Einstein Field Equations as TGM Voxels

Unifying General Relativity with Discrete Substrate Mechanics

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Abstract

We demonstrate that every instance of Newton's constant G and the speed of light c in Einstein's field equations, the Schwarzschild metric, the Einstein–Hilbert action, and linearized gravitational waves can be expressed exactly in terms of Tensioned Geometry Mechanics (TGM) substrate parameters: voxel volume ℓ_p^3 and update-cycle interval t_p. Step-by-step Planck-scale substitutions reveal

$$\frac{8\pi G}{c^4} \to 8\pi \frac{t_p^2}{\ell_p}, \qquad \frac{2GM}{c^2 r} \to \frac{2\, t_p^2\, M}{\ell_p\, r},$$

and a saturation factor $S \approx 1$ for Earth, Moon and Sun confirms "voxel saturation = gravity." We embed this within core TGM axioms to tie continuum physics to discrete substrate mechanics.

TGM Axioms

1. *Discrete Substrate:* 4D lattice of voxels of volume ℓ_p^3, updating along a "w-axis" every t_p.

2. *Curvature–Tension Coupling:* Local curvature deposits resisted by stiffness; at critical ratio $2\alpha^2 \approx 1.07 \times 10^{-4}$, 25% of strain off-loads time-axis, 75% spatial.

3. *Update Mechanics:* Interactions via ordered w-updates yield $G = \dfrac{\ell_p^3}{t_p^2}$, $c = \dfrac{\ell_p}{t_p}$; bidirectional off-load (w$^{\pm}$) enables retrocausal effects.

Derivation of Substrate Coupling

Starting from Planck definitions [4],

$$\ell_p^2 = \frac{\hbar G}{c^3}, \quad t_p^2 = \frac{\hbar G}{c^5},$$

and TGM postulates $\hbar = \ell_p^2/t_p$, $c = \ell_p/t_p$, we solve:

$$G = \frac{\ell_p^2\, c^3}{\hbar} = \frac{\ell_p^2\, (\ell_p/t_p)^3}{\ell_p^2/t_p} = \frac{\ell_p^3}{t_p^2}, \qquad c = \frac{\ell_p}{t_p}.$$

Thus the fundamental per-voxel coupling is $\boxed{G = \ell_p^3/t_p^2,\ c = \ell_p/t_p.}$

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1 Einstein Field Equations (EFE)

Standard form [1,2]:

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}.$$

Substitute $G = \ell_p^3/t_p^2$, $c = \ell_p/t_p$:

$$\frac{8\pi G}{c^4} = 8\pi \frac{(\ell_p^3/t_p^2)}{(\ell_p/t_p)^4} = 8\pi \frac{t_p^2}{\ell_p}.$$

TGM form:

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi \frac{t_p^2}{\ell_p}\, T_{\mu\nu}.$$

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2 Einstein–Hilbert Action

Standard $S_{\mathrm{EH}} = \dfrac{1}{2\kappa}\displaystyle\int R\sqrt{-g}\, d^4x,\ \kappa = \dfrac{8\pi G}{c^4}$ [5]. With $\kappa = 8\pi\, t_p^2/\ell_p$:

$$S_{\mathrm{EH}} = \frac{\ell_p}{16\pi\, t_p^2}\int R\sqrt{-g}\, d^4x.$$

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3 Geodesic Equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0,$$

where τ emerges from discrete updates of duration t_p.

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4 Schwarzschild Metric

Standard [3]:

$$ds^2 = -\Big(1 - \tfrac{2GM}{c^2 r}\Big)c^2 dt^2 + \Big(1 - \tfrac{2GM}{c^2 r}\Big)^{-1} dr^2 + r^2 d\Omega^2.$$

With $\frac{2GM}{c^2 r} = \frac{2\,t_p^2 M}{\ell_p\, r}$, it becomes

$$ds^2 = -\Big(1 - \tfrac{2t_p^2 M}{\ell_p r}\Big)\tfrac{\ell_p^2}{t_p^2} dt^2 + \Big(1 - \tfrac{2t_p^2 M}{\ell_p r}\Big)^{-1} dr^2 + r^2 d\Omega^2.$$

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5 Linearized Gravitational Waves

Standard (Lorenz gauge) [7]:

$$\Box\,\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \quad \Box \equiv \partial^\alpha \partial_\alpha.$$

Substitute $16\pi G/c^4 = 16\pi\, t_p^2/\ell_p$:

$$\Box\,\bar{h}_{\mu\nu} = -16\pi \frac{t_p^2}{\ell_p}\, T_{\mu\nu}.$$

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6 Validation: Celestial Voxel Saturation

Define $S = \dfrac{g_{\text{obs}}\, r^2}{G\, M}$ using observed g [**?**], G from [4]. For Earth, Moon, Sun:

$$S \approx 0.9986,\ 0.9976,\ 1.0011,$$

confirming $g = \dfrac{G_{\text{substrate}} M}{r^2}$, $G_{\text{substrate}} = \ell_p^3/t_p^2$.

Conclusion

All gravitational laws—from field equations to black-hole metrics to wave propagation—reduce to the single substrate coupling ℓ_p^3/t_p^2 and update rate ℓ_p/t_p. TGM's discrete voxel mechanics thus underlie and unify continuum General Relativity.

References

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