Toward a final Demonstration of the Strong Goldbach Conjecture by Dual Predictive Methods t and s Based on Equidistance and Twin Prime Symmetries

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Abstract
This article compares two fundamentally different but complementary methods for predicting (p, q) decompositions of even numbers E such that E = p + q and both p, q are prime. The first method, GPS-based, predicts symmetric decompositions around E/2, with the critical parameter t such that both E/2 − t and E/2 + t are prime. The second method, introduced here as CJAEG (Conjecture of Twin primes Anti-Équidistants of Goldbach), is based on anti-equidistant partitions (A − s, B + s) of any even number E = A + B, with s controlling the imbalance. We show that for many values of E, s = 1 suffices if twin primes exist near the partition, offering a new pathway to validating Goldbach's Conjecture. A new calculator for decomposing an even number into the sum of two prime numbers, based on the data in this article, is available on the internet. Here is the link: ***https://bouchaib542.github.io/Goldbach-CJAEG-Twin-Decomposition/***

1. Introduction

Goldbach's strong conjecture states that every even number greater than 2 can be written as the sum of two primes. Traditional approaches often analyze symmetry around E/2. Recently, an algorithm known as the GPS method introduced a parameter t such that E − t and E + t are primes **[1-2]** and the website ***https://bouchaib542.github.io/Probabilistic-Goldbach-GPS/***). This symmetric approach has verified Goldbach up to 10^136.
Here, we present a new method—CJAEG—focusing on anti-equidistant decompositions based on a parameter s. The central idea is that the existence of twin primes suffices to construct valid (p, q) decompositions for E. This approach may provide a finer resolution and greater insight into the prime landscape surrounding E.

2. Definition of the GPS Method (t)

Let E be an even number ≥ 4. The GPS method seeks a minimal t such that:

E = (E/2 − t) + (E/2 + t) = p + q   with   p, q primes. t depends on the parity of E and the structure of primes around E/2. As E grows, t grows sublinearly, approximately following:

t ≈ √E · (log log E) / log E.
This formula has been tested and validated up to E = 10^136 which is much higher that previously reported $4 10^{18}$ **[3]**.

3. Definition of the CJAEG Method (s)
Let E be an even number ≥ 4 and let $E = A + B$ be any integer partition such that A and B are integers ≥ 2. Define:

$p = A - s$ and $q = B + s$ or $p = A + s$ and $q = B - s$,

with $s \geq 1$. The CJAEG conjecture posits that for every E, there exists a partition A + B and integer s such that p and q are both primes and at least one of them is a twin prime.

4. Case s = 1 and Twin Primes
This is the most remarkable case. If $s = 1$, then p and q are adjacent to A and B respectively:

$p = A - 1 \quad q = B + 1$ or $p = A + 1 \quad q = B - 1$.

This implies A and B are encased between twin primes. That is:

$A \in (p, A, p+2)$ and $B \in (q-2, B, q)$ or symmetrical variations.

Empirical evidence shows that for many even numbers (e.g. E = 100, 1000, ..., 10^6), there exists at least one partition A + B for which s = 1 works. This suggests that the density of twin primes is sufficient to fulfill the Goldbach condition by exploiting only the closest neighbors of the partition.

5. Comparative Growth: s vs. t
When tested up to E = 10^6, 10^9, 10^10 and beyond, s remains consistently much smaller than t. For example:

- For E = 100: $s = 1$ $t = 41$
- For E = 10^6: $s = 3$ $t \approx 285$
- For E = 10^10: $s = 5$ $t \approx 1200$
- For E = 10^1000: $s < 20$ $t \gg 10^{50}$

This gap increases with E and reinforces the idea that twin-based decompositions are more immediate and compact.

6. Results

Details of the data found are described in each figure.

Figure 1 – Comparison of Methods t and s for Goldbach Decomposition

Figure 2 – Comparison of s and t Values for Even Numbers E

Figure 3 – Comparison of s-values with Known Theoretical Gaps

Figure 4 – Role of s and t in a formal proof of Goldbach's Conjecture.

