The Unified Gravitogenesis Theory (UGT): A Comprehensive Approach to Modified Gravity and Cosmological Phenomenology

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July 7, 2025

Abstract

The Unified Gravitogenesis Theory (UGT) proposes a reformulation of gravity that seeks to offer a unified explanation for phenomena attributed to dark matter and dark energy within the Standard Cosmological Model (ΛCDM). Grounded in a scalar field Lagrangian that describes the dynamics of fundamental fields, UGT postulates that the effective gravitational constant, G_{eff}, emerges as a function of the local energy density, $\kappa(\rho)$. This article details the theoretical principles of UGT, the mathematical development of field equations from a fundamental Lagrangian, and the subsequent computational implementation to simulate phenomena on galactic and cosmological scales. We present the results of simulations for galactic rotation curves and the spatial distribution of modified gravitational fields in 1D, 2D, and 3D, as well as the investigation of the cosmological dynamics of the effective scalar field. The numerical challenges encountered during the simulation of cosmic evolution are discussed, highlighting the inherent complexity in modeling modified gravity theories.

Keywords: *Modified Gravity, Dark Matter, Dark Energy, Scalar Field, Cosmology, Numerical Simulations, Field Theory.*

1. Introduction

The Standard Cosmological Model, ΛCDM, has been extraordinarily successful in describing the evolution of the universe across various scales, from the anisotropies of the Cosmic Microwave Background to the formation of large-scale structures (Planck Collaboration et al., 2020). However, this success is built upon the premise of the existence of two major components of unknown nature: dark matter, which accounts for approximately 27% of the universe's energy density and is essential for galactic dynamics and structure formation, and dark energy, which constitutes about 68% and is responsible for the observed accelerated expansion of the universe (Riess et al., 1998; Perlmutter et al., 1999). The lack of direct detection of these components and the absence of a fundamental explanation for their origin motivate the exploration of alternative theories of modified gravity (Clifton et al., 2012).

The Unified Gravitogenesis Theory (UGT) emerges in this scenario as a proposal that seeks to offer a unified explanation for phenomena attributed to dark matter and dark energy, without the need for additional exotic components. UGT distinguishes itself by postulating that Newton's gravitational constant, G_0, is not a universal constant, but rather a function that varies with the local energy density, ρ. This intrinsic variation of gravity is conceived as a manifestation of underlying fundamental fields, whose dynamics are governed by a scalar field Lagrangian.

This article aims to detail the theoretical foundations of UGT, from its most fundamental principles based on a scalar field Lagrangian, through the mathematical derivation of its equations, to its computational implementation for simulating cosmological and galactic phenomena. In Section 2, we present the theoretical principles of UGT, introducing the $\kappa(\rho)$ function and the role of the scalar field. Section 3 is dedicated to the detailed mathematical development, including the symbolic derivation of field equations from a fundamental Lagrangian and the transition to the effective cosmological model. Section 4 describes the computational implementation and the results of dark matter simulations (rotation curves and field visualization) and cosmological dynamics. Finally, Section 5 discusses the implications of UGT, its challenges, and prospects for future research.

2. Theoretical Foundations of UGT: From Field Theory to Effective Gravity

UGT postulates that the effective gravitational constant, G_{eff}, is not a constant, but rather a function of the local energy density, ρ. This dependence is expressed through a function $\kappa(\rho)$:

$$G_{\text{eff}}(\rho) = G_0 \cdot \kappa(\rho) \quad (1)$$

where G_0 is Newton's gravitational constant. The function $\kappa(\rho)$ is designed to exhibit behavior that approximates General Relativity at high densities and deviates from it at low densities, mimicking the effects of dark matter and dark energy. A common functional form for $\kappa(\rho)$ is:

$$\kappa(\rho) = \kappa_{\text{base}} + (\kappa_{\text{enhancement}} - \kappa_{\text{base}}) \exp\left(-\left(\frac{\rho}{\rho_{\text{char}}}\right)^{\beta_\kappa}\right) \quad (2)$$

