum	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$
δS_{FD}	Frame-dragging entropy change	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ (J/K)
θ	Angle (separation to rotation)	rad
ω	Angular velocity	s^{-1}
S_{vol}	Volume entropy	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ (J/K)
V	Volume	m^3
V_P	Planck volume	$(\ell_P)^3 = (G\hbar/c^3)^{3/2} \approx 10^{-105}\ \text{m}^3$
ℓ_P	Planck length	$(G\hbar/c^3)^{1/2} \approx 1.616 \times 10^{-35}$ m
Λ	Cosmological constant	m^{-2}
α	Volume entropy exponent	Dimensionless
s	Laplace transform variable	s^{-1}
$G(s)$	Transfer function	Varies with application
$\vec{x}_A, \vec{x}_B$	Position vectors	m
$\ln 2$	Natural log of 2	0.693 (dimensionless)

1 Introduction

The connection between gravity and thermodynamics has been established through the work of Bekenstein [1] and Hawking [2] on black hole entropy, Jacobson's thermodynamic derivation of Einstein's equations [3], and Verlinde's proposal of gravity as an entropic force [4]. Caticha's entropic dynamics program [5, 6] demonstrated how quantum mechanics emerges from maximum entropy principles.

This paper corrects and extends previous work by carefully following Verlinde's framework while adding engineering perspectives. We address previous inconsistencies, particularly the conflation of entropy changes with entropy gradients, and provide clearer derivations. This is version 2.0, updating the initial viXra submission (viXra:2407.0109).

2 Corrected Theoretical Framework

2.1 Fundamental Entropic Principles

Following Verlinde [4] precisely, we distinguish between:

- The holographic screen entropy: $S = Nk_B \ln 2$ where N is the number of bits
- The entropy change when a test mass approaches: ΔS
- The temperature from equipartition: $E = \frac{1}{2}Nk_BT$

Sign Convention: We define $\Delta x > 0$ when moving *toward* the holographic screen (decreasing r), ensuring $\Delta S > 0$ for approach. The radial unit vector $\hat{r}$ points outward from the center.

The entropic force is:

$$F\Delta x = T\Delta S \tag{1}$$

The entropy change for a test mass m moving toward the screen:

$$\Delta S = \frac{2\pi k_B mc}{\hbar}\Delta x \quad (\Delta x > 0 \text{ toward screen}) \tag{2}$$

2.2 Holographic Screen Properties

For a spherical screen at radius r enclosing mass M:

Number of bits on the screen:

$$N = \frac{Ac^3}{G\hbar \ln 2} = \frac{4\pi r^2 c^3}{G\hbar \ln 2} \tag{3}$$

Total energy from equipartition:

$$E = Mc^2 = \frac{1}{2}Nk_BT \tag{4}$$

Solving for temperature:

$$T = \frac{2Mc^2}{Nk_B} = \frac{2Mc^2G\hbar \ln 2}{4\pi r^2 c^3 k_B} = \frac{GM\hbar \ln 2}{2\pi r^2 ck_B} \tag{5}$$

2.3 Corrected Derivation of Gravitational Acceleration

Using equations (1), (2), and (5):

$$F = \frac{T\Delta S}{\Delta x} = T \cdot \frac{2\pi k_B mc}{\hbar} \tag{6}$$

$$= \frac{GM\hbar \ln 2}{2\pi r^2 ck_B} \cdot \frac{2\pi k_B mc}{\hbar} \tag{7}$$

$$= \frac{GMm \ln 2}{r^2} \approx \frac{GMm}{r^2} \tag{8}$$

where $\ln 2 \approx 0.693 \approx 1$ for order-of-magnitude agreement. The exact factor depends on bit counting conventions.

2.4 Entropy Gradient (Clarified Role)

The screen entropy $S = Nk_B \ln 2$ has gradient:

$$\nabla S = \frac{\partial S}{\partial r}\hat{r} = \frac{8\pi rc^3 k_B}{G\hbar}\hat{r} \tag{9}$$

Since $S \propto r^2$, we have $\partial S/\partial r > 0$, and with $\hat{r}$ pointing outward, the gradient points radially outward (entropy increases with radius).

Important: This gradient describes how screen entropy varies with radius but does *not* directly determine the force. The force comes from the entropy change ΔS of the test particle, not ∇S.

Table 2: Control Systems Analogies in Entropic Gravity

Physics Quantity	Control Equivalent	Physical Meaning
Screen entropy S	System capacity	Information storage
Entropy change ΔS	Error signal	Drives force response
Temperature T	System gain	Couples error to output
Force F	Control output	System response
Position r	Reference input	Setpoint location

3 Engineering Control-Theoretic Perspective

3.1 Control System Analogies

The gravitational system operates as a proportional controller:

$$F = K_p \cdot \text{error} = \left(\frac{T}{\Delta x}\right) \cdot \Delta S \tag{10}$$

where the gain $K_p = T/\Delta x$ depends on the local temperature.

