

A New Formula to Determine the Prime Number

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Abstract

I present a new analytical formulation to determine whether a given number p is prime. The method involves constructing a specific sequence of integer products and mapping it to a continuous function via Fourier representation. Using complex analysis, particularly the argument principle, we explore the number of zeroes of this function to infer the compositeness of p . This approach provides a new perspective on primality testing through continuous mathematics.

1 Introduction

The problem of efficiently determining whether a number is prime has intrigued mathematicians for centuries. While several algorithms exist — from trial division to modern probabilistic and deterministic tests like AKS — this work proposes an analytical method based on constructing a discrete matrix of potential factors and converting it into a continuous function. The behavior of this function, particularly its zeroes, reveals whether the original number is composite. This paper presents the construction of this function and demonstrates how complex analysis can be used to analyze it.

2 proof

lets think a number p .we want to determine is it a prime number or not

$$A = \begin{pmatrix} 2 \times 2 & 2 \times 3 & \dots & 2 \times \lfloor \frac{p}{2} \rfloor \\ 3 \times 2 & 3 \times 3 & \dots & 3 \times \lfloor \frac{p}{2} \rfloor \\ \vdots & \vdots & \ddots & \vdots \\ \lfloor \sqrt{p} \rfloor \times 2 & \lfloor \sqrt{p} \rfloor \times 3 & \dots & \sqrt{p} \times \lfloor \frac{p}{2} \rfloor \end{pmatrix}$$

if p is a composite number then the largest factor of p is less than or equal $\lfloor \frac{p}{2} \rfloor$ and there must exist a factor of p is less than or equal \sqrt{p} ,so i can say that p exists in A if p is composite

But if p is a prime number p doesn't exist in A

Now i want to make A as a series like that

2×2 is the 0th term of the series

2×3 is the 1st term

\vdots

$2 \times \lfloor \frac{p}{2} \rfloor$ is the $(\lfloor \frac{p}{2} \rfloor - 2)$ th term

3×2 is the $(\lfloor \frac{p}{2} \rfloor - 1)$ th term

3×3 is the $\lfloor \frac{p}{2} \rfloor$ th term

\vdots

$3 \times \lfloor \frac{p}{2} \rfloor$ is the $(2\lfloor \frac{p}{2} \rfloor - 3)$ h term

\vdots

$\lfloor \sqrt{p} \rfloor \times \lfloor \frac{p}{2} \rfloor$ is the $(\lfloor \sqrt{p} \rfloor \lfloor \frac{p}{2} \rfloor - \lfloor \sqrt{p} \rfloor - \lfloor \frac{p}{2} \rfloor)$ th term

So now I want to make a formula that can find the n th term of this above series

Each term of the series is factor of two numbers . The first number of each term changes after $\lfloor \frac{p}{2} \rfloor - 2$ term . So the first number of n th term is

$$(\lfloor \frac{n}{a} \rfloor + 2) \text{ where } a = \lfloor \frac{p}{2} \rfloor - 1$$

Now the second number of each term start from 2 and goes to $\lfloor \frac{p}{2} \rfloor$ and then again start from 2 and goes to $\lfloor \frac{p}{2} \rfloor$ and this process remain continue ,so the second number of n th term will be

$$(\lfloor n + 1 \rfloor - a \lfloor \frac{n}{a} \rfloor + 1)$$

Finally the n th term of the series will be

$$(\lfloor \frac{n}{a} \rfloor + 2) (\lfloor n + 1 \rfloor - a \lfloor \frac{n}{a} \rfloor + 1)$$

we know that

$$0 \leq n \leq (\lfloor \sqrt{p} \rfloor \lfloor \frac{p}{2} \rfloor - \lfloor \sqrt{p} \rfloor - \lfloor \frac{p}{2} \rfloor)$$

But i want to make $0 \leq n \leq 1$ to do this the series formula will be

$$(\lfloor bn \rfloor + 2) (\lfloor n + 1 \rfloor - a \lfloor bn \rfloor + 1)$$

$$\text{Here } b = \frac{(\lfloor \sqrt{p} \rfloor \lfloor \frac{p}{2} \rfloor - \lfloor \sqrt{p} \rfloor - \lfloor \frac{p}{2} \rfloor)}{\lfloor \frac{p}{2} \rfloor - 1}$$

n value is from 0 to 1 now we can say that

$$p - (\lfloor bn \rfloor + 2) (\lfloor n + 1 \rfloor - a \lfloor bn \rfloor + 1) \neq 0$$

if and only if p is a prime

Now i want to make this function continuous so i change the floor function by a fourier representation and i want to avoid the gibbs phenomena so i use a window function to reduce gibbs phenomena then the expression will be look like that

$$\lim_{K \rightarrow \infty} p^{-\left(bn + \frac{3}{2} + \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kbn)}{2\pi k} \right)} \cdot \left((1-ab)n + \left(\frac{3}{2} + \frac{a}{2} \right) + \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kn)}{2\pi k} \right) - a \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kbn)}{2\pi k} \neq 0$$

