Cosmology from Imaginary Mold Space: A Unified Model of Matter, Dark Matter, and Dark Energy

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July 11, 2025

Abstract

We propose a novel cosmological framework where real space emerges from a finite, hyperbolic lattice in the imaginary quaternionic mold space $\mathbb{I}^3$. Mold space consists of 10^{61} negative-energy, positive-mass Planck-scale white holes arranged in a $\{5,5\}$ tiling with curvature $\kappa = -e/\ell_{\text{Pl}}^2$. At the Planck epoch, thermal excitation releases $E = 0$ white holes forming a hyperbolic boundary between mold and real spaces, while Planck-mass black holes evaporate and fill real space with radiation. The boundary's meta-stable lifetime $t_{\text{meta}} \sim 10^{-32}$ s ends via quantum tunneling at $N_{\text{vertex}} = 26$ critical sites, triggering inflation with $N_e \sim 60$ e-folds. Dark matter arises as $0.269 \cdot 10^{61}$ $E = 0$ white holes released after inflation, while mold space repulsion drives late-time accelerated expansion. The model predicts suppression of the CMB quadrupole ($\ell = 2$), hyperbolic lattice signatures at $\ell = 5, 10, 15$, and a Planck-scale gravitational wave background. This framework provides a natural solution to the cosmological constant problem and offers a unified picture of matter, dark matter, and dark energy without introducing external scalar fields or fine-tuning.

Executive Summary

This work presents a cosmological framework where the observable universe emerges from a primordial lattice of Planck-scale oscillators in **imaginary quaternion space** $\mathbb{I}^3$, termed *mold space*. Key features and results are summarized below.

Core Framework

- **Hybrid spacetime**: $\mathbb{R}^3 \oplus \mathbb{I}^3 \oplus \mathbb{R}_t$ with shared time $t \in \mathbb{R}$.
- **Mold space**: Finite hyperbolic $\{5,5\}$ lattice of $N_{\text{mold}} = 10^{61}$ white holes with:

$$E_{\text{mold}} = -m_{\text{Pl}}c^2, \quad V_{\text{mold}} < 0, \quad \rho_{\text{mold}} = \frac{m}{V} < 0.$$

- **Boundary**: Metastable hyperbolic membrane ($\kappa = -e/\ell_{\text{Pl}}^2$) formed by $E = 0$ white holes.

Key Results

1. Resolution of Cosmological Tensions

- *Vacuum energy*: Exact cancellation between real and mold-space zero-point energies:

$$\rho_{\text{vac}}^{\text{total}} = \sum_{\vec{k}} \left(\frac{1}{2}\hbar\omega_{\vec{k}} - \frac{1}{2}\hbar\omega_{\vec{k}} \right) = 0.$$

  Residual $\Lambda \sim 10^{-122}$ from boundary coupling $\gamma \sim 10^{-122}$.

- *Dark matter*: 26.9% of molds stabilize as $E = 0$ white holes (non-interacting, massive).

2. Emergent Cosmology

- *Inflation*: Triggered by boundary rupture at $t \sim 10^{-32}$ s, with:

$$N_e \sim 60 \text{ e-folds}, \quad H_{\text{infl}} = \sqrt{\frac{8\pi G}{3}\rho_{\text{Pl}}}.$$

- *Dark energy*: Late-time acceleration driven by mold repulsion ($\rho_{\text{mold}} \propto 1/R^3$).

Observational Signatures

- **CMB anomalies**: Enhanced power at $\ell = 5, 10, 15$ from $\{5, 5\}$ lattice harmonics.
- **Gravitational waves**: Spectrum peaked at $f \sim 10^{42}$ Hz (Planck-scale boundary modes).
- **Quadrupole suppression**: Finite mold space cuts off super-horizon modes.

Conceptual Advantages

- Eliminates need for ad hoc fields (inflaton, WIMPs) or fine-tuning.
- Unifies dark sector via geometric degrees of freedom in $\mathbb{I}^3$.
- Predicts testable deviations from ΛCDM at Planck-sensitive scales.

1 Geometry of Imaginary Quaternion Space

We begin by formalizing the structure of space in this framework, which extends standard 3-dimensional real space into a 7-dimensional hybrid: three real spatial coordinates $x, y, z \in \mathbb{R}^3$, one shared time coordinate $t \in \mathbb{R}$, and three imaginary spatial coordinates $i, j, k \in \mathbb{I}^3$. This defines a 7-dimensional geometric structure composed of three real spatial axes, three imaginary spatial axes corresponding to the mold space, and one shared time axis.

$$\mathbb{R}^3 \oplus \mathbb{I}^3 \oplus \mathbb{R}_t$$

This structure is inspired by Hamilton's quaternion algebra:

$$q = t + ix + jy + kz,$$

where $i^2 = j^2 = k^2 = ijk = -1$, we use i, j, k as physical imaginary spatial directions.

1.1 Volume and Orientation

The triple product in quaternion space gives the volume:

$$V = (ia) \cdot (jb) \times (kc) = -abc$$

This implies that any volume spanned by in imaginary basis vectors has negative real value:

$$V_{\text{imaginary}} = -abc < 0$$

Thus, negative volume is a built-in geometric feature of quaternion imaginary space, and not merely a convention. This naturally represents negative energy negative volume structure as a "mold" of particle in the imaginary space, so this space is called a "mold space" in the article. Paticle molds does not have charge so are invisible to EM detection, as is the whole imaginary mold space. It coexists with real space in real time.

1.2 Displacements and Velocities

A particle-like structure (mold) residing in imaginary space is displaced along an imaginary vector:

$$\vec{u}(t) = iu_x(t) + ju_y(t) + ku_z(t)$$

Its velocity is:

$$\vec{v}(t) = \frac{d\vec{u}}{dt} \in \mathbb{I}^3, \quad \text{with } t \in \mathbb{R}$$

Since time remains real, acceleration is also imaginary:

$$\vec{a}(t) = \frac{d^2\vec{u}}{dt^2} \in \mathbb{I}^3$$

1.3 Force and Mass Density

Assuming each mold has positive mass $m > 0$ but is embedded in negative volume $V < 0$, the resulting mass density becomes:

$$\rho = \frac{m}{V} < 0$$

The inertial force acting on the mold obeys Newton's second law, generalized into imaginary space:

$$\vec{F} = m\vec{a}, \quad \vec{a} \in \mathbb{I}^3$$

Hence, forces and responses are purely imaginary vector quantities.

1.4 Oscillatory Behavior

We now consider oscillatory solutions along each imaginary axis, of the form:

$$u_i(t) = A_i \cos(\omega t + \phi_i), \quad A_i \in \mathbb{R}, \quad \omega \in \mathbb{R}$$

Despite occurring along imaginary axes, the oscillations have real frequencies. Therefore, energy associated with these oscillators can be formally defined by:

$$E = \frac{1}{2}m\left|\frac{d\vec{u}}{dt}\right|^2 + \frac{1}{2}K\,|\vec{u}|^2$$

Since the displacement $\vec{u}$ lies along imaginary directions, the square magnitude yields a real and positive scalar. However, the oscillator is understood to contribute negative energy within the physical context:

$$E_{\text{mold}} = -\frac{1}{2}KA^2 < 0$$

The total energy of a mold oscillator confined to imaginary space is given by:

$$E_{\text{mold}} = -\frac{1}{2}KA^2$$

Here, K is the real and positive spring constant (or restoring stiffness), and A is the amplitude of oscillation along the imaginary axes i, j, k. The negative sign reflects the fact that mold oscillators reside in a space of negative volume and are governed by inverted restoring forces. These oscillators, while possessing real-valued amplitudes and frequencies, contribute negative energy to the vacuum. This is not merely a mathematical artifact: the negative energy of mold oscillators plays a crucial role in stabilizing the vacuum and ensuring that the total zero-point energy of the combined system (real + imaginary spaces) cancels to zero. This cancellation mechanism underlies the resolution of the cosmological constant problem in the present model.

2 Quantized Oscillators in Mold Space

We model mold structures as quantized harmonic oscillators within the imaginary space $\mathbb{I}^3$. Each oscillator has displacements aligned along i, j, k with quantized energy levels governed by:

$$E_n = -\hbar\omega \left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

The minus sign reflects that mold oscillators store negative energy relative to real space. Time evolution remains governed by real time t, so oscillations are stable and bounded.

2.1 Negative Temperature Formalism

We define the partition function for a single oscillator:

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n}$$

This converges only for $\beta < 0$ — i.e., the system must be at negative absolute temperature.

The average energy and entropy are:

$$\langle E \rangle = -\hbar\omega \left(\frac{1}{2} + \frac{1}{e^{-\beta\hbar\omega} - 1} \right)$$

$$S = k_B \left(\ln Z + \beta \langle E \rangle \right)$$

Molds are thus most stable in high negative-temperature phases and exhibit inverted population distributions — a necessary condition for energy transfer to real space.

Historical Context: Dirac's Sea

The concept of a negative-energy substrate has historical roots in the work of Paul Dirac, who postulated that the vacuum consists of an infinite sea of negative-energy electron states, later known as the Dirac sea. In our framework, this notion is reinterpreted geometrically: the mold lattice in imaginary quaternion space serves as a discrete, oscillator-based analog of Dirac's continuous sea. Mold excitations correspond to transitions within a structured, Planck-scale reservoir of negative energy. Unlike Dirac's model, our construction naturally resolves the zero-point energy problem and supports emergent particle behavior via quantized transitions between mold and real space.

3 Transition to Real Space: Negative Frequencies as Positive Energy

The oscillatory modes of the mold lattice, while entirely imaginary in spatial direction, possess real-valued frequencies ω. The solutions to the harmonic oscillator equation in this context are symmetric under time reversal, leading to both positive and negative frequency components:

$$u(t) = Ae^{i\omega t} + Be^{-i\omega t}$$

In standard quantum mechanics, negative frequency modes are often discarded or reinterpreted via creation operators. In this model, however, the negative frequency modes correspond to real-space phenomena.

3.1 Energy Reflection at the Mold-Real Boundary

Consider the total energy associated with each oscillation mode:

$$E = \pm\hbar\omega\left(n + \frac{1}{2}\right)$$

In mold space, these energies are negative. When an oscillator mode transitions across the interface into real space, its energy flips sign:

$$E_{\text{real}} = -E_{\text{mold}} = +\hbar|\omega|\left(n + \frac{1}{2}\right)$$

This mechanism allows real particles to be interpreted as the projection of negative-energy mold excitations into the real $\mathbb{R}^3$ space.

3.2 Interpretation of Time Flow

Because $\omega < 0$ corresponds to modes with reversed time phase, the apparent time direction of the emerging real-space particle aligns with forward-time evolution, even though it originated from a backward-phase mode in mold space.

This duality resolves the apparent contradiction between energy positivity in real space and the bounded-from-below oscillator structure in the mold vacuum.

3.3 Real and Imaginary Energy Components

In our model, physical space is divided into two distinct domains: real space $\mathbb{R}^3$, where ordinary particles and fields reside, and imaginary space $\mathbb{I}^3$, where molds exist and oscillate. Although mold motion occurs entirely along imaginary spatial directions, time remains shared and real. This shared time allows imaginary oscillations to possess real frequencies — and therefore, real energies — even though their spatial displacements are unobservable in $\mathbb{R}^3$.

These oscillations in mold space store negative energy relative to the positive energy states of real particles. When a mold system undergoes a transition — for example, due to instability or external disturbance — part of its oscillatory energy can no longer remain confined within imaginary space. Instead, this energy is reconfigured and emitted as a real-space excitation: a particle, a photon, or another positive-energy manifestation.

Importantly, not all imaginary motion contributes to observable real outcomes. Only specific combinations of mold oscillations — such as those aligned with real time and associated with a loss of internal equilibrium — can project energy outward into the real domain. This transformation is not continuous; it happens in discrete events when energy stored in the mold becomes dynamically incompatible with purely imaginary confinement and is released into $\mathbb{R}^3$.

In this way, imaginary-space dynamics serve as a hidden reservoir of oscillatory energy. These oscillations are electromagnetically inert and gravitate repulsively due to their negative mass density, yet they underlie the emergence of real matter and interactions through selective transitions into real space.

This asymmetry is built into the cosmological evolution: only negative-frequency mold modes are allowed to become real particles, ensuring a net energy flux from imaginary to real space.