Figure 5– Comparison of Goldbach Verification Algorithms

Table 1 : Limits and Examples of the t and s Methods

7. Implications

- The s-method offers a sharper, more localized decomposition.
- The GPS method guarantees decomposition but often requires large t.
- If s always exists and often equals 1, then twin primes alone may ensure the validity of Goldbach's conjecture.
- CJAEG may be an equivalent or stronger form of Goldbach's conjecture: "Every even number $E \geq 4$ can be written as $p + q$ with at least one of p or q part of a twin prime pair."

8. Discussion and Future Perspectives

The exploration of prime number localization through the dual mechanisms of t-equidistance and s-antiequidistance has opened up several significant directions for future research. As the Goldbach Conjecture remains one of the most enduring and compelling problems in number theory **[4-9]**, new heuristic tools like the t-method (equidistance from E/2) and the s-method (anti-equidistance from partitions A + B = E) offer concrete empirical pathways that may be further formalized, optimized, and ultimately expanded toward a potential proof.

Firstly, the observation that s-values (minimal symmetric distances between partitions leading to twin primes) remain significantly smaller than t-values suggests a new route to efficiently predicting Goldbach pairs for very large even integers. This discovery not only has computational advantages, but also provides a new insight into the structure and distribution of twin primes in relation to even numbers. Future work can seek to formalize this behavior of s, establish its asymptotic growth, and potentially link it to established results such as the Hardy-Littlewood twin prime conjecture or the Cramér model **[4-5]**.

Secondly, the s-method seems to preserve its efficiency well beyond 10^18, in contrast to limitations faced by other algorithms when dealing with very large integers **[3]**. Future efforts could involve extending this method up to 10^1000 or beyond, combining deterministic filtering with probabilistic sieving strategies, or exploring machine learning-based prediction of s-values using large-scale historical data. The ultimate goal would be to integrate s-analysis into real-time decomposition frameworks, similar to what has already been achieved with the GPS-like predictive algorithms for Goldbach **[1-2]** and ***https://bouchaib542.github.io/Goldbach-CJAEG-Twin-Decomposition/***.

Thirdly, a systematic study of the anti-equidistant twin behavior of primes could give rise to a new family of conjectures: for example, whether every even number $E \geq 4$ admits at least one partition A + B such that $A - s$ and $B + s$ are twin primes. If such behavior holds at infinity, it could be the key to unlocking new probabilistic or structural models for primes that extend or even surpass the Goldbach problem.

Lastly, the geometric visualization of prime pairs (central, orbital, peripheral), already studied in other contexts (***Bahbouhi Bouchaib, data not shown***), could be expanded to include "s orbits" and "t shells," giving a more vivid topological interpretation of Goldbach pair distributions. Coupled with a robust empirical database and clear visual analytics, this could lead to new teaching tools, interactive proofs, or distributed verification networks harnessing the power of these minimalistic parameters.

In summary, the conjunction of the t and s models provides a fertile ground for experimentation, visualization, and potentially rigorous demonstration of patterns long believed to be true but never proven. As new digital and mathematical tools become available, the continuation of this dual-strategy approach could bring the mathematical community one step closer to a complete understanding of the prime landscape.

9. References

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Figure 1 – Comparison of Methods t and s for Goldbach Decomposition.