In this formulation, κ_{base} is the value of κ at high densities (typically 1), $\kappa_{\text{enhancement}}$ is the enhancement factor at low densities, ρ_{char} is a characteristic density that defines the transition scale, and β_κ is a parameter that controls the "sharpness" of the transition. The phenomenological parameters used in our galactic simulations were: $\kappa_{\text{base}} = 1.0$, $\kappa_{\text{enhancement}} = 2.0$, $\rho_{\text{char}} = 1.0 \times 10^{-24}\ \text{kg/m}^3$, and $\beta_\kappa = 2.0$.
The origin of this variation in G_{eff} and the dynamics driving dark energy phenomena are attributed to a fundamental scalar field, ϕ. In UGT, this field is not an *ad hoc* entity but emerges from a more fundamental field theory. While the complete theory may involve multiple interacting fields, for cosmological purposes, an effective scalar field ϕ is often sufficient to capture the essence of gravitational modification. The dynamics of this field ϕ are governed by a potential $V(\phi)$, which can take various forms. An "optimized Mexican hat" potential is particularly interesting for its ability to generate dynamic "dark energy" and its relationship with scalar field models in cosmology:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 - \beta_{\text{potential}}\phi^2 \exp\left(-\frac{\phi^2}{\sigma^2_{\text{potential}}}\right) + \Lambda_{\text{cos}}$$

Here, m^2 is the mass term, λ is the self-interaction constant, $\beta_{\text{potential}}$ and $\sigma_{\text{potential}}$ are parameters that modulate the potential's shape, and $\Lambda_{\text{cosmological}}$ is an intrinsic cosmological constant term. The cosmological parameters adopted in our simulations were: $\alpha = 5.0$, $\phi_0 = 1.0$, $m^2 = -1.0 \times 10^{-11}$, $\lambda = 1.0 \times 10^{-10}$, $\beta_{\text{potential}} = 1.0 \times 10^{-5}$, $\sigma_{\text{potential}} = 20.0$, and $\Lambda_{\text{cosmological}} = 2.0 \times 10^{-4}$.

3. Mathematical Development: From Fundamental Lagrangian to Effective Cosmology

UGT can be formally constructed from a scalar field Lagrangian that describes the interaction of fundamental fields. This approach allows for a consistent derivation of the equations of motion and provides a theoretical basis for observed phenomena. To illustrate the complexity and structure of the underlying field theory, we present a "toy" Lagrangian that involves multiple fields and their interactions.

3.1. Fundamental Lagrangian and Symbolic Derivation

We consider a total Lagrangian density $\mathcal{L}_{\text{total}}$ that includes a scalar field Φ, an auxiliary field Ψ, and a field GG (which can be interpreted as a mediator of gravitational interaction or a curvature-related field), all dependent on a spatial variable x:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\Phi} + \mathcal{L}_{\Psi} + \mathcal{L}_{GG} + \mathcal{L}_{GG-\text{Matter}} \quad (4)$$

The individual terms of the Lagrangian are defined as:

Lagrangian of Field Φ:

$$\mathcal{L}_{\Phi} = \frac{1}{2}(\nabla\Phi)^2 - V(\Phi) \quad (5)$$

with the potential $V(\Phi) = a_v(\Phi^2 - b_v)^2$, where a_v and b_v are constants.

Lagrangian of Field Ψ:

$$\mathcal{L}_{\Psi} = \frac{1}{2}(\nabla\Psi)^2 - \frac{1}{2}m_{\Psi}^2\Psi^2 \quad (6)$$

where m_{Ψ} is the mass of field Ψ.

Lagrangian of Field GG:

$$\mathcal{L}_{GG} = \frac{1}{2}(\nabla GG)^2 + \frac{1}{2}\lambda_{GG}(GG - \rho_{\text{quantum}})^2 \quad (7)$$

where λ_{GG} is a coupling constant and $\rho_{\text{quantum}} = \frac{1}{2}(\Phi^2 + \Psi^2)$ is an energy density of quantum or field origin.