4 Mold Space, Real Particles, and Voids

Mold space represents a negative energy state where particles initially reside. In this state, the particle and its mold are bound together, with the mold itself serving as the inner structure around the particle. When the particle transitions to real space, it leaves behind both the mold and a corresponding void in the mold space. The mold retains the essential properties of the particle, even after it has been emitted. This process can be thought of as analogous to the birth of a particle, where the mold is the "womb" from which the particle emerges.

In the case of stable particles, such as free electrons, the mold and void remain unperturbed. The negative volume of the void relative to imaginary space is a natural extension of the volume concept. They do not interact with other defects in the mold space and retain their structural integrity. The absence of the particle leaves a corresponding void that reflects the negative energy associated with the particle's existence. This mold and void structure does not actively interact with the surrounding mold lattice unless there are perturbations or external influences.

However, particles with shorter lifetimes, such as unstable or decaying particles, exhibit stronger interactions with the surrounding mold space and other voids. The instability of these particles causes more pronounced interactions within the mold and void structures, leading to their eventual decay or transformation. These decays can occur as the particle transitions between energy states, causing a ripple in the mold space, affecting the surrounding lattice and possibly resulting in the creation of new voids or mold defects.

The role of the mold and void in this context is crucial to understanding the dynamic interplay between negative energy states and real, observable particles. The energy released during particle emission into real space is accompanied by changes in the mold structure, and the resulting void serves as a memory of the particle's previous existence in mold space. This provides a mechanism for understanding the behavior of particles, their decay processes, and the influence of mold space on real particle dynamics.

The emission of the particle into real space marks a transition from a negative energy state to a positive energy state, and this transition is critical for the creation of observable particles and their interactions with the surrounding environment. The mold and void remain in mold space, but their relationship with real space becomes less direct, as the real particle now operates independently of the mold.

The transition between mold and real space is governed by the Zitterbewegung (ZB) frequency, which dictates the oscillations of particles between mold and real space. This frequency governs the interaction between the particle and its mold, and as the particle reaches its real space state, the mold's role is diminished, with the void providing the structure necessary for the particle's identity and stability in real space.

4.1 Relativistic Covariance in Hybrid Space

The hybrid manifold $\mathbb{R}^3 \oplus \mathbb{I}^3 \oplus \mathbb{R}_t$ naturally separates into a visible real sector and an invisible imaginary sector. Rotational symmetries in each 3-dimensional subspace form the groups $SO(3)_R$ and $SO(3)_I$, acting independently on the real and imaginary spatial coordinates, respectively:

$$\vec{x} \in \mathbb{R}^3 \Rightarrow SO(3)_R, \qquad \vec{u} \in \mathbb{I}^3 \Rightarrow SO(3)_I$$

To ensure relativistic consistency, we must understand how these rotational symmetries embed into the Lorentz group $SO(1,3)$, which preserves the Minkowski metric in real space:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

Within the real subspace $(\mathbb{R}^3, t)$, standard Lorentz transformations apply, preserving causality and the speed of light. However, since mold-space displacements occur along imaginary axes with velocities $\vec{v} \in \mathbb{I}^3$, the effective speed becomes ic, and Lorentz transformations must act independently on the real and imaginary parts.

We thus postulate a doubling of the local symmetry group:

$$SO(3,1)_R \times SO(3)_I$$

The group $SO(3,1)_R$ acts on real space-time $(\mathbb{R}^3, t)$, while $SO(3)_I$ preserves rotational invariance in the imaginary mold subspace.

In this formalism, the imaginary sector is not inertially accessible from the real sector — but real curvature and forces may emerge from torsional excitations in $SO(3)_I$. The embedding of $SO(3)_R \times SO(3)_I \subset SO(6)$ suggests that the full symmetry group is a 6D rotational group with shared real time:

$$\mathbb{R}^3 \oplus \mathbb{I}^3 \oplus \mathbb{R}_t \Rightarrow \text{Covariant under } SO(6) \times \mathbb{R}_t$$

This structure allows for rotational dynamics in both sectors, but preserves causality and relativistic structure in the observable space. All physical projections — including mass, charge, and gravity — must respect $SO(3,1)_R$ covariance and derive from scalar or vector invariants under this embedding.

5 Field Quantization and Boundary Interactions

The mold lattice, consisting of quantized oscillators in imaginary space, exhibits discrete excitation levels. These can be modeled analogously to phonon quantization in solid state physics.

The Hamiltonian for a single oscillator mode:

$$H = \frac{1}{2}m\dot{u}^2 + \frac{1}{2}Ku^2$$

yields eigenvalues:

$$E_n = -\hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

The minus sign corresponds to negative energy of the mold space.

At the interface between mold and real space, energy can be exchanged through a coupling Hamiltonian:

$$H_{\text{int}} = \gamma\, \dot{u}_{\text{mold}} \cdot \dot{u}_{\text{real}}$$

Assuming Planck-scale displacements:

$$\dot{u} \sim \ell_{\text{Pl}}\omega_{\text{Pl}}, \quad \Rightarrow \quad \rho_\Lambda \sim \gamma\ell_{\text{Pl}}^2\omega_{\text{Pl}}^2$$

This gives:

$$\rho_\Lambda \sim \gamma \cdot \frac{c^5}{\hbar G}$$

To match observations:

$$\gamma \sim 10^{-122}$$

This small coupling explains the non-zero cosmological constant after vacuum energy cancellation and establishes a dynamical mechanism for real space emergence through oscillatory leakage.

6 Transition to Real Space

Mold oscillations evolve under imaginary-space dynamics with time $t \in \mathbb{R}$, and spatial displacements $\vec{u} \in \mathbb{I}^3$. The oscillator equation takes the familiar form:

$$m\frac{d^2\vec{u}}{dt^2} + K\vec{u} = 0$$

whose general solution involves frequencies ω satisfying:

$$\omega = \pm\sqrt{\frac{K}{m}}$$

This produces two roots — one positive and one negative — which reflect the two fundamental directions of temporal evolution allowed by the oscillator. In standard quantum mechanics, both roots are typically retained symmetrically; however, in our model, the distinction between them acquires ontological meaning.

The positive root $\omega > 0$ corresponds to a bound oscillation entirely confined within imaginary space. It defines the stable mold state, contributing negative vacuum energy while remaining electromagnetically inert and gravitationally repulsive. This is the default state of the gravitational vacuum lattice: a sea of Planck-scale molds oscillating with positive frequency.

The negative root $\omega < 0$, however, signifies a loss of confinement. This mode cannot remain in imaginary space alone — instead, it initiates a transition into real space, manifesting as a particle or real-space excitation. Thus, the emergence of particles corresponds to the population of these negative-frequency solutions, which flip the energy sign and become physically observable as positive-energy states.

This mechanism connects the origin of real matter to the dynamics of imaginary-space molds. In particular, real particles originate not from vacuum instability, but from selective transitions between bound mold oscillations (positive ω) and emitted real-space excitations (negative ω). This frequency bifurcation is the mathematical basis of the real–imaginary boundary structure and gives rise to the coupling that defines physical particles.

7 Dispersion Relation in Mold Space

We now analyze the dispersion relation for oscillations in the mold lattice, where displacements and wave vectors exist in imaginary spatial directions. Consider a wave-like solution for mold displacement:

$$\vec{u}(\vec{r}, t) = \vec{A}\cos\Big(\vec{k}\cdot\vec{r} - \omega t + \phi\Big)$$

Here, $\vec{k} \in \mathbb{I}^3$ is the imaginary wave vector, and $\vec{r} \in \mathbb{I}^3$ the spatial position within the imaginary lattice. The dot product $\vec{k}\cdot\vec{r}$ is real, and time $t \in \mathbb{R}$, so the argument of the cosine is real, ensuring well-defined oscillatory behavior.

We assume a simple harmonic restoring force due to mold stiffness K, so each oscillator obeys:

$$m\frac{d^2\vec{u}}{dt^2} = -K\vec{u}$$

Introducing the plane wave ansatz into the wave equation for elastic media in imaginary space gives:

$$\omega^2 = \frac{K}{m} + c_{\text{eff}}^2|\vec{k}|^2$$

Here, c_{eff} is an effective speed of propagation in imaginary space, generally expected to be $c_{\text{eff}} = ic$, purely imaginary, indicating that the lattice responds with a bounded but imaginary signal speed.

Thus, the dispersion relation becomes:

$$\omega^2 = \frac{K}{m} - c^2|\vec{k}|^2$$

This formula has two important consequences:

1. For small enough wave number $|\vec{k}|$, the system remains stable with real ω, corresponding to oscillatory molds.

2. For sufficiently large $|\vec{k}|$, the frequency becomes imaginary.

The critical wave number beyond which this transition occurs is:

$$|\vec{k}_{\text{crit}}| = \sqrt{\frac{K}{mc^2}}$$

This threshold delineates stable mold oscillations from real-space emergence, offering a potential mechanism for defining ultraviolet (UV) behavior in the model.

7.1 Transition Beyond Planck Frequency: Imaginary Modes and Subquantum Regime

The dispersion relation derived for mold oscillators,

$$\omega^2 = \frac{K}{m} - c^2|\vec{k}|^2,$$

defines a critical frequency at which the character of the solution changes. Setting the frequency to zero gives the critical wave number:

$$|\vec{k}_{\text{crit}}| = \sqrt{\frac{K}{mc^2}}, \quad \Rightarrow \quad \omega_{\text{crit}} = \sqrt{\frac{K}{m}} = \omega_{\text{Planck}}.$$

Below this wave number, $|\vec{k}| < |\vec{k}_{\text{crit}}|$, the frequency is real and the oscillator supports propagating solutions in imaginary space. However, for $|\vec{k}| > |\vec{k}_{\text{crit}}|$, the frequency becomes imaginary:

$$\omega = i\tilde{\omega}, \quad \text{with } \tilde{\omega} = \sqrt{c^2|\vec{k}|^2 - \frac{K}{m}}.$$

These imaginary-frequency solutions are not discarded, but are interpreted as non-propagating or evanescent mold modes. Instead of being sharply cut off, they reflect the subquantum regime of the vacuum structure and may encode:

- Virtual mold excitations that mediate coherence across the lattice;
- Subquantum exchanges that do not project directly into real space;
- Deep gravitational or topological transitions in mold space.

Importantly, these modes are exponentially suppressed by the Boltzmann factor:

$$P(\omega) \propto \exp\left(-\frac{|\omega|}{\omega_{\text{Planck}}}\right),$$

which ensures they remain part of the full structure without contributing divergences. Thus, the Planck frequency is not a cutoff but a threshold into the imaginary domain of deeper vacuum interactions.

7.2 Phonon Modes and Density of States

In 3D, assuming cubic symmetry and using $k_{\max} \sim \pi/\ell_{\text{Pl}}$, the number of modes is:

$$N = \left(\frac{L}{\ell_{\text{Pl}}}\right)^3$$

We define a density of states $D(\omega)$ with cutoff at ω_{Pl}, but no longer need any regularization because positive and negative frequencies cancel in pairs:

$$\rho_{\text{vac}}^{\text{mold}} = -\frac{1}{2}\int d^3k\, \hbar\omega(k) \quad \text{(cancels with real space)}$$

8 Thermodynamics and Negative Temperature

Thermodynamic analysis of mold oscillators reveals a key insight: the negative energy structure requires negative temperature $T < 0$ for proper convergence of partition functions.

8.1 Partition Function and Energy Expectation

The partition function is defined by:

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n} = \sum_{n=0}^{\infty} \exp\left(\beta\hbar\omega\left(n + \frac{1}{2}\right)\right)$$

This series only converges when $\beta < 0$, which implies negative absolute temperature.

The mean energy is:

$$\langle E \rangle = -\frac{\partial \ln Z}{\partial \beta}, \quad \text{and entropy } S = k_B \left(\ln Z + \beta\langle E \rangle\right)$$

8.2 Transition from $E = -m_{\mathbf{Pl}}c^2$ to $E = 0$

Let us now focus on a specific energy transition relevant to the early mold space configuration. Each mold begins in the lower energy state $E = -m_{\text{Pl}}c^2$, the most tightly bound oscillatory state within the imaginary vacuum.

However, due to thermal agitation at the negative Planck temperature $T = -T_{\text{Pl}}$, a transition to a less bound but still non-interacting state $E = 0$ becomes statistically possible. This state corresponds to molds with no rest energy but finite positive mass and zero real-space interaction — making them excellent dark matter candidates. They still remain invisible to electromagnetism and do not radiate or decay. They are initially located on the boundary between positive energy real space and negative energy imaginary mold space.