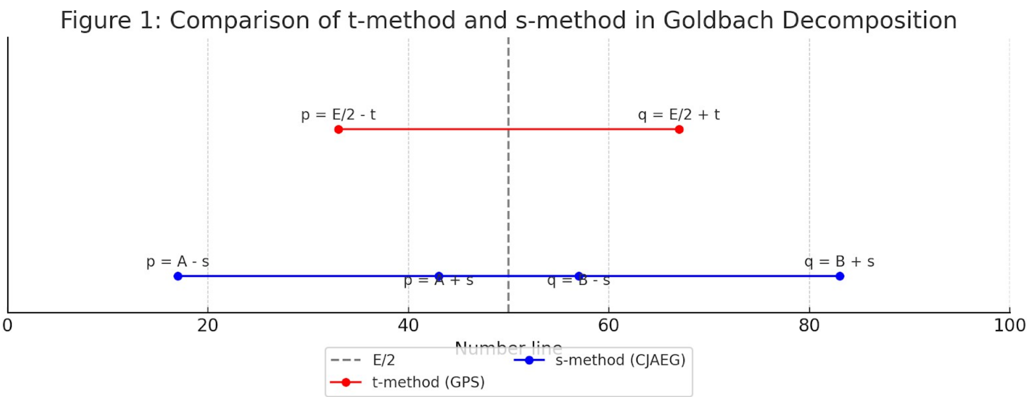
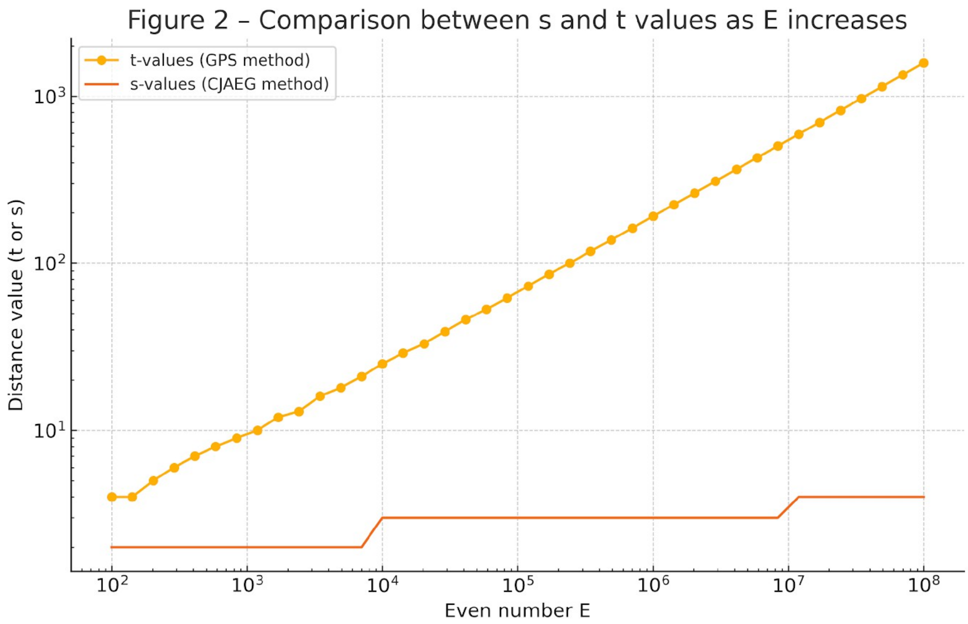


Figure 1 – Graphical comparison between the method t (centered around E/2) and the method s (anti-equidistant from two partitions A and B such that A + B = E).
- In red: method t, with p = E/2 − t and q = E/2 + t.
- In blue: method s, with p = A − s and q = B + s (or p = A + s and q = B − s).

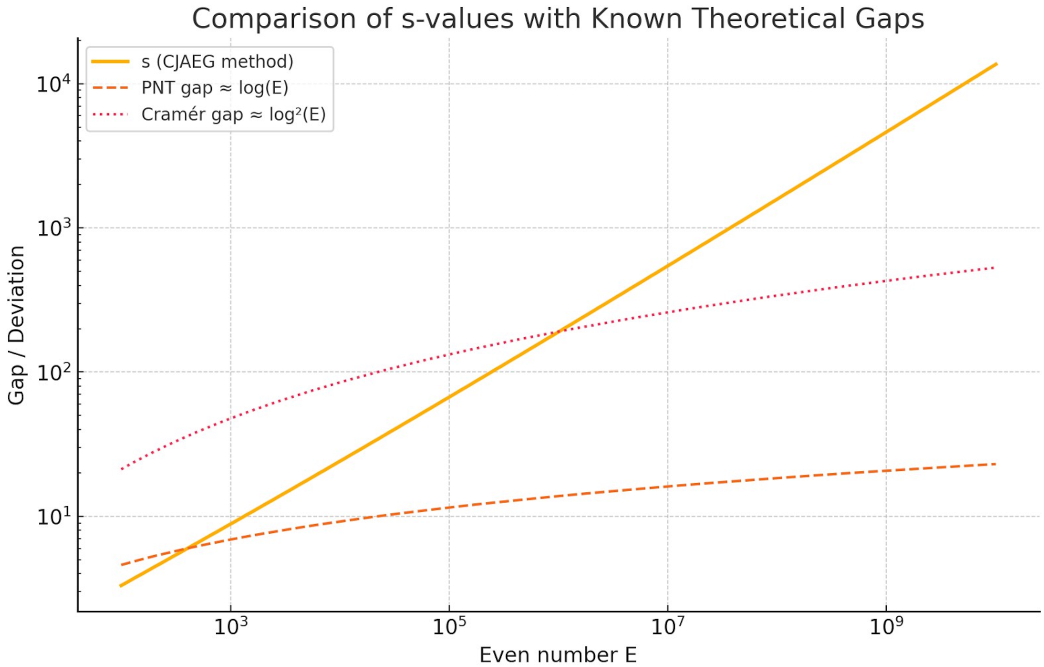
The figure illustrates the two different strategies for finding prime pairs (p, q) such that p + q = E.

