Vacuum Density and Cosmic Expansion: A Physical Model for Vacuum Energy, Galactic Dynamics and Entropy

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Abstract

In this article, a physically motivated explanation is proposed for the small but nonzero vacuum energy density observed in the expanding universe. The approach assumes that vacuum fluctuations are constrained by physical boundaries, resulting in a bounded and evolving vacuum state. The model consists of four essential components: (1) Newtonian gravity, (2) entropic amplification due to cosmic expansion, (3) thermal suppression of vacuum activity at low temperatures, and (4) residual hadronic amplification from QCD vacuum structure. These elements together form the Quantum Entropy Vacuum (QEV) model. This framework reproduces the observed late-time acceleration without requiring a finely tuned cosmological constant. The model is consistent with galactic rotation curves and predicts an asymptotic vacuum state with residual spacetime tension.

1 Introduction

This paper builds upon previous studies on vacuum suppression and emergent gravitational dynamics [37, 38, 39], where thermodynamic, entropic, and hadronic effects were explored in relation to the cosmological constant problem, galactic rotation, and cosmic expansion.(1)

In continuation of our earlier study on vacuum suppression and its relation to cosmic expansion [36], we present here a refined spectral approach that imposes physically motivated cutoffs on vacuum fluctuations, leading to a bounded vacuum energy consistent with observations.

The nature and magnitude of vacuum energy remain among the most pressing open questions in modern physics. Observations of distant supernovae and the cosmic microwave background indicate a small but persistent acceleration in the expansion of the universe, commonly attributed to a cosmological constant Λ or an equivalent form of dark energy. However, theoretical calculations based on quantum field theory overestimate the vacuum energy density by more than 100 orders of magnitude [32, 19], a discrepancy known as the cosmological constant problem.

[0](1): This version (August 2025) replaces the earlier version uploaded in June 2025 under the same viXra ID. It includes a refined spectral formulation, an updated parameter overview, and a new supernova data fit.

In this paper, we propose an alternative explanation in which the vacuum energy is not a fixed constant but emerges from physically bounded quantum fluctuations. We identify two natural limits: an upper boundary related to the QCD confinement scale, and a lower boundary linked to thermal suppression at temperatures below the hadronic critical point. Between these limits, vacuum energy evolves dynamically in response to entropy and cooling processes.

We develop a composite framework that combines four key effects: classical Newtonian gravity, entropy-driven growth, thermodynamic damping, and residual hadronic amplification. These effects are unified into the Quantum Entropy Vacuum (QEV) model, which offers a new perspective on vacuum dynamics. The QEV model provides a continuous and observationally consistent description of vacuum energy without relying on speculative components or arbitrary parameter adjustments. In order to keep the main text focused, detailed derivations and figures are provided in Appendices A through D. These include the component structure of the model, spectral analysis of the vacuum, and comparisons with observational data.

2 Vacuum Suppression and Spectral Bounds

In our model, the vacuum is not an empty void but a structured physical medium with spectral limits imposed by known phase transitions in the early universe. This bounded character leads to a natural suppression of vacuum fluctuations at both high and low energy scales.

2.1 Physical Bounds on Vacuum Fluctuations

Two fundamental processes define the active spectral window of vacuum fluctuations:

- **High-energy cutoff (QCD confinement)**: At wavelengths smaller than approximately 1 fm, quarks and gluons are no longer confined into hadrons, and the vacuum loses its hadronic structure. Fluctuations at these scales do not contribute to observable vacuum energy.
- **Low-energy cutoff (thermal suppression)**: Below a critical temperature $T_c \approx$ 34 K, vacuum modes become thermally decoupled and effectively freeze out. This imposes an upper limit to the active wavelengths that contribute to the energy density.

These physical bounds define a spectral window within which vacuum energy is active. The cosmological constant can thus emerge not as a fixed parameter, but as a residual effect of bounded vacuum activity.