GG-Matter Coupling Lagrangian:

$$\mathcal{L}_{GG-\text{Matter}} = -gg_{\text{coupling}} GG^2 \rho_{\text{quantum}} \quad (8)$$

where gg_{coupling} is the coupling constant between the GG field and the quantum density.
The field equations for Φ, Ψ, and GG are obtained by applying the Euler-Lagrange equations:

$$\frac{\partial}{\partial x}\left(\frac{\partial \mathcal{L}_{\text{total}}}{\partial(\partial_x \chi)}\right) - \frac{\partial \mathcal{L}_{\text{total}}}{\partial \chi} = 0 \quad (9)$$

where χ represents Φ, Ψ, or GG. The symbolic derivation of these equations, performed computationally, yields the following expressions:
Field Equation for Φ:

$$-\frac{\partial^2}{\partial x^2}\Phi(x) - 2a_v\Phi(x)(\Phi(x)^2 - b_v) - \lambda_{GG}\Phi(x)\left(\frac{1}{2}\Phi(x)^2 + \frac{1}{2}\Psi\right.$$

Field Equation for Ψ:

$$-\frac{\partial^2}{\partial x^2}\Psi(x) - m_\Psi^2\Psi(x) - \lambda_{GG}\Psi(x)\left(\frac{1}{2}\Phi(x)^2 + \frac{1}{2}\Psi(x)^2 - GG(\right.$$

Field Equation for GG:

$$-\frac{\partial^2}{\partial x^2}GG(x) + \lambda_{GG}\left(\frac{1}{2}\Phi(x)^2 + \frac{1}{2}\Psi(x)^2 - GG(x)\right) - 2gg_{\text{coupl}}$$

These equations demonstrate the complex interconnection between the fundamental fields of UGT.

3.2. Transition to the Effective Cosmological Model

The fundamental Lagrangian and field equations derived in Section 3.1 establish the theoretical basis for UGT. In the cosmological context, the

dynamics of these fundamental fields, especially in a homogeneous and isotropic universe, can be effectively described by a single scalar field, ϕ, and its interaction with the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. The field ϕ used in the cosmological formulation (Equation 3) is, therefore, an effective representation or a dominant component that emerges from the complex dynamics of the fundamental fields on larger scales.
The Friedmann equations, which describe the evolution of the scale factor $a(t)$ in a homogeneous and isotropic universe, are modified in UGT to incorporate the effective gravitational constant $G_{\text{eff}}(\rho)$ and the energy-pressure density of the scalar field ϕ. The system of coupled differential equations governing the cosmological evolution of UGT is given by:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\text{eff}}(\rho_{\text{total}})}{3}\rho_{\text{total}} \quad (13)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{\text{eff}}(\rho_{\text{total}})}{3}(\rho_{\text{total}} + 3P_{\text{total}}) \quad (14)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (15)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_{total} is the total energy density (matter, radiation, and scalar field), P_{total} is the total pressure, and $V'(\phi)$ is the derivative of the scalar field potential with respect to ϕ. The energy density and pressure of the scalar field are given by:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (16)$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (17)$$

This system of coupled equations is at the core of UGT's cosmological simulation, allowing us to investigate how the scalar field's dynamics and the variation of G_{eff} influence the expansion of the universe.

4. Computational Implementation and Simulation Results

UGT was computationally implemented in Python, using a modular environment to manage the different aspects of the theory and simulations. The `numpy` (for numerical operations), `scipy.integrate` (for ODE integration), and `sympy` (for symbolic derivation) libraries were the main tools.

4.1. Dark Matter Simulations (Galactic Scales)

The `TGUMetricSolver` module was used to model matter distribution in galaxies and calculate UGT's response to these distributions.

4.1.1. Galactic Rotation Curves (1D)

Objective: To demonstrate UGT's ability to reproduce the observed flat rotation curves in spiral galaxies, without the need for a non-baryonic dark matter halo.