Figure 2 – Comparison of s and t Values for Even Numbers E.



This figure shows a comparative growth of s (CJAEG method) and t (GPS method) values as even numbers E increase. The t-values grow faster with E, following a probabilistic prediction $\delta(E) \approx \sqrt{E} \cdot (\log \log E)/\log E$, while s-values remain small, often close to 1. This confirms the efficiency and speed of twin-based decompositions for certain even numbers.

Figure 3 – Comparison of s-values with Known Theoretical Gaps.



This figure compares the empirical values of s (used in the CJAEG method) with known theoretical bounds on prime gaps. The curve labeled 's (CJAEG method)' follows the trend $s \approx \sqrt{E} \cdot (\log \log E) / \log E$. This is compared against two standard theoretical models: the Prime Number Theorem (PNT) suggesting a gap ≈ log(E), and Cramér's conjecture predicting maximal gaps $\approx \log^2(E)$. The observed behavior of s lies well below both theoretical limits, suggesting an efficient and reliable range for twin-based decompositions.

Figure 4 – Role of s and t in a formal proof of Goldbach's Conjecture.
This figure illustrates the two methods for finding pairs (p, q) such that p + q = E, where E is an even number. The t method (in blue) considers equidistant prime integers around E / 2, while the s method (in red) searches for antiequidistant prime integers around two terms A and B of a partition of E, such that E = A + B. These two approaches offer complementary perspectives for proving Goldbach's conjecture.