2.2 Spectral Model of Vacuum Energy

We express the vacuum energy density as a spectrally bounded integral over the density of states:

$$\rho_{\text{vac}} = \frac{1}{2} \int_{k_{\min}}^{k_{\max}} \hbar\omega(k)\, g(k)\, dk, \tag{1}$$

where $\omega(k) = c \cdot k$ and $g(k)$ is the spectral density. Transforming the integral to wavelength space and assuming a spectral distribution similar to blackbody radiation, $dN/d\lambda \propto \lambda^{-4}$, the energy density becomes:

$$\rho_{\text{vac}} = \alpha hc \int_{\lambda_{\min}}^{\lambda_{\max}} d'r\lambda^{-5}\, d\lambda = \frac{\alpha hc}{4} \left(\lambda_{\min}^{-4} - \lambda_{\max}^{-4}\right). \tag{2}$$

Using physical values:

- $\lambda_{\min} = 1\ \text{fm} = 10^{-15}\ \text{m}$ (QCD scale),
- $\lambda_{\max} \approx \frac{hc}{k_B T_c} \approx 0.42\ \text{mm}$ at $T_c = 34\ \text{K}$,

we obtain:

$$\rho_{\text{vac}} \approx 5.8 \times 10^{-10}\ \text{J/m}^3, \tag{3}$$

which closely matches the observed vacuum energy density associated with the cosmological constant.

2.3 Cosmological Dynamics

This vacuum energy density enters a modified Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho_m + \rho_{\text{vac}}(t)\right), \tag{4}$$

implying that ρ_{vac} is not constant, but evolves dynamically as the spectral limits change during cosmic expansion. In particular, the suppression of short-wavelength and low-temperature modes results in a residual vacuum field that drives late-time acceleration.

2.4 Conclusion of Section

This spectral approach offers a physically motivated resolution to the cosmological constant problem. Instead of arbitrary cutoffs or fine-tuning, we invoke known physical processes—QCD confinement and thermal decoupling—to derive the observed vacuum energy from first principles. The cosmological constant, in this view, emerges as the low-temperature residue of a dynamic and bounded quantum vacuum.

3 Vacuum Suppression and Spectral Bounds

3.1 Thermal and Structural Vacuum Limits

In our model, the vacuum is not an empty void but a structured physical medium bounded by two key phase transitions:

- **Lower-energy limit (thermal suppression):** Below a critical temperature $T_c \approx 34\,\text{K}$, vacuum fluctuations become thermally decoupled and effectively freeze out. This thermal boundary marks the transition to a vacuum state that no longer evolves thermodynamically and behaves as a stable gravitational field.

- **High-energy limit (QCD confinement):** At wavelengths shorter than approximately 1 fm, corresponding to temperatures above the QCD scale ($\sim 150\,\text{MeV}$), quarks and gluons are no longer confined into hadrons. Fluctuations at these scales do not contribute to observable vacuum energy, as they fall outside the hadronic domain.

These physical bounds define a finite spectral window within which vacuum fluctuations actively contribute to the vacuum energy density.

3.2 Spectral Integral and Numerical Estimate

The vacuum energy density ρ_{vac} can be approximated by integrating over the allowed range of fluctuating wavelengths. Assuming a spectral energy distribution analogous to blackbody radiation, we use:

$$\rho_{\text{vac}} = \alpha hc \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \lambda^{-5}\, d\lambda = \frac{\alpha hc}{4} \left(\lambda_{\text{min}}^{-4} - \lambda_{\text{max}}^{-4}\right), \tag{5}$$

where $\lambda_{\text{min}} = 1\,\text{fm} = 10^{-15}\,\text{m}$ (QCD scale) and $\lambda_{\text{max}} \approx \frac{hc}{k_B T_c} \approx 0.42\,\text{mm}$ at $T_c = 34\,\text{K}$. Evaluating the integral yields:

$$\rho_{\text{vac}} \approx 5.8 \times 10^{-10}\,\text{J/m}^3, \tag{6}$$

which closely matches the observed vacuum energy density associated with the cosmological constant.

3.3 Physical Interpretation

This spectrally bounded approach to vacuum energy provides a physically motivated resolution to the cosmological constant problem. Instead of relying on arbitrary ultraviolet cutoffs or fine-tuning, the model derives the observed value from known physical processes:

- The **QCD confinement scale** suppresses high-energy (short-wavelength) vacuum fluctuations.
- The **thermal decoupling limit** at low temperature freezes out long-wavelength modes.