Now think it is a function

$$f(n) = \lim_{K \rightarrow \infty} p^{-\left(bn + \frac{3}{2} + \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kbn)}{2\pi k} \right)} \cdot \left((1-ab)n + \left(\frac{3}{2} + \frac{a}{2} \right) + \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kn)}{2\pi k} \right) - a \sum_{k=1}^K \frac{(1 - \cos(\frac{\pi k}{K})) \sin(2\pi kbn)}{2\pi k}$$

we know n is from 0 to 1 ,If i apply the complex argument principle of f(n) on the region of x from 0 to 1 and y from -h to h where h tends to 0 .then we will get how many zeroes f(n) have because f(n) have no poles on this region but we know if p is prime $f(n) \neq 0$ now i use z instead of n now we can say that

$$\frac{1}{2\pi i} \oint_{\gamma} \frac{f'(z)}{f(z)} dz = \begin{cases} 0 & \text{if p is prime} \\ > 0 & \text{if p is composite} \end{cases}$$

γ is a region in complex plane and that is $x \in [0, 1]$ and $y \in [-h, h]$ where $h \rightarrow 0^+$

But actually there is a problem if p is prime then also there are some zeroes of f(z) because by using fourier series we make it a continuous function .

we know $n > 2 \times \lfloor \frac{n}{2} \rfloor$ if n is a odd integer . so we can say that in the continuous function the line cross the x axis $(\lfloor \sqrt{p} \rfloor - 2)$ times twice

so now the conditions will be

$$2(\lfloor \sqrt{p} \rfloor - 2) - \frac{1}{2\pi i} \oint_{\gamma} \frac{f'(z)}{f(z)} dz = \begin{cases} 0 & \text{if p is prime} \\ < 0 & \text{if p is composite} \end{cases}$$

But this formula work well for odd number not for even number but thats not problem because we know all even number is composite except 2

Now if i differentiate f(z) then we get

$$\begin{aligned}
 f'(z) = \lim_{K \rightarrow \infty} - \left[\right. & \left(b + b \sum_{k=1}^K \left(1 - \cos \left(\frac{\pi k}{K} \right) \right) \cos(2\pi k b z) \right) \cdot \left((1 - ab)z + \left(\frac{3}{2} + \frac{a}{2} \right) \right. \\
 & + \sum_{k=1}^K \frac{(1 - \cos \left(\frac{\pi k}{K} \right)) \sin(2\pi k z)}{2\pi k} - a \sum_{k=1}^K \frac{(1 - \cos \left(\frac{\pi k}{K} \right)) \sin(2\pi k b z)}{2\pi k} \left. \right) \\
 & + \left(b z + \frac{3}{2} + \sum_{k=1}^K \frac{(1 - \cos \left(\frac{\pi k}{K} \right)) \sin(2\pi k b z)}{2\pi k} \right) \cdot \left((1 - ab) \right. \\
 & \left. \left. + \sum_{k=1}^K \left(1 - \cos \left(\frac{\pi k}{K} \right) \right) \cos(2\pi k z) - ab \sum_{k=1}^K \left(1 - \cos \left(\frac{\pi k}{K} \right) \right) \cos(2\pi k b z) \right) \left. \right]
 \end{aligned}$$

Its a convergent series so it's not needs to K be very big

3 Analysis and Limitations

This method is highly analytical and demonstrates an elegant connection between number theory and continuous analysis. However:

- It does not work for even numbers (except 2), though this is a minor issue.
- The method involves computation of Fourier series terms, which could be computationally intensive.
- The computational cost grows with K , though convergence reasonably fast
- Because we approximate floor functions with Fourier series, small numerical errors or spurious zeros may appear — handled by adjusting the region and applying the argument principle with care.

4 Conclusion

I propose a novel, continuous function built from matrix factorizations and Fourier analysis, whose analytic properties are tightly linked to the primality of a given number. This approach provides a unique theoretical tool and opens pathways to further analytical study in prime detection.

References

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