Methodology: A typical galaxy was modeled with three baryonic matter components: a bulge (Hernquist profile), a disk (exponential disk), and a halo (NFW profile, whose mass is the target of UGT's explanation). The galaxy parameters were: $M_{\text{bulge}} = 1.5 \times 10^{10} M_{\odot}$, $a_{\text{bulge}} = 0.5 \text{ kpc}$, $M_{\text{disk}} = 6.0 \times 10^{10} M_{\odot}$, $R_{\text{disk}} = 3.0 \text{ kpc}$, $z_0 = 0.3 \text{ kpc}$, $M_{\text{halo}} = 1.0 \times 10^{12} M_{\odot}$, $R_{\text{vir}} = 200.0 \text{ kpc}$, $c_{\text{nfw}} = 10.0$. The total baryonic matter density was calculated as a function of radius, and the $\kappa(\rho)$ function (Equation 2) was applied to determine G_{eff} at each point. Rotation velocities were then calculated under UGT and compared with Newtonian predictions.

Results: Simulations showed that in the central, high-density regions of the galaxy, where $\rho \gg \rho_{\text{char}}$, the $\kappa(\rho)$ function approaches 1, and UGT's rotation curve coincides with the Newtonian prediction. However, in the galactic outskirts, where baryonic matter density significantly decreases ($\rho \lesssim \rho_{\text{char}}$), $\kappa(\rho)$ increases towards $\kappa_{\text{enhancement}}$. This increase in G_{eff} results in rotation velocities that remain high at large radii, mimicking the observed flat rotation curves and eliminating the need for dark matter. UGT, therefore, offers a gravitational explanation for the dark matter problem on galactic scales.

4.1.2. Field Visualization in 2D and 3D

Objective: To provide an intuitive spatial representation of how matter density, the $\kappa(\rho)$ function, and the effective gravitational constant G_{eff} vary within a galaxy.

Methodology: 2D (in the galactic plane) and 3D (slices) grids were created for a modeled galaxy. For each grid point, the total matter density was calculated, followed by the determination of $\kappa(\rho)$ and G_{eff}. Heatmaps (2D) and slice visualizations (3D) were generated to display

the distribution of these fields.
Results: Visualizations confirmed that in dense regions (bulge and inner disk), $\kappa(\rho)$ remains close to 1, and G_{eff} is essentially G_0. As the distance from the galactic center increases and density decreases, the visualizations clearly show an increase in $\kappa(\rho)$ and, consequently, in G_{eff}. This pattern graphically illustrates how UGT modifies gravity non-uniformly in space, with the most pronounced modification occurring in low-density regions.

4.2. Cosmological Simulations

The `TGUFieldEquations` module was used to simulate the evolution of the universe under UGT and compare it with the ΛCDM model.

4.2.1. Symbolic Derivation of Field Equations

Objective: To validate the correctness of the field equations (Euler-Lagrange) for the fundamental fields of the toy Lagrangian, ensuring the mathematical consistency of the theory.
Methodology: Using the `sympy` library, the total Lagrangian (Equation 4) was symbolically defined. The Euler-Lagrange equations (Equation 9) were symbolically applied to derive the equations of motion for fields Φ, Ψ, and GG.
Results: The symbolic field equations (Equations 10, 11, 12) were successfully obtained, confirming the mathematical derivation. This step is crucial for UGT's theoretical robustness, as it validates the equations describing the dynamics of the fundamental fields from which modified gravity emerges.

4.2.2. Cosmological Evolution of the Effective Scalar Field (UGT vs. ΛCDM)

Objective: To simulate the evolution of the scale factor $a(t)$ and the dynamics of the effective scalar field $\phi(t)$ under UGT's equations (Equations 13-15), and compare with the ΛCDM model's evolution.
Methodology: The system of ODEs for UGT (a, $\dot{a}$, ϕ, $\dot{\phi}$) and for ΛCDM (a, $\dot{a}$) was numerically integrated using `scipy.integrate.solve_ivp`. Initial conditions for a, $\dot{a}$, ϕ, and $\dot{\phi}$ were set at an initial time of $0.5\ \mathrm{Gyr}$. The initial expansion rate $\dot{a}$ was calculated from the radiation-dominated Friedmann equation, ensuring consistency of units (Gyr^{-1}) with the rest of the system.
Results and Numerical Challenges: The simulation of UGT's and ΛCDM's cosmological evolution revealed the system's extreme sensitivity to numerical perturbations and initial conditions. Although the mathematical framework was correctly implemented and units were consistent, the highly non-linear and coupled nature of UGT's equations, particularly the dependence of G_{eff} on the total density, posed significant challenges for obtaining a robust and long-term (on the order of billions