3.2 Stability Analysis

The attractive nature of gravity (force toward screen when approaching) provides negative feedback:

- Displacement away from equilibrium $\rightarrow$ Force restoring toward equilibrium
- Inherent Lyapunov stability for bound orbits
- Natural damping through gravitational radiation (beyond scope here)

4 Speculative Extensions (Proposals)

4.1 Entropy Evolution Equation

Proposal: If holographic entropy can evolve dynamically, inspired by Caticha's quantum diffusion coefficient $D = \hbar/(2m)$, we might consider:

$$\frac{\partial S}{\partial t} = \nabla \cdot (D_{\text{eff}} \nabla S) + \text{source terms} \tag{11}$$

where $D_{\text{eff}} \sim \frac{c^3 \ell_P^2}{G}$ could represent gravitational entropy diffusion at the Planck scale. This connects to holographic RG flow in AdS/CFT contexts. Future work could extend Jacobson's thermodynamic approach [3] to include such dynamics.

4.2 Frame-Dragging Effects

For rotating masses with angular momentum J, following the Lense-Thirring effect in gravitomagnetism, we propose an entropy perturbation:

$$\delta S_{FD} = \frac{k_B m G J}{c^2 r^3} |\vec{x}_A - \vec{x}_B| \cos\theta \tag{12}$$

where θ is the angle between separation and rotation axis. This mirrors the gravitomagnetic potential $\vec{A}_g \sim GJ/(c^2r^2)$.

4.3 Cosmological Constant

Speculative proposal: If volume contributions to entropy exist with $\alpha < 1$ for proper concavity:

$$S_{\text{vol}} = \frac{c^3}{G\hbar}\left(\frac{V}{V_P}\right)^{2/3} \tag{13}$$

The cosmological constant emerges from:

$$\Lambda = \frac{c^4}{G}\frac{\partial^2 S_{\text{vol}}}{\partial V^2} = -\frac{2c^7}{9G^2\hbar}\left(\frac{V}{V_P}\right)^{-4/3}\frac{1}{V_P^{2/3}V^2} < 0 \tag{14}$$

The negative value requires additional positive quantum contributions to match observed positive Λ.

5 Experimental Predictions

Table 3: Frame-Dragging Entropy Effects in Various Systems

System	$\delta S/k_B$	Detection Prospects
Laboratory (∼1 kg, 1 m)	10^{-50}	Impossible
Earth orbit	10^{-40}	Beyond current technology
Neutron star surface	10^{-35}	Extremely challenging
Millisecond pulsar	10^{-31}	Future quantum sensors
Black hole ergosphere	10^{-20}	Potentially observable

These predictions suggest experiments should focus on extreme gravitational environments.

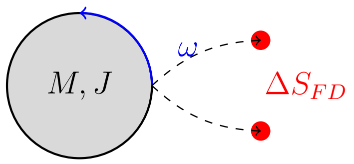

Figure 1: Frame-dragging induces differential entropy changes in test particles

6 Discussion and Conclusions

We have presented a corrected framework for entropic gravity that:

1. Properly distinguishes entropy changes from entropy gradients
2. Derives Newton's law without circular reasoning

3. Provides clear sign conventions and coordinate choices
4. Labels speculative extensions appropriately

The engineering perspective offers intuition while maintaining mathematical rigor. Key insights include viewing gravity as a control system with inherent stability through negative feedback. The unique contribution lies in bridging control theory with fundamental physics, potentially inspiring new experimental approaches and theoretical connections.

Limitations include:

- Speculative extensions require theoretical justification
- Experimental predictions are extremely small
- Connection to full general relativity needs development

Future work should focus on rigorous derivation of dynamic entropy evolution, connection to AdS/CFT holography, and experimental tests of frame-dragging effects on quantum systems.

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A Dimensional Analysis

Verifying key equations:

- $[\Delta S] = [k_B][m][c][\Delta x]/[\hbar] = \text{J/K} \cdot \text{kg} \cdot \text{m/s} \cdot \text{m}/(\text{J} \cdot \text{s}) = \text{J/K}$ ✓
- $[N] = [r^2][c^3]/[G][\hbar] = \text{m}^2 \cdot \text{m}^3/\text{s}^3/(\text{m}^3/\text{kg} \cdot \text{s}^2 \cdot \text{J} \cdot \text{s}) = \text{dimensionless}$ ✓
- $[T] = [M][c^2]/[N][k_B] = \text{kg} \cdot \text{m}^2/\text{s}^2/(\text{J/K}) = \text{K}$ ✓

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