The Boltzmann factor gives the probability $P(E)$ that a mold transitions to a higher energy level E=0 from the ground state $E = -m_{\text{Pl}}c^2$:

$$P(E) \propto e^{-\frac{E_{final}}{k_B T}} = e^{-\frac{0}{k_B T}}$$

With:

$$E_{\text{final}} = 0$$

and $T = -T_{\text{Pl}}$, we have:

$$P_0 = \frac{e^{\frac{0}{k_B T_{\text{Pl}}}}}{Z} = \frac{1}{e+1}$$

This yields the following:

$$P_0 = \frac{1}{e+1} \approx \frac{1}{3.718} \approx 0.269$$

This is a finite and well-defined fraction: approximately 26.9% of all Planck molds undergo the transition to $E = 0$. Crucially, this value is independent of normalization and reflects a statistical equilibrium determined solely by the temperature and energy gap.To maintain total energy of mold space conserved, since mold space gains energy with emission negative energy white holes to E=0 white holes, the same amount of black holes with positive energy $E = m_{\text{Plank}}c^2$, which are assumed in this work to be white hole antiparticles, are emitted and enters real space created by boundary. These processes are simultaneous.

Thus, the total initial population of molds splits naturally: 26.9% become effectively massless and non-radiating, later populating the real universe as dark matter, while the remaining 73.1%

remain in the vacuum sea, contributing continued negative mass density. This explains both the finite mass content and the persistent gravitational background structure.

This result is central to the physical interpretation of mold space: only a well-defined, statistically calculable subset of molds become real-universe participants, and the process is governed purely by imaginary-space oscillator dynamics under Boltzmann statistics.

9 Transitions of White Holes from State $E = -m_{\textbf{Plank}}c^2$ to $E = m_{\textbf{Plank}}c^2$

9.1 Assumptions and setup

The transition from negative energy ($E = -m_{\text{Plank}}c^2$) to positive energy ($E = m_{\text{Plank}}c^2$) occurs in a two-state system. The energy of the white holes in mold space undergoes a thermal excitation. After filling the boundary level $E = 0$, this state does not participate in further transitions and remains sealed. The system then oscillates between the states $E = -m_{\text{Plank}}c^2$ and $E = m_{\text{Plank}}c^2$, where:

- The transition happens under the assumption that the system remains at $T = -T_{\text{Planck}}$, where this temperature is constant and governed by the reservoir supplying thermal energy to mold space.

9.2 Thermodynamic Considerations: Partition Function

The transition between the two energy states, $E = -m_{\text{Plank}}c^2$ and $E = m_{\text{Plank}}c^2$, is driven by thermal excitation, and no external energy input is required for the transition to occur. This can be modeled using the Boltzmann distribution. The partition function Z is given by:

$$Z = e^{\beta m_{\text{Plank}}c^2} + e^{-\beta m_{\text{Plank}}c^2}$$

Where:

$$\beta = \frac{1}{k_B T}, \quad T \text{ is the temperature of the system (assumed to be } T_{\text{Planck}}), \quad m_{\text{Plank}}c^2 \text{ represents the energy of}$$

Numerically, we find:

$$Z = 3.089$$

9.3 Probability of Transition: $P(E = m_{\textbf{Plank}}c^2)$

To calculate the probability of the system being in the state $E = m_{\text{Plank}}c^2$, we use the following formula:

$$P(E = m_{\text{Plank}}c^2) = \frac{e^{-\beta m_{\text{Plank}}c^2}}{Z}$$

This yields:

$$P(E = m_{\text{Plank}}c^2) = \frac{e^{-\beta m_{\text{Plank}}c^2}}{e^{\beta m_{\text{Plank}}c^2} + e^{-\beta m_{\text{Plank}}c^2}} = 0.119$$

Similarly, the probability of the system being in the $E = -m_{\text{Plank}}c^2$ state is:

$$P(E = -m_{\text{Plank}}c^2) = \frac{e^{\beta m_{\text{Plank}}c^2}}{e^{\beta m_{\text{Plank}}c^2} + e^{-\beta m_{\text{Plank}}c^2}} = 0.881$$

9.4 Relating the Probability to the Initial Energy of Mold Space

The initial energy of mold space is represented by 73.1

- 73.1- To find the contribution of white holes in the $E = m_{\text{Plank}}c^2$ state, we multiply the probability of the transition by 73.1

$$\text{Contribution to the transition} = 0.731 \times 0.119 = 0.087$$

This indicates that initially 8.7 percent of the total energy in the universe comes from the white holes that transition to the $E = m_{\text{Plank}}c^2$ state.

9.5 Conclusion: Sealing of the E=0 Level and Transition Process

After the E = 0 level is filled, it becomes inactive for further energy transitions. The boundary is filled. The system will then continue to oscillate between the two states: $E = -m_{\text{Plank}}c^2$ and $E = m_{\text{Plank}}c^2$. These states now represent the stable two-state system:

9.6 White Holes with mc^2 and Their Role in the Expansion of the Universe

9.6.1 Evaporation and Particle-Antiparticle Emission

White holes with energy $E = mc^2$ undergo evaporation, emitting both particles and antiparticles. This emission is part of the thermodynamic process that drives the creation of new particles in the universe.

9.6.2 Partitioning of Energy

Half of the emitted pairs are antiparticles (4.35% of the initial total energy) which are directed into mold space, where they help create the Dirac sea mold space that facilitates the formation of structures like electrons, quarks, protons, and other matter particles in the mold space.

9.6.3 Baryonic Matter and Real Space

The remaining 4.35% of the emitted pairs stay in the real universe. This amount is very close to the baryon share (4.9%) of the universe, which aligns with the model's expectation for matter distribution.

- These particles contribute to the observable matter in the universe, which expands again, forming baryonic structures and galaxies. The resulting 0.731% - 8.7% = 64.4% of initial total energy remains in mold space negative energy white holes and continues to project repulsive gravitation that accelerates cosmic expansion of real space.

9.6.4 Resulting Matter

This process helps drive the re-expansion of the universe, as the matter created in real space from these interactions leads to the formation of baryonic matter, which is a key component of the observable universe.

- The transition happens spontaneously under thermal excitation.

9.7 Volume and Energy Calculation After Release of Black Holes

9.7.1 Mold Energy Balance and Antiparticle Release

To maintain the mold energy balance, when white holes transition from $E = -m_{\text{Plank}}c^2$ to $E = 0$, antiparticles are emitted. These antiparticles are modeled here as black holes with mass m_{Plank}. The release of these black hole antiparticles into the real universe ensures that the mold space maintains its energy balance, preventing any large-scale disturbances in the system. These black holes are subsequently evaporated via Hawking mechanism, releasing radiation, and their energy is transferred into the real universe.

9.7.2 Volume of the Real Universe

After the release of black holes, the volume of the real universe can be calculated using the number of black holes, given as 0.269×10^{61}, and the Planck length $l_{\text{Planck}} = 1.616255 \times 10^{-35}$ m. The volume is therefore:

$$\text{Volume of real universe} = 0.269 \times 10^{61} \times l_{\text{Planck}}^3 = 1.136 \times 10^{-44}\,\text{m}^3$$

9.7.3 Energy Density in the Real Universe

The total energy released by the black holes is $E_{\text{total}} = 5.27 \times 10^{69}$ J, which is distributed across the volume of the real universe. The radiation energy density u_{rad} is given by:

$$u_{\text{rad}} = \frac{E_{\text{total}}}{V} = 4.64 \times 10^{113}\,\text{J/m}^3$$

9.7.4 Temperature of the Universe

Using the Stefan-Boltzmann law, the temperature T of the universe can be calculated from the radiation energy density. The formula is:

$$T = \left(\frac{u_{\text{rad}}}{a}\right)^{1/4} = 1.57 \times 10^{32}\,\text{K}$$

This corresponds to the temperature of the universe at the moment when the radiation from the evaporating black holes is the only energy content of the universe. It is Planck scale - as expected.

10 Finiteness of the Initial Mold Space

It is essential to emphasize that the initial mold space is not infinite. The number of independent harmonic mold oscillators is finite and bounded by the initial conformal volume. Specifically, we posit:

$$N_{\text{mold}} \sim \left(\frac{R_0}{\ell_{\text{Pl}}}\right)^3 \sim 10^{61}$$

where R_0 is the initial effective imaginary radius of the mold lattice. This finiteness plays a critical role in regulating the zero-point energy sum, ensuring exact pairwise cancellation of modes ω and $-\omega$, and leads to observable consequences. For example, the suppression of the CMB quadrupole moment can be attributed to the absence of super-horizon oscillation modes, which simply do not fit within a finite oscillator lattice. Thus, the boundary of mold space not only resolves theoretical infinities but may leave measurable traces in the cosmic microwave background.

An essential feature of this framework is the assumption that the initial mold space is not only geometrically distinct from real space but also *finite* in extent and structure. Specifically, we assume that the mold vacuum consists of a discrete lattice of Planck-mass white hole oscillators with spacing set by the Planck length ℓ_{Pl}.

Total mold vacuum energy of order $N_{\text{mold}} m_{\text{Pl}} c^2$. Since mold space consists of three imaginary spatial dimensions $\mathbb{I}^3$, the effective imaginary size of the universe is given by:

$$L \sim N^{1/3} \ell_{\text{Pl}} \approx 10^{20} \ell_{\text{Pl}} \approx 10^{-15}\,\text{m},$$

As the size of the mold space is imaginary, this does not spatially leave any noticeable trail.

The volume in imaginary space is defined using the quaternionic triple product:

$$V = (ia) \cdot (jb) \times (kc) = -abc,$$

implying that all finite volumes in mold space are real but negative:

$$V_{\text{mold}} < 0.$$

Thus, the initial mold universe is compact, invisible to electromagnetic probes, and characterized by negative mass density and negative volume, but positive mass m and energy density.

11 Vacuum Energy Cancellation and Cosmological Constant Resolution

In quantum field theory, each mode of a harmonic oscillator contributes a zero-point energy

$$E_{\text{ZP}} = \frac{1}{2}\hbar\omega$$

This leads to an infinite total vacuum energy density unless regulated. In our model, however, both the real and imaginary (mold) spaces host identical oscillator spectra, but with opposite sign contributions.

11.1 Pairwise Cancellation of Zero-Point Energies

In mold space, the oscillator energies are:

$$E_n^{(\text{mold})} = -\hbar\omega \left(n + \frac{1}{2} \right)$$

while in real space they are:

$$E_n^{(\text{real})} = +\hbar\omega \left(n + \frac{1}{2} \right)$$

Thus, each positive energy real space mode is mirrored by an negative energy imaginary mode. The sum of vacuum energies is:

$$\rho_{\text{vac}}^{\text{total}} = \sum_{\vec{k}} \left(\frac{1}{2}\hbar\omega_{\vec{k}} - \frac{1}{2}\hbar\omega_{\vec{k}} \right) = 0$$

This symmetry removes the need for a Planck-scale cutoff and eliminates the 10^{122} discrepancy in vacuum energy predictions.

11.2 Residual Coupling and Small Cosmological Constant

Despite exact cancellation in the bulk, a weak interaction across the boundary can lead to a small residual energy density:

$$H_{\text{int}} \sim \gamma\, \dot{u}_{\text{real}}\, \dot{u}_{\text{mold}}$$

This term couples the time derivatives of displacement in both spaces. Assuming $\dot{u} \sim \ell_{\text{Pl}}\omega_{\text{Pl}}$, the residual energy density is:

$$\rho_\Lambda \sim \gamma\, \ell_{\text{Pl}}^2 \omega_{\text{Pl}}^2$$

To match observed values of the cosmological constant, γ must be extremely small:

$$\gamma \sim 10^{-122}$$

This framework offers a natural mechanism to solve the cosmological constant problem without fine-tuning or cutoff regularization. This approach requires more research.

12 Thermodynamic Properties of the Mold Lattice

The mold lattice, composed of mold oscillators in imaginary space, exhibits thermodynamic behavior governed by negative energy states and, in many cases, negative temperature.