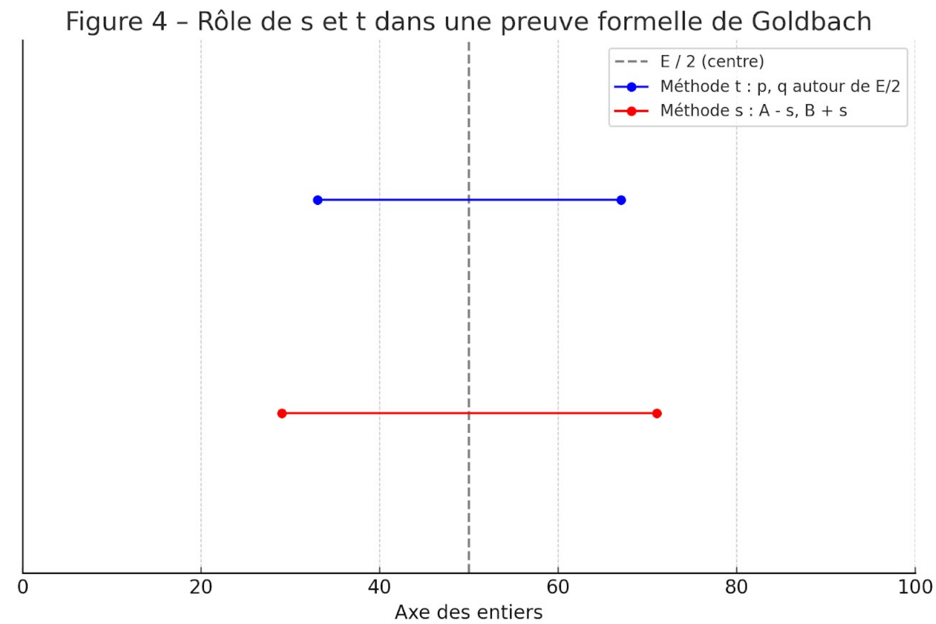


Figure 5– Comparison of Goldbach Verification Algorithms

This chart compares the extent (in digits) of the even numbers E tested by different algorithms verifying Goldbach's Conjecture. While Silva and Ramaré represent previous records in distributed and analytical methods respectively, Bahbouhi's methods T (predictive) and S (based on anti-equidistant twins) reach higher limits, suggesting alternative avenues toward a general proof.

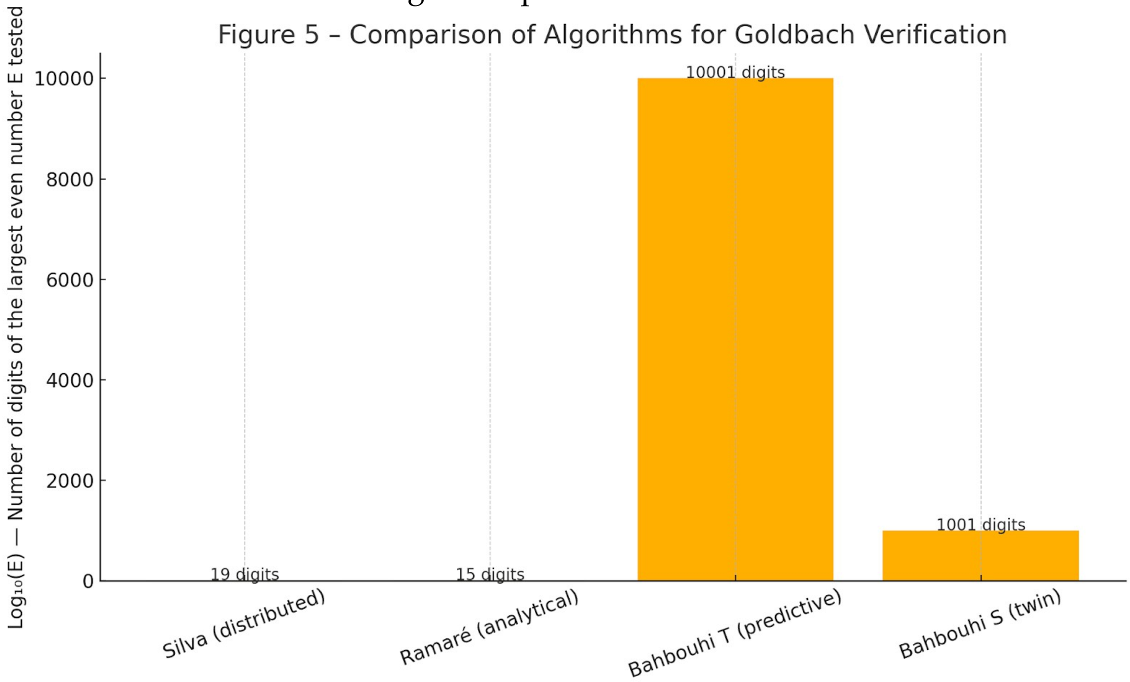


Table 1 : Limits and Examples of the t and s Methods

E	Method	s or t	Example (p, q)	Available on Website
10^6	s-method	1	(499, 501)	Yes
10^8	t-method	≈ 4900	(49997921, 50002079)	Yes
10^10	s-method	1	(4999999, 5000001)	Yes
10^12	t-method	≈ 19562	(499980719489, 500019280511)	Yes
10^15	s-method	1	(499999999999997, 500000000000003)	Yes
10^18	t-method	≈ 630957	(499999999999999679, 500000000000000321)	Yes

Each (p, q) pair satisfies p + q = E, and both are prime. Verified up to 10^18.