of years) numerical integration.
Numerical Instability: A persistent tendency towards instability was observed, where the values of derivatives (especially $\ddot{a}$) reached extreme magnitudes or resulted in non-numerical values (`NaN` , `inf`) after a short integration period. This prevented the simulation from progressing for long cosmic timescales, making it unfeasible to obtain a complete and stable cosmological history.
Extreme Sensitivity: Small variations in the initial conditions of the scalar field (ϕ, $\dot{\phi}$) or in the potential parameters could lead to divergent and unstable behaviors. This sensitivity makes it difficult to calibrate the theory with observational data and to systematically explore the parameter space to find physically reasonable solutions.
Convergence: Obtaining a convergent and stable solution over long periods would require extremely tight numerical tolerances (`rtol` , `atol`) and/or the use of more robust and adaptive integration methods, with very small time steps, which would significantly increase computational cost and execution time.
Despite these challenges, the computational structure allowed for the verification of equation consistency and the exploration of scalar field dynamics over short time intervals, providing *insights* into the complexity of the interaction between the ϕ field, energy density, and modified gravity.

5. Discussion and Conclusions

The Unified Gravitogenesis Theory (UGT) presents a fundamental approach to the challenges of dark matter and dark energy, proposing that gravity itself is a dynamic interaction that varies with local energy density. This variation emerges from an underlying scalar field theory, whose equations of motion were symbolically derived, confirming the mathematical consistency of the theoretical framework.
Computational simulations on galactic scales demonstrated UGT's potential to explain phenomena such as the flat rotation curves of spiral galaxies. By allowing G_{eff} to increase in low-density regions, UGT offers a mechanism to mimic the effects of dark matter without the need for exotic particles. The 2D and 3D visualizations of density fields, $\kappa(\rho)$, and G_{eff} clearly illustrate how gravitational modification manifests spatially, with the most pronounced modification occurring in low-density regions.
In the cosmological domain, UGT seeks to explain the accelerated expansion of the universe through the dynamics of the effective scalar field ϕ. Although the mathematical formulation for cosmological evolution was implemented, the simulation faced significant numerical challenges. The highly non-linear and coupled nature of UGT's equations, coupled with sensitivity to initial conditions and the dependence of G_{eff} on total

density, resulted in instabilities that prevented robust and long-term integration of the system. This highlights the inherent complexity in modeling modified gravity theories with dynamic couplings and the need for extremely sophisticated numerical methods to effectively explore their parameter space.

UGT remains an active and promising area of research in cosmology. To advance the validation and refinement of this theory, future work should focus on:

Numerical Optimization and Stabilization: Developing and applying more advanced numerical integration methods and error control strategies to overcome instabilities and enable long-term cosmological simulations.

Comprehensive Parameter Space Exploration: Conducting a systematic exploration of UGT's vast parameter space to identify regions that yield a cosmological history consistent with current observations.

Detailed Observational Predictions: Once simulations become stable, UGT can generate detailed predictions for a wide range of cosmological observables, such as the universe's expansion history, large-scale structure formation, and the evolution of density fluctuations, allowing for direct and rigorous comparisons with observational data.

UGT offers an elegant and unified alternative to the unknown components of the universe, but the complexity of its numerical modeling is a significant challenge that needs to be overcome for its predictions to be fully explored and tested against the real universe.

References

Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. (2012). Modified Gravity and Cosmology. *Physics Reports*, 513(1-3), 1-189.

Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., ... & White, R. L. (1999). Measurements of Ω and Λ from 42 High-Redshift Supernovae. *The Astrophysical Journal*, 517(2), 565.

Planck Collaboration et al. (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641, A6.

Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., ... & Schmidt, B. P. (1998). Astronomical Journal, 116, 1009.