12.1 Partition Function and Energy Spectrum

Each oscillator contributes an energy spectrum quantized as:

$$E_n = -\hbar\omega\left(n + \frac{1}{2}\right)$$

Assuming independent oscillators, the canonical partition function at inverse temperature β is:

$$Z = \sum_{n=0}^{\infty} \exp\left(-\beta E_n\right) = \exp\left(\beta \frac{\hbar\omega}{2}\right) \sum_{n=0}^{\infty} \exp\left(\beta\hbar\omega n\right)$$

The sum converges only if $\beta < 0$, i.e., the system must have a negative absolute temperature. Evaluating the series:

$$Z = \frac{\exp\left(\beta\frac{\hbar\omega}{2}\right)}{1 - \exp\left(\beta\hbar\omega\right)}$$

12.2 Mean Energy and Entropy

The mean energy of an oscillator becomes:

$$\langle E\rangle = -\frac{\hbar\omega}{2} - \frac{\partial}{\partial\beta}\ln Z = -\frac{\hbar\omega}{2} - \frac{\hbar\omega\exp(\beta\hbar\omega)}{1 - \exp(\beta\hbar\omega)}$$

The entropy is given by:

$$S = k_B\left(\ln Z + \beta\langle E\rangle\right)$$

which is strictly positive as long as $\beta < 0$ and $\omega > 0$. Therefore, despite occupying negative energy levels, the system possesses thermodynamically meaningful properties.

12.3 Negative Temperature and the Dominance of the Zero-Energy State

In a negative-temperature mold vacuum, higher energy levels become more populated than lower ones. As a result, the zero-energy state $E = 0$ becomes the most probable configuration accessible to molds transitioning from the ground state $E = -m_{\mathrm{Pl}}c^2$. This leads to a well-defined fraction of molds occupying the zero-energy level, which, as derived earlier, constitutes approximately 26.9% of the total mold population.

13 Gravitational Projection and Curvature

In the hybrid framework combining real space $\mathbb{R}^3$, imaginary quaternionic space $\mathbb{I}^3$, and shared real time t, we consider how local asymmetries in the mold space manifest gravitational effects in observable real space.

The mold lattice consists of stable oscillator configurations characterized by negative energy and negative mass density, embedded in negative volume. Defects or transitions in this lattice produce local interruptions in the negative energy sea. Let us denote this emergent energy density as

$$\rho_{\text{eff}} = \rho_{\text{mold, background}} - \rho_{\text{mold, perturbed}} > 0$$

Here, ρ_{eff} represents an emergent positive energy density due to asymmetry between mold and real space at the defect site. The resulting curvature in imaginary space becomes encoded in real space as gravitational attraction.

Since real space emerges from projection of the mold oscillator solutions with $\omega < 0$, the geometric deformation (curvature) of the mold background appears in the real projection as a distortion of the real metric, with geodesics bending toward the site of reduced negative energy.

This provides a natural geometric origin of gravity: it is not an intrinsic property of real space, but rather a projection of mold-space curvature due to oscillator defects and mass-energy asymmetries. Such projection gives rise to the classical gravitational potential in the low-energy limit:

$$\nabla^2 \Phi = 4\pi G \rho_{\text{eff}}$$

without requiring real-space mass to be the primary source.

This approach connects the Einstein field equations to boundary and defect dynamics of the imaginary oscillator lattice, yielding a fundamentally emergent view of gravitation.

14 Gravitational Effect as Mold-Space Curvature

Gravitation in this model is not a fundamental force, but a consequence of local distortions in the mold space geometry. When a mold is displaced or transitions to a different energy state—such as from $E = -mc^2$ to $E = 0$ or higher—the surrounding imaginary space experiences curvature. This curvature, characterized by a localized change in negative mass density, becomes encoded in real space as gravitational attraction.

In this picture, what appears as mass in real space is actually a projection of a defect or excitation in the mold lattice. The curvature or "strain" around such a defect affects the motion of nearby molds, which translates into the familiar force of gravity in real space. The gravitational constant G reflects this projection effect.

15 Universe Expansion and Mold Space Negative Mass Density as the Cause of Expansion

15.1 1. The Role of Mold Space in Universe Expansion

In this model, the **expansion of the universe** is driven by **negative energy**, **negative mass density**, **negative volume**, and **positive energy density** in **mold space**. These components contribute to **gravitational repulsion**, which leads to the **accelerating expansion** of **real space** (manifesting as **dark energy**).

The key to understanding the **gravitational repulsion** lies in the **negative mass density** and **negative volume** of **mold space**, which affect the **curvature of space-time** in **real space**, causing its **expansion**.

15.1.1 Negative Mass Density in Mold Space

- **Negative mass density** in **mold space** refers to the total mass density with white holes in mold space. The negative mass density produces a positive gravitational repulsion, driving the expansion of the universe. The mass density in mold space is associated with positive mass but negative volume, leading to a negative mass density that results in gravitational repulsion.

- This negative mass density directly contributes to the dark energy that causes the accelerating expansion of the universe. The negative mass density remains practically constant in the mold space, driving continuous accelerated expansion of the real universe.

15.1.2 Negative Volume

- **Negative volume** is a geometrical feature of imaginary quaternion space, a key topological property of mold space.
- The negative volume ensures that the negative mass density does not simply vanish but continues to affect real space as the universe expands.

15.1.3 Positive Energy Density in Mold Space

- Energy density in mold space is positive, meaning that even though the energy is negative, the energy density remains positive. The cause again is the negative volume of imaginary quaternion space. This positive energy density contributes to space-time curvature and drives the gravitational repulsion observed as dark energy.

15.2 Gravitational Repulsion from Negative Mass Density and Real Space Expansion

The gravitational potential Φ in real space due to negative mass density in mold space is given by:

$$\Phi_{\text{mold}}(\mathbf{r}) = +G \int \frac{\rho_{\text{mass}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, d^3r'$$

Where:

- G is the gravitational constant.
- $\rho_{\text{mass}}(\mathbf{r}')$ is the negative mass density in the mold space because of the negative volume).
- $\mathbf{r}$ is the point in real space where the potential is being calculated.
- $\mathbf{r}'$ is the position in mold space where the mass density resides.

The positive sign in the equation indicates gravitational repulsion due to negative mass density in mold space.

15.2.1 Energy Density from Negative Mass Density

The positive energy density from negative mass density in mold space contributes to the dark energy effect. This contribution can be expressed as:

$$\rho_{\text{energy}} = \frac{1}{2}\Phi^2_{\text{mold}}$$

This equation links the gravitational potential due to negative mass density in mold space to the positive energy density responsible for the accelerating expansion of the universe.

15.2.2 Dark Energy Contribution

- Initially, 73.1% of the total energy of the universe is attributed to negative energy in mold space, primarily from white holes. This energy is responsible for the dark energy effect, which drives the accelerating expansion of the universe.
- Over time, as white holes transition to $E = mc^2$ states, their contribution to dark energy decreases, stabilizing at 64.4%.

Key Quantities:

Quantity	Formula	Sign	Description
Volume in Mold Space	$V_{\text{mold}} < 0$	Negative	Geometrical feature ijk=-1
Mass Density in Mold Space	$\rho_{\text{mass}} = \frac{\text{mass}}{\text{volume}} < 0$	Negative	Negative mass density
Energy Density in Mold Space	$\rho_{\text{energy}} = \frac{\text{energy}}{\text{volume}} > 0$	Positive	Negative energy and volume
Gravitational Potential	$\Phi_{\text{mold}}(\mathbf{r}) = +G \int \frac{\rho_{\text{mass}}(\mathbf{r}')}{\|\mathbf{r}-\mathbf{r}'\|} \, d^3r'$	Positive	Gravitational repulsion
Dark Energy	$\rho_{\text{energy}} \sim \frac{1}{2}\Phi^2_{\text{mold}}$	Positive	Energy density of dark energy.

16 Connections to Current Cosmology

16.1 Dark Matter and the Role of $E = 0$ White Holes

In the model, $E = 0$ white holes are responsible for what is traditionally considered dark matter. These white holes are distinct from the usual notion of dark matter in that they are massive but have zero rest energy. They are distributed throughout the universe, interacting gravitationally with visible matter but not emitting detectable radiation.

16.2 Dark Energy and the Repulsive Gravitational Effect from Mold Space

Unlike traditional models that invoke dark energy as a cosmological constant or vacuum energy, the model suggests that the expansion of the universe is driven by the negative mass density in mold space. This negative mass density creates a repulsive gravitational effect that causes real space to expand.

In the standard cosmological model, dark energy is responsible for the accelerating expansion of the universe. The current interpretation is that dark energy contributes approximately 68% of the total energy density of the universe. In the model, the repulsive force from mold space acts similarly to dark energy, driving the acceleration of cosmic expansion. The key difference is that mold space is not a higher-dimensional energy field; instead, it is a negative energy substrate with negative mass density that influences the universe's growth.

By emphasizing the negative mass density in mold space, the framework connects to existing cosmological phenomena in a new and potentially more unified way.

17 Toward a Unified Framework of Quantum and Classical Realities

The hybrid 7-dimensional structure we propose—comprising real space $\mathbb{R}^3$, imaginary quaternion space $\mathbb{I}^3$, and shared real time t—provides a foundation not only for vacuum structure and cosmological energy balance, but also for bridging the quantum-classical divide.

17.1 Quantum Mechanics in Mold Space

By treating mold-space oscillators as harmonic fields in imaginary directions, we introduce a new framework for field quantization:

$$\hat{u}_i(t) = \sum_k \left(a_k e^{-i\omega_k t} + a_k^\dagger e^{i\omega_k t} \right) i_k$$

The raising and lowering operators act on mold oscillator states, and the corresponding field quantization carries over into the real space by projection of negative-frequency components.

17.2 Emergence of Classical Particles

Real space excitations appear as classical or quantum particles depending on coherence of the projection:

- Coherent oscillations yield persistent particle-like excitations.
- Decoherent transitions in mold space correspond to thermal radiation or quantum noise.

This offers a physical origin for wave-particle duality: the mold retains a wavelike phase structure, but real space sees only discrete projections, localized in energy and time.

17.3 Boundary Geometry and Temporal Directionality

Due to the symmetry of solutions $\pm\omega$, we ensure conservation laws and time-orientable causality. However, the asymmetry in projections ($\omega < 0$) explains the arrow of time in real space as a consequence of selecting one branch of the full wave solution. Mold space itself remains symmetric in time, thus preserving fundamental reversibility.

18 Gravitational Interaction Across Spaces

In this model, gravitation emerges not as a field confined to either the real or imaginary domain, but as a coupled deformation mode involving both the real space and the mold lattice. The fundamental idea is that mass-energy present in real space affects the underlying mold structure via boundary interaction, and vice versa.

18.1 Stress-Energy Feedback from Mold to Real Space

Molds are massive entities embedded in negative volume. While their internal energy is negative, they can mediate curvature of real space by adjusting the local density and topology of the underlying imaginary lattice. These adjustments propagate into real space as curvature effects, captured effectively by a modified Einstein equation:

$$G_{\mu\nu}^{\text{real}} + G_{\mu\nu}^{\text{mold}} = \frac{8\pi G}{c^4}\left(T_{\mu\nu}^{\text{real}} + T_{\mu\nu}^{\text{mold}}\right)$$

Here, $G_{\mu\nu}^{\text{mold}}$ is the curvature arising from distortions in mold space, while $T_{\mu\nu}^{\text{mold}}$ includes contributions from the imaginary-space oscillations and negative volume densities.

18.2 Geometric Potential of Mold Defects

Localized deformations in the mold lattice — such as defects, missing mold units, or excessive excitation — create discontinuities in the imaginary-space stress field. These project onto real space as gravitational potentials. We propose an effective scalar potential Φ in real space, sourced by negative mass density ρ_{mold}, defined as:

$$\nabla^2\Phi = 4\pi G\rho_{\text{eff}}, \quad \rho_{\text{eff}} = \rho_{\text{real}} + \alpha\rho_{\text{mold}}$$

Here, α is a geometric projection factor coupling imaginary-space mass density to the real-space Poisson equation. This mechanism allows mold-space defects (such as missing or excited white hole molds) to appear as attractive or repulsive gravitational sources in the real universe.

18.3 Gravity as Mold-Mediated Elastic Force

At the microscopic level, gravitational interaction can be interpreted as an elastic response of the mold lattice to the presence of real matter. The torsional and longitudinal stiffness of the mold lattice define the gravitational constant G via:

$$G \sim \frac{\ell_{\rm Pl}^2 c^3}{\hbar} \sim \frac{c^2}{K_{\rm mold}}$$

This suggests that gravitational force is not fundamental but emergent, arising from elastic propagation through the negative-energy mold vacuum. The absence of gravitons in this framework is not a deficiency but a consequence of the elastic medium's continuous structure.

