Beyond the Horizon (Part II): Inside-Out Cosmology

Radial Inversion and Conformal Structure in a VSL Framework

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Abstract

We extend a previously proposed cosmological model in which the interior of a black hole is treated as an independent universe governed by a variable speed of light (VSL). In this Part II, we develop the "inside-out" geometry suggested by the original framework through coordinate transformations that invert standard causal and geometric structures. Using a radial inversion and a conformal time rescaling, we reframe the black hole horizon as a geometric origin surface analogous to the Big Bang. This transformed view regularizes the singularity, enables a novel formulation of cosmological expansion, and embeds the VSL profile directly in the spacetime metric. While still speculative, this approach offers a clearer mathematical scaffolding for interpreting black hole interiors as cosmogenic domains.

1 Introduction

The standard cosmological model begins from a singular spacetime point, with expansion described by the FLRW metric under the assumptions of homogeneity and isotropy. In contrast, the black hole interior model proposed in Part I treats the horizon not as a coordinate artifact but as a physical, causal, and thermodynamic boundary. Here, expansion originates not from a point but from a surface—the event horizon—and proceeds inward in proper distance and forward in local time.

To formalize this "inside-out" idea, we apply a radial inversion transformation $R = r_H^2/r$ and a conformal time coordinate η defined by $d\eta = (c(r)/r_H)dt$. This reparameterization regularizes the geometry at $r = 0$, positions the horizon as a temporal origin, and allows reinterpretation of the interior as an expanding closed universe.

2 Radial and Temporal Coordinate Transformations

We define the radial inversion:

$$R = \frac{r_H^2}{r}, \tag{1}$$

so that $r \to 0$ maps to $R \to \infty$, and the horizon $r = r_H$ maps to $R = r_H$. This inversion effectively swaps the center and exterior, placing the horizon at a geometric pivot.

Next, we introduce conformal time:

$$d\eta = \frac{c(r)}{r_H}dt, \tag{2}$$

which diverges as $c(r) \to 0$ near the horizon, pushing $\eta \to -\infty$. This turns the horizon into a null-like past boundary, analogous to the Big Bang.

Assuming a simple exponential VSL profile $c(r) = c_0 \exp(-\alpha|r - r_H|)$, this transforms into a conformal profile $c(\eta) = c_0 \exp(\gamma\eta)$ as $\eta \to -\infty$, for some constant $\gamma > 0$. This shows consistency with the causal origin interpretation.

3 Transformed Metric and Interpretation

Applying these transformations to a Schwarzschild-like metric with VSL:

$$ds^2 = -c(r)^2 dt^2 + \left(1 - \frac{r_H}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{3}$$

we obtain:

$$ds^2 = \left(\frac{r_H}{R}\right)^4 \left[-d\eta^2 + \frac{dR^2}{(R/r_H)^2 - 1} + r_H^2 d\Omega^2\right]. \tag{4}$$

This metric is conformal to a closed FLRW-like geometry, with conformal factor $(r_H/R)^4$. The interior, now represented by $R > r_H$, expands away from the horizon surface.

4 Modified Friedmann Equations

Assuming an effective scale factor $a(\eta) = (r_H/R)^2$, the modified Friedmann equations in this transformed frame become:

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{eff}} a^2 - \frac{c^2(\eta)}{r_H^2}, \tag{5}$$

$$\frac{a''}{a} = -\frac{4\pi G}{3}(\rho_{\text{eff}} + 3p_{\text{eff}}) + 2\left(\frac{c'}{c}\right)\left(\frac{a'}{a}\right), \tag{6}$$

where primes denote derivatives with respect to η. These equations demonstrate that a decaying $c(\eta)$ contributes a term that can drive inflation-like behavior without an inflaton field.

The accretion-expansion link is reinforced by noting:

$$\frac{da}{d\eta} = \frac{\partial}{\partial\eta}\left(\frac{r_H^2}{R^2}\right) = 2\frac{r_H}{R^2}\frac{dr_H}{d\eta} = 2\frac{a}{r_H}\frac{dr_H}{d\eta}, \tag{7}$$

which shows explicitly that $\dot{a}/a \propto \dot{r}_H/r_H$.

5 Discussion and Future Work

The inside-out coordinate transformation reframes the black hole interior as an expanding universe whose causal structure originates at the horizon. This inversion naturally regularizes the singularity at $r = 0$ and embeds the VSL profile geometrically. The resulting Friedmann-like equations suggest that variations in c can serve as a driver of both early inflation and late-time acceleration.

As $\eta \to -\infty$, $c(\eta) \to 0$, implying that the entropy density $S \propto c^3(\eta) \to 0$, consistent with the low-entropy initial condition proposed in Part I.

Moreover, since the scale factor is inversely related to R, which itself is defined via r_H^2/r, we can interpret accretion-driven growth of r_H as driving an increase in $a(\eta)$. This links the interior expansion directly to ongoing mass influx from the parent universe. Preliminary analysis confirms radial null geodesics satisfy $dR/d\eta \propto c(\eta)$, enforcing causal isolation at $\eta \to -\infty$.

Several future directions suggest themselves: exploration of geodesic behavior in these coordinates; observational consequences such as dipolar power asymmetries in the CMB from asymmetric accretion (distinct from cosmic variance); entropy evolution near the horizon and possible links to the holographic principle; Penrose-style diagrams of the geometry; scalar-tensor Lagrangians for $c(\eta)$; numerical simulations of $a(\eta)$; and refined derivations of the $\dot{a}/a$ to $\dot{r}_H/r_H$ relationship.

6 Conclusion

We have introduced a geometric formalism to support the inside-out cosmology proposed in Part I. Through radial and conformal transformations, we interpret the black hole horizon as a temporal and causal origin analogous to the Big Bang. This construction embeds the VSL dynamics into the metric and yields modified Friedmann equations suggesting inflation-like behavior without exotic fields. While further development is needed, this framework provides a mathematically motivated platform for reinterpreting black hole interiors as cosmogenic domains.

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