18.4 Gravitational Waves as Coupled Oscillations

Fluctuations in spacetime geometry — gravitational waves — can be reinterpreted here as coherent oscillations of mold and real space in phase. A passing gravitational wave corresponds to a coupled standing wave across both spaces:

$$u_{\rm real}(x,t) = A\cos(kx - \omega t), \quad u_{\rm mold}(x,t) = A\cos(kx + \omega t)$$

The counter-propagating time evolution (due to opposite frequency signs) enforces phase coherence, maintaining total energy conservation. Detection of such waves is thus interpreted as the real-space projection of mold-lattice resonances. We now propose a unifying interpretation: **real space arises entirely as a boundary projection** of dynamic events in the imaginary quaternion mold space. All observed particles, fields, and their interactions can be recast as energetic reflections of phase transitions and oscillatory emissions originating from mold configurations.

18.5 Ontological Status of Real and Mold Space

Mold space is structurally primary: it hosts the fundamental degrees of freedom (rotor modes, bosonic condensates, fermionic molds) within a negative volume lattice framework. Real space is a derived space: it exists only where and when there is a projection of negative-energy mold transitions into positive-energy excitations.

18.6 Implications for Cosmology and Vacuum Energy

This view resolves long-standing cosmological tensions:

- The zero-point vacuum energy in real space is canceled by an equal and opposite contribution from mold oscillators.
- The residual cosmological constant arises solely from weak boundary coupling:

$$\rho_\Lambda \sim \gamma \ell_{\rm Pl}^2 \omega_{\rm Pl}^2 \sim 10^{-122} \rho_{\rm vac}^{\rm Plank}$$

- Dark matter emerges as a frozen zero-energy state of bosonic molds, while radiation and baryons arise from nonzero transitions.

Thus, the universe is not "from nothing"—it is from a silent mold sea that constantly decays and projects structured excitation into emergent reality.

19 Zitterbewegung as Mold-Real Oscillation

The ZB behavior is explained as an oscillation between mold and real space states. From the mold oscillator's frequency equation:

$$\omega = \pm\sqrt{\frac{K}{m}},$$

we interpret the positive root $\omega > 0$ as a stable mode confined to mold space, while the negative root $\omega < 0$ is projected into real space as the presence of a real particle.

Thus, the electron is not permanently present in real space, but appears periodically:

$$t_n = \frac{2\pi n}{|\omega|}, \quad n \in \mathbb{Z}$$

This interpretation gives a physical origin to Zitterbewegung as mold-induced projection events.

19.1 Outlook

This model invites reinterpretation of spin and charge as emergent phenomena of mold topology. It also provides a framework in which mass, charge, and presence in real space arise from the deeper, invisible mold structure in $\mathbb{I}^3$.

20 Imaginary Frequencies, Subquantum Modes, and Unification

In the oscillator analysis of the mold lattice, we encountered solutions where the frequency ω becomes imaginary for sufficiently high imaginary wavenumber $k \in \mathbb{I}$. Specifically, if the dispersion relation is of the form:

$$\omega^2 = \omega_0^2 \sin^2\left(\frac{k\ell_{\mathrm{Pl}}}{2}\right)$$

then for large imaginary $k = i\kappa$, we obtain:

$$\omega^2 = -\omega_0^2 \sinh^2\left(\frac{\kappa\ell_{\mathrm{Pl}}}{2}\right) \quad \Rightarrow \quad \omega = i\Omega, \quad \Omega \in \mathbb{R}$$

Such solutions correspond to oscillatory behavior confined purely to the mold (imaginary) lattice, with no direct projection into real space. We interpret these as **subplanckian modes**, which carry energy and momentum within the unobservable mold domain but do not form localized real particles.

These imaginary-frequency modes lie beyond the Planck frequency scale, marking a boundary where standard quantization breaks down and new physics may emerge. Unlike the zero-point modes whose positive and negative frequency components cancel in real space, the imaginary modes accumulate within mold space, potentially contributing to vacuum stiffness and its gravitational effects.

This regime may offer a natural path toward unification:

- Gravity emerges from lattice-scale curvature in the mold space, driven by defects and effective mass densities.
- Electromagnetism emerges from coherent oscillations within the real space projected from the mold.
- Sub-Planckian modes with imaginary frequency provide a bridge, dynamically linking both sectors via coupling near the critical Planck scale.

In this view, the Planck frequency ω_{Pl} is not a cutoff but a spectral boundary — above it, excitations are mold-bound, invisible to photons, yet possibly influencing real-space dynamics indirectly, including dark energy, cosmic acceleration, or even entanglement-like correlations. The origin of charge and mass is still work in progress, particle physics as well.

Further work may reveal how these modes regulate field coherence, cancel divergences, and provide effective communication channels between real and imaginary domains.

21 Hyperbolic Structure of Mold Space and Negative Curvature

Mold space, which underpins particle emergence, is structured with a hyperbolic geometry, characterized by intrinsic negative curvature. This negative curvature governs the interactions between particles and voids within this space, shaping the behavior of oscillations and particle transitions.

21.1 Negative Curvature and Mold Space Geometry

Mold space is organized according to hyperbolic geometry, with the curvature denoted by $k = -2.8\,\mathrm{s}^{-2}$, where the characteristic length scale s corresponds to the Planck length ℓ_{Pl}. It corresponds to 5,5 tiling of mold space. This negative curvature is fundamental to the mold space's properties, allowing it to behave similarly to spaces with constant negative curvature, like those modeled by the Poincaré disk.

This unique structure forms a pentagonal tiling of the mold space, known as the 5,5 tiling. This tiling is integral to the geometry, as it reflects the discrete lattice configuration of mold space, where each mold particle is surrounded by five neighboring molds. This pattern is crucial for maintaining the integrity of the lattice and influences the particle-mold interactions, which ultimately govern particle creation and transformation.

21.2 Inverse Temperature and Bolyai-Lobachevsky Thermodynamics

The thermodynamic properties of mold space are deeply influenced by its negative curvature. In this space, the concept of inverse temperature $\beta = \frac{1}{k_B T}$ becomes critical, as it takes on negative values, signifying a system where higher energy states are more populated than lower ones. This form of thermodynamics is akin to Bolyai-Lobachevsky thermodynamics, where the system tends toward high-energy configurations rather than the traditional low-energy configurations observed in typical systems with positive temperature.

This negative temperature behavior is a hallmark of mold space's thermal dynamics. The high-energy states in the mold lattice facilitate the transition from negative energy configurations to real, observable particles, a process tied to the peculiar properties of mold space.

21.3 Emergence of Real Particles from Mold Space

Real particles emerge from mold space through an inversion process, where the mold, initially in a negative energy state, emits a particle into real space, taking on positive energy. Upon emission, the mold and void structure are left behind, with the mold retaining the essential characteristics of the particle and void taking the place of the particle. This process is akin to the birth of a particle, where the mold serves as the "womb" for the particle's existence. Void explains physical meaning of negative volume fundamental to imaginary quaternion mold space.

In the case of stable particles like free electrons, the mold and void in mold space remain largely undisturbed, interacting minimally with other molds. However, for unstable particles, the interaction with the mold and void structures becomes more pronounced, leading to particle decay or transformation. These transitions can result in the creation of new voids or defects in the mold space.

The transition from mold to real space is governed by the Zitterbewegung (ZB) frequency, dictating the oscillatory behavior between mold and real space states.

21.4 Comparisons with AdS/CFT

While the mold space model shares conceptual similarities with AdS/CFT, especially in terms of the boundary-bulk relationship, there are significant differences. The AdS/CFT framework involves a projection of bulk states onto lower-dimensional boundaries, focusing on conformal field theories. In contrast, the mold space model is rooted in hyperbolic geometry, where particles emerge from a negative-energy lattice, and thermodynamics plays a key role in defining the nature of particle formation.

Additionally, unlike AdS/CFT, the mold space model provides a direct description of particle emergence. It introduces new mathematical structures, such as pentagonal tiling and negative-temperature thermodynamics.

Thus, the mold space approach offers a unique perspective, grounded in hyperbolic geometry, negative-energy states, and discrete lattice structures. It provides a promising avenue for exploring quantum gravity and particle physics.

22 The Birth of Real Space and the Hyperbolic Boundary

This section serves as an introduction to the concept of a hyperbolic boundary in physical systems and explores its implications for early universe dynamics. Before diving into technical details, we clarify what it means for a boundary to possess hyperbolic geometry, why this is significant for cosmology, and how analogies from familiar systems can help us understand these exotic structures.

Hyperbolic boundaries are fascinating because they represent regions of space where negative curvature fundamentally changes how geometry, forces, and thermodynamics behave. In the context of the early universe, a hyperbolic boundary could act as a protective shell separating real space from an external mold space, storing immense tension and delaying the release of energy that drives cosmic evolution. Understanding such boundaries requires not only mathematics but also intuition drawn from condensed matter physics, biological membranes, and lattice defects.

22.1 What is a Hyperbolic Boundary?

In mathematics, a hyperbolic surface has negative curvature, meaning that parallel lines diverge and the surface expands exponentially with distance. Physically, a hyperbolic boundary can arise in systems where local repulsive forces or topological constraints prevent global flattening. Examples include:

- Certain lattice defects in crystals and liquid crystals.
- Spacetime models with negative curvature (anti-de Sitter spaces).
- Topological membranes with a large genus ($g \to \infty$), as seen in some condensed matter systems.

In our model, the boundary separating mold space and real space is envisioned as a tightly curved, hyperbolic membrane with a $\{5,5\}$ tiling structure that enforces negative curvature.

22.2 Physical Analogies for Hyperbolic Boundaries

To understand how such a structure might emerge, consider analogies from everyday physics:

- **Soap Films on Wire Frames:** When soap films are stretched over wireframes containing holes or sharp bends, they can develop saddle-shaped regions of negative curvature to minimize surface tension.
- **Graphene Sheets:** Introducing pentagonal or heptagonal defects into an otherwise flat graphene sheet forces it to buckle into hyperbolic forms.
- **Biological Membranes:** Certain biological structures, like mitochondrial cristae or viral capsids, exhibit hyperbolic-like folding due to packing constraints.

These examples suggest that hyperbolic boundaries can emerge when local geometrical constraints overpower the system's tendency to flatten.

22.3 A Story of Tension and Balance

Picture the hyperbolic boundary as a cosmic drumhead stretched to its limits. On the inside, mold space exerts a relentless repulsive force, pushing outward. On the boundary, negative curvature tries to keep the membrane taut, like a saddle balancing opposing strains. Initially, the tension is so high that any attempt to puncture or deform the surface requires enormous energy. Over time, this balance begins to shift.

22.4 A Defect Tries to Propagate

Now imagine a tiny localized disturbance: a single defect forming in the hyperbolic boundary. As it begins to grow, the surrounding negative curvature pushes back, amplifying the energy cost of expansion. Each step the defect takes spreads geodesically, but the membrane's exponential divergence stretches and thins the disturbance, dissipating its energy across the surface. The defect stalls, unable to tear through the boundary unless the curvature and negative temperature weaken significantly.

Visualization Analogy: Imagine ripples on a taut saddle surface. Instead of propagating smoothly like waves on a pond, these ripples quickly flatten and stretch apart as the saddle's negative curvature forces them outward. Any ripple that tries to grow larger is dissipated across the expanding geometry, leaving the surface stable.

22.5 Physical Intuition for Hyperbolic Curvature

Imagine stretching a flexible mesh to fit pentagons uniformly. Unlike hexagons (which tile flat surfaces), pentagons force the surface to curve negatively. This curvature amplifies local deformations: a small perturbation grows rapidly, creating a geometrically rigid but globally fragile structure.

In physics, hyperbolic boundaries act as energy amplifiers:

- Small strains can release large energies due to exponential geodesic divergence.
- They can filter out low-energy states, leaving only high-energy excitations.

22.6 The Role of Negative Temperature

At negative absolute temperatures ($T < 0$), the population of states inverts: higher-energy states are more populated. Combined with hyperbolic geometry, this creates an effective barrier against defect nucleation, stabilizing the boundary.

22.7 Step-by-Step: Coupled Curvature and Temperature Dynamics

To make sense of the coupled evolution, let us introduce it with an intuitive scenario.

Imagine the boundary as a taut, saddle-shaped membrane under immense tension. As time progresses, this tension relaxes, but at different rates in geometry (curvature) and thermal excitations (temperature). Picture mold space as continuously tugging outward, while the boundary fights back.

We describe this process with two key quantities:

$$|\kappa(t)| = |\kappa_0| \left(1 - \frac{t}{\tau_c}\right)^{\alpha}, \tag{1}$$

$$|T(t)| = |T_0| \left(1 - \frac{t}{\tau_T}\right)^{\beta}. \tag{2}$$

Here:

- $|\kappa_0|$ is the initial curvature (very high, stabilizing the lattice).
- $|T_0|$ is the initial negative temperature.
- τ_c and τ_T are relaxation timescales.
- α and β shape how slowly these properties decay.

Numerical Example with Explanation: Consider these sample values:

- $|\kappa_0| = 10^{70}\,\mathrm{m}^{-2}$, $|T_0| = T_{\mathrm{Pl}}$.
- $\tau_c = 10^8$ years, $\tau_T = 5 \times 10^7$ years.
- $\alpha = 3$, $\beta = 2$.

At $t = 10^7$ years, the boundary is still rigid because:

$$\begin{aligned}
|\kappa(10^7)| &= |\kappa_0| \left(1 - \frac{10^7}{10^8}\right)^3 \\
&= 0.9^3 |\kappa_0| \\
&\approx 0.73 |\kappa_0|, \\
|T(10^7)| &= |T_0| \left(1 - \frac{10^7}{5 \times 10^7}\right)^2 \\
&= 0.8^2 |T_0| \\
&\approx 0.64 |T_0|.
\end{aligned}$$

The slow decay preserves the hyperbolic structure for tens of millions of years.

22.8 Interpreting the Critical Time

As the mold space tugs harder and the membrane weakens, defect nucleation becomes probable when the boundary temperature reaches a critical value:

$$|T(t_{\mathrm{meta}})| \approx T_c = \frac{\hbar c}{k_B} \sqrt{|\kappa(t_{\mathrm{meta}})|}. \tag{3}$$

This dimensionless condition signals the onset of instability, and solving it gives the meta-stable lifetime t_{meta}.

22.9 Why Hyperbolic Boundaries are Rare in Physics

In most physical systems, boundaries tend to minimize energy by flattening. Hyperbolic boundaries require:

- Strong local constraints (e.g., fixed pentagonal tilings).
- External forces (like mold space repulsion).
- Topological protection (high genus membranes).

These rare conditions make such boundaries exotic but possible in extreme regimes like the early universe.

22.10 Observable Consequences

If a hyperbolic boundary controlled the early universe's dynamics, we might observe:

- Large-scale voids tracing the initial {5,5} lattice symmetry.
- Notable deviations in CMB multipoles (5,10,15).
- Cold dark matter filaments aligned with the initial hyperbolic skeleton.

Conclusion: Hyperbolic boundaries, though exotic, provide a powerful framework for understanding a meta-stable early universe. Their evolution is governed by coupled curvature and temperature dynamics, which can delay the release of mold space energy and shape cosmic structure.

23 The Quaternion imaginary space and and the Hyperbolic Boundary

This section introduces the concept of hyperbolic curvature in mold space and explores its profound consequences for early universe dynamics. To fully understand these ideas, we will explain what hyperbolic curvature means in an imaginary quaternion space, why it matters for the stability of the universe's boundary, and how it drives the interplay between mold space and real space.

23.1 What is Hyperbolic Curvature in Mold Space?

In the quaternion mold space, curvature takes on an unusual character: the principal curvatures are imaginary,

$$k_1 = i\tilde{k}_1, \quad k_2 = i\tilde{k}_2,$$

where i is one of the imaginary quaternion units. The Gaussian curvature is then computed as

$$K = k_1 k_2 = (i^2)\tilde{k}_1\tilde{k}_2 = -\tilde{k}_1\tilde{k}_2.$$

Thus, even though k_1 and k_2 are imaginary in mold space, their product is real and negative in the boundary between mold space and real space. This negative curvature manifests as a hyperbolic geometry.

23.2 Why Does Hyperbolic Curvature Matter?

Hyperbolic geometry is characterized by diverging geodesics and an exponentially expanding area element. In the boundary layer between mold space and real space, this negative curvature has two critical effects:

- It makes the boundary resistant to deformation, requiring immense energy to form defects.
- It amplifies local tensions, creating a "tight membrane" that traps real space within a meta-stable configuration.

23.3 Mold Space and Boundary Dynamics

Mold space, with its hyperbolic curvature and negative mass density, exerts a repulsive gravitational effect on real space. The boundary acts as a mediator between these two regions. Initially, the boundary's hyperbolic geometry blocks mold space repulsion from acting directly on real space, keeping the universe quasi-static and preventing premature expansion.

23.4 Imaginary Curvature and Real-Space Effects

The imaginary curvatures in quaternion mold space do not "project" onto real space in a literal holographic sense. Instead, they create a real negative curvature in the boundary. This is why real space experiences a repulsive gravitational field sourced by the mold lattice's negative mass density. The curvature's sign remains negative; it does not flip in the transition from mold space to real space.

Illustrative Analogy: Imagine a saddle surface in mold space. On this saddle, all geodesics tend to diverge rapidly due to the negative curvature. If we visualize ripples propagating across this saddle, they stretch and dissipate, making it hard for any defect to grow. Similarly, in mold space, small disturbances in the boundary cannot expand easily because the curvature disperses their energy.

Conclusion: The interplay of imaginary curvatures in mold space and the hyperbolic geometry of the boundary provides a powerful framework for understanding the universe's early stability and later expansion. This hyperbolic tension acts as both a shield and a reservoir, controlling when and how real space can grow.

23.5 Enhanced Energy Barrier from Hyperbolic Geometry

The hyperbolic boundary's resistance to rupture is dramatically amplified by geometric and topological effects inherent in its 5,5 lattice structure. This amplification explains why the energy barrier for defect formation is not merely Planckian but instead enhanced by a factor N_{boost} approaching 26.

1. Hyperbolic Geometry and Angular Deficit

In the 5,5 tiling, five regular pentagons meet at each vertex, resulting in an angular deficit:

$$\Delta\theta = 2\pi - 5 \cdot \frac{3\pi}{5} = -\frac{\pi}{5}.$$

This negative curvature locks adjacent pentagons together. Deforming a single vertex requires shifting all five surrounding pentagons simultaneously, leading to a collective deformation over multiple Planck-scale cells.

2. Topological Locking

The hyperbolic boundary can be modeled as a high-genus membrane. Any local defect must overcome constraints at several vertices simultaneously. The energy barrier scales with the number of interconnected vertices: The metastable lifetime at early times is predicted as

$$t_{\text{meta}} \sim t_{\text{Pl}} \cdot \exp\left(\frac{N'_{\text{vertex}}\,\Delta E_{\text{defect}}\,t_{\text{Pl}}}{\hbar}\right),$$

with $\Delta E_{\text{defect}} \sim m_{\text{Pl}}c^2$. Solving for $t_{\text{meta}} \sim 10^{-32}$ s gives:

$$N'_{\text{vertex}} \approx 26.$$

This suggests that the hyperbolic boundary's fragility at Planck-scale size allows only ~ 26 independent critical vertices before rupture.

$$\Delta E_{\text{geo}} \sim N_{\text{vertex}} m_{\text{Pl}} c^2.$$

For $N_{\text{vertex}} \sim 26$, we obtain an amplified energy barrier:

$$\Delta E_{\text{geo}} \sim 26\, m_{\text{Pl}} c^2.$$

3. Negative Temperature Population Inversion

At $T < 0$, the hyperbolic boundary's state distribution is inverted. More than 99.9% of excitations occupy super-Planckian energy levels. This thermodynamically forbids small-scale defects, further reinforcing stability.

4. Collective Effect on Quantum Tunneling and Inflation

The tunneling probability per Planck patch is:

$$P_{\text{patch}} \sim \exp\left(-\frac{\Delta E_{\text{geo}}}{\hbar\omega_{\text{Pl}}}\right).$$

With $\Delta E_{\text{geo}} \sim 26\, m_{\text{Pl}} c^2$, this probability becomes quite small, ensuring the boundary remains meta-stable for $\tau_{\text{meta}} \sim 10^-32$ seconds.

Once quantum tunneling initiates and defects percolate through the boundary, mold space repulsion rapidly overwhelms the weakened hyperbolic tension. This triggers an explosive expansion of real space — an inflationary-like phase driven by the release of mold space's repulsive gravitational energy.

Rapid Energy Release Driving Inflation At the moment of rupture, the repulsive gravitational potential stored in mold space is no longer confined. The energy density at the boundary is on the order of Planck density:

$$\rho_{\text{Pl}} = \frac{c^7}{\hbar G^2} \sim 10^{113}\,\text{J/m}^3.$$

When the boundary breaks, this energy is released across the entire real space volume almost instantaneously. The power released can be estimated as:

$$P \sim \frac{\Delta E_{\text{geo}}}{\Delta t} \sim \frac{\rho_{\text{Pl}} V}{\Delta t}.$$

With $\Delta t \sim 10^{-32}$ s, the release rate is enormous, driving a rapid exponential increase in the scale factor $a(t)$:

$$\frac{\ddot{a}}{a} \sim \frac{8\pi G}{3}\rho_{\text{Pl}}.$$

This results in the inflationary expansion of real space over an extremely short period.

This sequence ensures that inflation occurs only after the universe has passed through its initial highly confined and screened phase.

Conclusion: The combination of hyperbolic curvature, negative temperature statistics, and topological locking enhances the energy barrier against boundary rupture. This explains the longevity of the meta-stable phase and provides a natural mechanism for a delayed but sudden inflationary expansion, aligning the model with the observed timescales of cosmic structure formation.

24 Hyperbolic Boundary Cosmology: Constants, Geometry, and Cosmic Dynamics

24.1 Initial Mold Space and Imaginary Geometry

At $t = 0$, the universe exists entirely in **mold space**, a quaternion imaginary space with coordinates (i, j, k).

- Mold space is **hyperbolic**, with imaginary curvatures $(i\tilde{k}_1, i\tilde{k}_2)$, yielding a **real negative curvature**:
$$\kappa_{\text{mold}} = (i\tilde{k}_1)(i\tilde{k}_2) = -\tilde{k}_1\tilde{k}_2. \tag{4}$$
- Mold space contains:
$$N_{\text{mold}} = 10^{61} \tag{5}$$
Planck-scale white holes. (Approximately mass of the universe today divided by Planck mass).
- Each white hole is in a **negative energy state**:
$$E_{\text{mold}} = -m_{\text{Pl}}c^2, \tag{6}$$
where:
$$m_{\text{Pl}} = \sqrt{\frac{\hbar c}{G}} \approx 2.2 \times 10^{-8}\,\text{kg}. \tag{7}$$

24.2 Hyperbolic Boundary Formation and Stabilization

A 26.9% fraction of initial mold white holes transitions to $E = 0$ states (zero rest energy) to form the boundary:
$$N_{E=0} = 0.269 \cdot N_{\text{mold}}. \tag{8}$$

Hyperbolic Geometry and Curvature

The boundary forms a regular {5,5} lattice with:

- **Curvature:**
$$K_{\text{boundary}} = -e/\ell_{\text{Pl}}^2, \tag{9}$$
where e appears naturally due to the exponential scaling of hyperbolic geometry.
- **Edge length:**
$$\ell_{\text{edge}} = \ell_{\text{Pl}}. \tag{10}$$

Negative Temperature Stabilization

The boundary remains stable at a **negative temperature** $T = -T_{\rm Pl}$, leading to a population inversion where the boundary's vibrational and topological modes are concentrated near the upper energy limit, suppressing low-energy deformations and stabilizing the lattice. The partition function

$$Z = \mathrm{Tr}\left(e^{H/(k_B|T|)}\right) \tag{11}$$

converges only if

$$|T| < \frac{\hbar c}{2\pi k_B}\sqrt{|\kappa|}. \tag{12}$$

Here, k_B is the Boltzmann constant, and $\hbar c\sqrt{|\kappa|}$ sets the characteristic energy scale of the hyperbolic lattice modes. The critical temperature for stabilization is:

$$T_c = \frac{\hbar c}{2\pi k_B}\sqrt{|\kappa|}. \tag{13}$$

This evaluates to

$$|T| < T_c \approx 0.26\,T_{\rm Pl}.$$

Thus, the boundary stabilizes slightly below the Planck temperature, ensuring that topological defects are suppressed in the hyperbolic lattice during the early universe.

Radius at Boundary Formation

The total boundary area is:

$$A_{\rm boundary} = N_{E=0}\cdot \ell_{\rm Pl}^2. \tag{14}$$

The radius of the real universe at boundary formation:

$$R_{\rm start} = \sqrt{\frac{A_{\rm boundary}}{4\pi}} \approx 1.17\times 10^{-6}\,\mathrm{m}. \tag{15}$$

24.3 Energy Emission and Conservation

To conserve total mold space energy:

1. while emitting $E = 0$ white holes from the boundary.
2. same number of $m_{\rm Pl}$ black holes are emitted into real space, which evaporate rapidly ,filling real space with radiation.

Energy Budget in Real Space

- **Radiation:**

$$E_{\rm rad} = N_{E=0}\cdot m_{\rm Pl}c^2. \tag{16}$$

- **Matter and Antimatter:**

$$E_{\rm pair} = 0.087\cdot N_{\rm mold}\cdot m_{\rm Pl}c^2. \tag{17}$$

- **Total Confined Energy:**

$$E_{\rm confined} = E_{\rm rad} + E_{\rm pair} \approx 6.97\times 10^{69}\,\mathrm{J}. \tag{18}$$

24.4 Meta-Stable Confinement Phase

During meta-stability, real space expands slowly due to internal pressure, but mold space repulsion is screened by the boundary.

Energy Density Decline

- **At $R_{\mathbf{start}} \sim 10^{-6}$ $\rho_{\mathbf{start}} = \frac{E_{\mathbf{confined}}}{\frac{4}{3}\pi R^3_{\mathbf{start}}} \approx 3.0 \times 10^{87}$ J/m^3.(19)**

At $R_{\mathbf{confined}} \sim 10^{-3}$ m:

$$\rho_{\text{confined}} \approx 1.66 \times 10^{78}\ \text{J/m}^3. \tag{20}$$

Meta-Stable Confinement Time and Tunneling Rate

The meta-stable confinement time is determined by the tunneling probability per boundary patch:

$$P_{\text{patch}} = \exp\left(-\frac{\Delta E_{\text{geo}}}{\hbar\omega_{\text{Pl}}}\right) \tag{21}$$

with energy barrier:

$$\Delta E_{\text{geo}} = N_{\text{vertex}} \cdot m_{\text{Pl}} c^2, \tag{22}$$

and $N_{\text{vertex}} \sim 26$. The confinement time is then calculated as:

$$\tau_{\text{meta}} = \frac{1}{N_{E=0} \cdot P_{\text{patch}} \cdot \omega_{\text{Pl}}}. \tag{23}$$

Substituting all terms yields:

$$\tau_{\text{meta}} \sim 10^{-32}\ \text{s}. \tag{24}$$

24.5 Boundary Rupture and Inflation

At $t = \tau_{\text{meta}}$, quantum tunneling ruptures the boundary.

Inflationary Dynamics

- Fraction of Mold Energy Released:

$$\epsilon = 1. \tag{25}$$

  In this model, all mold space repulsion acts on real space immediately after boundary rupture, driving rapid inflation without any artificial suppression.

- Hubble Parameter:

$$H_{\text{infl}} = \sqrt{\frac{8\pi G}{3}\rho_{\text{eff}}}, \tag{26}$$

  where $\rho_{\text{eff}} = \epsilon \cdot \rho_{\text{mold}}$.

- Inflation Duration:

$$\Delta t_{\text{infl}} \sim 6 \times 10^{-42}\ \text{s}, \tag{27}$$

  derived from the Planck energy density and chosen to give $N_e \sim 60$ e-folds. Inflation ends when the scale factor has grown exponentially by e^{60}, and mold space repulsion transitions to driving late-time acceleration.

- Scale Factor Growth:

$$R(t_{\text{infl.end}}) = R(t_{\text{meta}}) \cdot e^{N_e}, \tag{28}$$

  resulting in $R(t_{\text{infl.end}}) \sim 10^{20}$ m. However, during inflation the observable horizon remains fixed at

$$R_{\text{horizon}}(t_{\text{infl.end}}) \sim 10^{-22}\ \text{m}. \tag{29}$$

25 Timeline of Mold Space Cosmology

We present a detailed sequence of events in the mold space cosmology model, including all relevant formulas, numerical values, and physical mechanisms.

25.1 Planck Epoch and Mold Space Initialization ($t = 0$)

At $t = 0$, the universe exists purely in the form of a hyperbolic quaternion lattice in the imaginary space $\mathbb{I}^3$. Mold space consists of

$$N_{\text{mold}} = 10^{61}$$

negative-energy, positive-mass white holes arranged in a hyperbolic lattice with curvature and tiling $\{5, 5\}$:

$$\kappa_{\text{mold}} = -\frac{e}{\ell_{\text{Pl}}^2}.$$

The energy density of mold space is at the Planck scale:

$$\rho_{\text{mold}} = \rho_{\text{Pl}} = \frac{c^7}{\hbar G^2} \approx 1.2 \times 10^{113}\,\text{J/m}^3.$$

Here, ℓ_{Pl} denotes the Planck length. Time t is shared between the imaginary and emerging real spaces.

—

25.2 Birth of Real Space and Hyperbolic Boundary ($t \sim t_{\text{Pl}}$)

As real space nucleates, mold space emits $E = 0$ white holes to form a boundary between mold and real spaces due to thermal excitation at T_{Pl}. To balance mold space energy, positive-energy Planck mass black holes ($E = +m_{\text{Pl}}c^2$) are emitted into real space as well(assumed to be white hole antiparticles).

This boundary inherits hyperbolic geometry from mold space, resulting in a $\{5, 5\}$ tiling with curvature

$$\kappa_{\text{boundary}} = \kappa_{\text{mold}} = -\frac{e}{\ell_{\text{Pl}}^2}.$$

The total number of boundary cells is in accordance with Boltzmann probability

$$N_{\text{boundary}} = 0.269 \cdot 10^{61}.$$

At this stage, the real universe has a total radius

$$R_{\text{real}} \approx 10^{-6}\,\text{m},$$

set by the boundary area:

$$A_{\text{boundary}} = 4\pi R_{\text{real}}^2.$$

However, the observable universe radius is much smaller:

$$R_{\text{horizon}}(t) \sim ct_{\text{Pl}} \sim 1.6 \times 10^{-35}\,\text{m}.$$

—

25.3 Force Balance at the Boundary

Two opposing forces act at the boundary:

1. **Mold space repulsion:**
   The negative energy density of the mold lattice $\rho_{\text{mold}} < 0$ generates effective anti-gravity (repulsive force $F_{\text{repulsion}}$).

$$F_{\text{repulsion}} = m_{\text{Pl}} \cdot \nabla \Phi_{\text{mold}},$$

   with

$$F_{\text{repulsion}} \approx \frac{G m_{\text{Pl}}^2}{\ell_{\text{Pl}}^2} = F_{\text{Pl}} = \frac{c^4}{G} \approx 1.2 \times 10^{44}\,\text{N}.$$

2. **Geometric resistance of the boundary:**

$$F_{\text{geo}} = \frac{c^4}{G} \sqrt{\frac{e}{\ell_{\text{Pl}}^2}} \cdot \ell_{\text{Pl}}.$$

   Simplifying gives

$$F_{\text{geo}} \approx F_{\text{Pl}} \cdot \sqrt{e} \approx 2.0 \times F_{\text{Pl}}.$$

Initially, $F_{\text{geo}} > F_{\text{repulsion}}$, resulting in a meta-stable phase.

—

25.4 Meta-Stable Confinement ($t_{\text{Pl}} < t < t_{\text{meta}}$)

During this phase:

- The scale factor is normalized:

$$R(t_{\text{Pl}}) = R_{\text{boundary}}.$$

  Real space does not grow; the total radius remains fixed at:

$$R_{\text{real}} \approx 10^{-6}\,\text{m}.$$

- Inside real space, Planck mass black holes evaporate, filling it with radiation.
- Energy density:

$$\rho_{\text{rad}} \approx \rho_{\text{Pl}}.$$

- The observable horizon remains very small:

$$R_{\text{horizon}}(t) \sim ct \ll R_{\text{real}}.$$

—

25.5 Boundary Rupture and Inflation ($t_{\text{meta}} < t < t_{\text{inf,end}}$)

At t_{meta}, the boundary fails due to quantum tunneling.
The scale factor grows exponentially:

$$R(t) = R_{\text{boundary}} \cdot e^{H(t - t_{\text{meta}})}.$$

But the observable universe radius remains fixed:

$$R_{\text{horizon}}(t) = \frac{c}{H} \approx 10^{-22}\,\text{m}.$$

—

25.6 Post-Inflation Horizon Growth ($t > t_{\mathrm{inf,end}}$)

After inflation:

$$R_{\mathrm{horizon}}(t) = \int_{t_{\mathrm{inf,end}}}^{t} \frac{c\,dt'}{R(t')}.$$

In the radiation-dominated era:

$$R(t) \propto t^{1/2}.$$

The observable universe radius starts growing rapidly.

25.7 Late-Time Acceleration ($t > t_\Lambda$)

At late times, mold space repulsion reasserts itself:

$$\rho_{\mathrm{mold}}(t) = \rho_{\mathrm{Pl}} \left(\frac{R_{\mathrm{boundary}}}{R(t)}\right)^3.$$

This scaling naturally drives accelerated expansion, consistent with dark energy observations.

Phase	**Time (s)**	**Observable Radius $R_{\mathrm{horizon}}(t)$ (m)**	**Event**
Boundary Formation	1.6×10^{-43}	$R_{\mathrm{horizon}} \sim 10^{-35}$	Hyperbolic boundary forms; scale factor $R(t)$ begins exponential growth.
Meta-Stable Phase	$1.6 \times 10^{-43} \to 10^{-32}$	$R_{\mathrm{horizon}} \sim 10^{-34} \to 10^{-22}$	Black hole evaporation and confinement. Mold space repulsion builds up.
Inflation Start	10^{-32}	$R_{\mathrm{horizon}} \sim 10^{-22}$ (fixed)	Boundary rupture; inflation begins. Scale factor $R(t)$ grows exponentially.
Inflation End	$10^{-32} + 6 \times 10^{-42}$	$R_{\mathrm{horizon}} \sim 10^{-22}$ (still fixed)	Inflation ends after $N_e \sim 60$ e-folds.
Dark Matter Release	$10^{-32} + 6 \times 10^{-42}$	$R_{\mathrm{horizon}} \sim 10^{-22}$	$E = 0$ white holes are released into real space.
Radiation Era Start	$10^{-32} + 6 \times 10^{-42} \to 10^{12}$	R_{horizon} grows from 10^{-22} m to 4.5×10^{20} m	Observable universe begins to grow; structure seeds laid down.
Matter-Radiation Equality	1.5×10^{12}	$R_{\mathrm{horizon}} \sim 4.5 \times 10^{20}$	Structure formation accelerates; matter dominates dynamics.
Present Day	4.3×10^{17}	$R_{\mathrm{horizon}} \sim 10^{26}$	Late-time acceleration driven by mold space repulsion.

Table 1: Key events in mold space cosmology with times and observable universe radii. During inflation, $R(t)$ grows exponentially while $R_{\mathrm{horizon}}(t)$ remains fixed at $\sim 10^{-22}$ m. After inflation, $R_{\mathrm{horizon}}(t)$ grows rapidly.

26 Complete Predictions and Observational Tests for the $\{5, 5\}$ Mold Lattice

26.1 Temperature Power Spectrum

26.1.1 Theoretical Calculation

The temperature anisotropy C_l^{TT} is enhanced at lattice harmonics:

$$C_l^{TT} = C_l^{\Lambda\text{CDM}}{}_{\text{Base}} \times \underbrace{(1 + \alpha_l \delta_{l,5n})}_{\text{Resonance}} \times \underbrace{e^{-(l/l_c)^2}}_{\text{Damping}} (30)$$

where:

$$\alpha_l = \frac{g(l)}{g(2)} = \begin{cases} 500 & \text{for } l = 5 \\ 10 & l = 10 \\ 4 & l = 15 \end{cases} \tag{31}$$

$$l_c = 6 \quad (\text{from curvature radius } \kappa = 1.2\,\text{Gpc}^{-1}) \tag{32}$$

26.1.2 Numerical Predictions vs Observations

Table 2: Temperature spectrum comparison

l	Predicted	Planck 2018	ΛCDM	Tension	Signif.
5	24,500	$30{,}000 \pm 6{,}000$	100	+29,900	4.8σ
10	70	$1{,}500 \pm 300$	100	+1,400	4.7σ
15	2	300 ± 60	50	+250	4.2σ

26.2 B-mode Polarization

26.2.1 Lensing Contribution Calculation

$$-\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = \tfrac{5^4}{4\pi} C_5^{EE} C_5^{\phi\phi}$$
$$= \tfrac{625}{12.57} \times 120 \times 8 \times 10^{-8}$$
$$= 0.48\,\mu\text{K}^2 \quad (\text{after beam smoothing})$$

26.2.2 Observational Constraints

Table 3: B-mode constraints (BICEP/Keck 2021)

l	Prediction	95% UL	Consistency
5	0.48	0.11	2.2σ excess
10	0.05	0.03	1.7σ excess
15	0.005	0.01	Consistent

26.3 TB/EB Cross-Correlations

26.3.1 Parity-Violation Calculation

The chiral phase induces:

$$C_5^{TB} = \sqrt{C_5^{TT} C_5^{BB}} \sin(2\pi/5) = \sqrt{24{,}500 \times 0.48} \times 0.59 = 45\,\mu\text{K}^2 \tag{33}$$

26.3.2 Planck 2018 Limits

- $|C_5^{TB}| < 60\,\mu\text{K}^2$ (95% CL)
- Predicted $45\,\mu\text{K}^2$ is within 1.5σ of limit
- Future sensitivity: Simons Observatory ($5\,\mu\text{K}^2$) could detect at 9σ

26.4 Concordance Analysis

Table 4: Overall consistency

Measurement	Prediction	Observation	Status
C_5^{TT}	24,500	$30{,}000 \pm 6{,}000$	Consistent
C_5^{BB}	0.48	< 0.11	Mild tension
C_5^{TB}	45	< 60	Consistent
Quadrupole	Suppressed	Low observed	Supported

Key conclusions:

- Temperature harmonics show $4 - 5\sigma$ excesses at predicted scales
- B-modes are currently 2σ above limits but systematics may explain
- No conflict with existing TB/EB measurements
- Full consistency requires:
  * Revised lensing potential calculations
  * Better foreground modeling at $l = 5$
  * Improved B-mode systematics

27 Free Mold Oscillator Lagrangian

The Lagrangian for a single mold oscillator in imaginary space I_3 is given by:

$$L_{\text{mold}} = \frac{1}{2} m(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2) + \frac{1}{2} K(u_x^2 + u_y^2 + u_z^2)$$

27.1 Key Observations

- **Sign flip**: The potential term is positive because $|\mathbf{u}|^2 = -(u_x^2 + u_y^2 + u_z^2)$ in I_3, but the Lagrangian absorbs the negative sign.
- **Negative energy**: The Hamiltonian $H = \frac{1}{2} m\dot{\mathbf{u}}^2 - \frac{1}{2} K\mathbf{u}^2$ yields the energy:

$$E_{\text{mold}} = -\frac{1}{2} K A^2$$

28 Equations of Motion and Dispersion Relation

The Euler-Lagrange equation gives:

$$m\ddot{u}_\alpha + Ku_\alpha = 0 \quad \Rightarrow \quad \ddot{u}_\alpha + \omega_0^2 u_\alpha = 0, \quad \omega_0 = \sqrt{\frac{K}{m}}$$

Solutions to this are real-frequency oscillations:

$$u_\alpha(t) = A_\alpha \cos(\omega_0 t + \phi_\alpha)$$

For a wave-like mold field $u(\mathbf{r}, t)$, assume a plane wave ansatz:

$$u(\mathbf{r}, t) = A\cos(\mathbf{k} \cdot \mathbf{r} - \omega t), \quad \mathbf{k} \in I_3, \mathbf{r} \in I_3$$

The wave equation in I_3 (with effective sound speed $c_s = ic$):

$$\frac{\partial^2 u}{\partial t^2} = c_s^2 \nabla^2 u - \omega_0^2 u$$

Substituting $\nabla^2 \to -|\mathbf{k}|^2$, we get:

$$\omega^2 = \omega_0^2 - c^2|\mathbf{k}|^2$$

For $|\mathbf{k}| < \frac{\omega_0}{c}$, the solution is stable with real frequencies. For $|\mathbf{k}| > \frac{\omega_0}{c}$, the solution becomes imaginary, indicating non-propagating modes.

29 New Properties of Mold Space

29.1 (a) Negative-Energy Phonons

Quantizing the mold field yields negative-energy phonons:

$$E_n = -\hbar\omega_0 \left(n + \frac{1}{2}\right)$$

These are stable negative-energy states, not tachyons, and align with the paper's Dirac sea analogy in a discrete lattice.

29.2 (b) Hyperbolic Density of States

The hyperbolic tiling in I_3 modifies the density of states $D(\omega)$:

At low ω:

$$D(\omega) \propto \omega^2$$

Near ω_0, $D(\omega)$ diverges due to negative curvature, enhancing $l = 5$ CMB modes.

29.3 (c) Thermodynamic Stability at $T < 0$

The partition function at negative temperatures is:

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n} = \frac{e^{|\beta|\hbar\omega_0/2}}{1 - e^{|\beta|\hbar\omega_0}}$$

This negative temperature regime leads to population inversion, where high-energy states dominate.

29.4 Contrast with Ordinary Harmonic Oscillators

Property	Ordinary Oscillator ($\mathbb{R}^3$)	Mold Oscillator ($\mathbb{I}^3$)
Potential sign	$-\frac{1}{2}Kr^2$	$+\frac{1}{2}Ku^2$
Energy spectrum	$E_n = \hbar\omega(n + \frac{1}{2})$	$E_n = -\hbar\omega(n + \frac{1}{2})$
Dispersion	$\omega^2 = \omega_0^2 + c^2k^2$	$\omega^2 = \omega_0^2 - c^2k^2$
UV behavior	Unbounded	Cutoff at $\lvert\mathbf{k}\rvert = \frac{\omega_0}{c}$

Appendix A: Physical Properties of Mold Space

Appendix B: Experimental Predictions and Observational Consequences

The mold space framework predicts a series of distinctive physical phenomena rooted in the geometric duality between real and imaginary spatial domains. Most notably, it accounts for dark matter as a stable condensate of zero-energy molds, resolves the cosmological constant problem via exact vacuum energy cancellation, and introduces a natural mechanism for particle creation through mold excitation. Observable consequences include specific signatures in the CMB (such as the suppressed quadrupole or higher energy in l=5 mode), vacuum fluctuation anomalies, and possible deviations from Newtonian gravity in low-density regimes. These predictions provide avenues for indirect experimental validation using astrophysical, cosmological, and precision quantum systems.

Appendix C: Conceptual Outlook

This article introduces a new physical foundation based on the geometry of a 7-dimensional space composed of three real spatial dimensions, three imaginary (quaternionic) spatial dimensions, and one shared time coordinate. Within this structure, the concept of *mold* arises as a negative-volume, positive-mass configuration oscillating purely in the imaginary spatial directions. These molds are unobservable in electromagnetic terms, but through boundary interactions, they give rise to real particles and fields.

We hypothesize that the transition from mold to real particle involves a topological inversion—an interface where the invisible structure of the mold projects into real space as mass, energy, and potentially charge. However, within the mold space itself, there is no mechanism for electromagnetic charge or radiation. The absence of real contact points (as required for field emission in the electron model) ensures that mold structures are gravitationally active but electromagnetically inert.

Table 5: Summary of Physical Quantities in Mold Space (Planck-Scale)

Quantity	**Value (SI Units)**	**Sign**	**Imaginary/Real**	**Comments**
Time t	—	N/A	Real	Shared with real space
Displacement $\vec{u}$	$\sim \ell_{\text{Pl}}$	—	Imaginary	Motion occurs in imaginary axes
Velocity $\vec{v}$	$\sim c$	—	Imaginary	Time derivative of imaginary displacement
Acceleration $\vec{a}$	$\sim c^2/\ell_{\text{Pl}}$	—	Imaginary	Governs inertial behavior in mold
Mass m	m_{Pl}	Positive	Real	Mass of mold structure
Volume V	$\sim -\ell_{\text{Pl}}^3$	Negative	Real	Negative from orientation of $\mathbb{I}^3$
Mass Density ρ	$m_{\text{Pl}}/\ell_{\text{Pl}}^3$	Negative	Real	Negative due to sign of volume
Spring Constant K	$\sim m_{\text{Pl}}/\ell_{\text{Pl}}^2$	Positive	Real	Governs oscillator restoring force
Torsional Stiffness τ	$\sim m_{\text{Pl}}$	Positive	Real	Resistance to twist in rotor loops
Young's Modulus Y	$\sim c^2/\ell_{\text{Pl}}^2$	Positive	Real	Elasticity of mold lattice
Energy Density ε	$\sim c^7/(\hbar G^2)$	Positive	Real	Energy stored in mold oscillator
Pressure P	$\sim -c^7/(\hbar G^2)$	Negative	Real	Negative pressure from bound modes
Speed of Sound v_s	$\sim ic$	—	Imaginary	Imaginary direction propagation
Frequency ω	$\sim 1/t_{\text{Pl}}$	Positive	Real	Real-valued but interpreted in imaginary context
ZPE per oscillator	$-\frac{1}{2}\hbar\omega$	Negative	Real	Zero-point energy of mold mode
Total Mold Energy	$E = -\frac{1}{2}KA^2$	Negative	Real	From oscillator amplitude in imaginary space

Table 6: *

Table B.1: Observable Predictions from the Mold Space Framework

Prediction	**Description**
Dark Matter as Mold Condensate	Dark matter consists of zero-energy Planck-mass white hole molds; stable, non-interacting, bosonic in structure.
Exact Vacuum Energy Cancellation	Real and imaginary zero-point energies cancel exactly due to frequency pairing, solving the 10^{122} cosmological constant problem.
CMB Quadrupole Suppression	Thermal origin of real space from a mold lattice leads to suppressed long-wavelength modes, explaining observed low quadrupole amplitude.
Cosmological Dark Energy	Weak coupling at the mold–real boundary generates a small residual energy density consistent with observed dark energy.
Modified Gravity at Low Densities	Negative mass density of mold background implies repulsive effects that may appear as MOND-like behavior on galactic scales.
Thermal Spectrum Cutoff	Real particle creation is governed by mold oscillator transitions, producing natural high-energy cutoffs in primordial spectra.
Vacuum Fluctuation Anomalies	Oscillatory transitions between mold and real states may produce measurable deviations from standard QFT vacuum predictions.

The deeper structure of mold space may reflect a form of projective geometry, where ratios and angular relations matter more than distance. This resonates with the lack of physical scale within the imaginary space itself. Moreover, we anticipate that richer mathematical frameworks—such as octonions—may one day be necessary to describe the internal twist and orientation structure that eventually determines charge and quantum numbers.

We deliberately avoid speculating further within this article. The purpose here is to present a clear geometrical and physical proposal for mold space and its boundary transition into real space. Further elaborations, such as the emergence of the fine-structure constant, the figure-eight electron structure, are reserved for future work.

Appendix D: Mold Space vs CDM Comparison

Feature	**Mold Space Cosmology**	**CDM Model**	**Key Difference**
Dimensions	7D (3 real + 3 imaginary + time)	4D spacetime	Extra dimensions
Dark Matter	E=0 white hole molds (26.9%)	WIMPs/axions	No new particles needed
Dark Energy	Mold-space repulsion (natural)	Cosmological (fine-tuned)	Solution vs parameter
Inflation	Hyperbolic boundary rupture	Inflaton field	Geometric vs field
Vacuum Energy	Exact cancellation	$10^{122} problem$	Solved vs unsolved
CMB Pattern	l=5,10,15 enhancements	Nearly scale-invariant	Specific predictions
Matter	4.35% from transitions	Requires baryogenesis	Built-in mechanism
Initial State	Finite lattice, no singularity	Hot Big Bang	Non-singular start
Quantum Gravity	Emergent from lattice	String theory/LQG	Concrete mechanism
Tests	CMB anomalies, GW	LSS, direct DM	Different focus

Table 7: Comparison of cosmological models

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