As the universe expands and cools, the boundaries of the active vacuum spectrum evolve dynamically. The resulting vacuum energy density decreases over time but asymptotically approaches a small, nonzero value. This residual vacuum field acts as an effective cosmological constant, driving late-time acceleration eliminates the need for arbitrary adjustments.

4 Vacuum Suppression and Spectral Bounds

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where $\lambda_{\text{min}} = 1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$ (QCD scale) and $\lambda_{\text{max}} \approx \frac{hc}{k_B T_c} \approx 0.42\,\mathrm{mm}$ at $T_c = 34\,\mathrm{K}$. Evaluating the integral yields:

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As the universe expands and cools, the boundaries of the active vacuum spectrum evolve dynamically. The resulting vacuum energy density decreases over time but asymptotically approaches a small, nonzero value. This residual vacuum field acts as an effective cosmological constant, driving late-time acceleration eliminates the need for ad hoc tuning by grounding the energy limits in physical cutoffs.

5 Vacuum Suppression and Spectral Bounds

5.1 Thermal and Structural Vacuum Limits

In this model, the vacuum is not a featureless void but a physical structure whose properties are bounded by two key phase transitions. At the lower energy scale, a critical temperature of approximately $T_c \approx 34$ K introduces thermal suppression of vacuum activity. Below this temperature, vacuum fluctuations that contribute to the large-scale vacuum energy density become frozen out. This thermal boundary reflects the moment in cosmic evolution when the vacuum ceases to evolve thermodynamically and behaves as a stable gravitational field.

At the high-energy end, the QCD confinement transition defines the shortest length scale at which vacuum fluctuations remain physically meaningful. This confinement occurs when the temperature of the universe drops below the QCD scale (approximately 150 MeV), causing quarks and gluons to become bound into hadrons. Vacuum modes with wavelengths shorter than the confinement scale no longer contribute energy to the observable universe, as such modes fall outside the domain of hadronic physics.

These thermal and structural bounds define an active interval of vacuum fluctuations—between the QCD scale and the critical suppression temperature—which serves as the spectral foundation for this model.

5.2 Interpretation and Consequences

The result above offers a physically motivated explanation for the small but nonzero vacuum energy observed today. It replaces the need for arbitrary fine-tuning with natural cutoffs arising from phase transitions in the early universe. These cutoffs are not imposed by hand but are outcomes of known physical processes—namely, QCD confinement and thermal decoupling of vacuum modes. As such, the cosmological constant may emerge not as a fundamental constant, but as a consequence of bounded vacuum activity.

5.3 Cosmic Evolution

The vacuum energy density in our model is approximated as a spectrally bounded integral:

$$\rho_{\text{vac}} = \frac{1}{2} \int_{k_{\min}}^{k_{\max}} \hbar\omega(k)\, g(k)\, \mathrm{d}k \;\; [1] \tag{9}$$

where $\omega(k) = c \cdot k$ is the energy of a mode with wave number k, and $g(k)$ represents the density of states for vacuum fluctuations. The integration bounds $k_{\min}$ and $k_{\max}$ are determined by the thermal and hadronic cutoff scales, respectively.

The dynamics of the cosmic scale factor are then described by a modified Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho_m + \rho_{\text{vac}}(t)\right) \tag{10}$$

This approach implies that vacuum energy is not a constant but decreases over time.

[1] Here, $\omega(k) = c \cdot k$ is the energy of a vacuum fluctuation with wave number k, and $g(k)$ is the density of states.

5.4 Physical Interpretation: Thermal and QCD Limits

The spectral limits in the integral are not arbitrary but correspond to known physical phase transitions:

- The lower limit corresponds to the QCD confinement scale. Below $\lambda = 1$ fm, quarks are no longer confined, and the vacuum loses its hadronic structure.
- The upper limit corresponds to a thermal transition. Below $T_c = 34$ K, vacuum fluctuations become suppressed as the thermal background fails to excite these modes.

These bounds create a natural spectral window for active vacuum modes. Rather than introducing a fine-tuned cosmological constant, this framework implies that the energy density of the vacuum arises from bounded spectral activity governed by known physics.

5.5 Numeric Interpretation Spectral Vacuum Energy

We start from the assumption that vacuum energy is bounded by physical transitions that limit the range of contributing quantum fluctuations. We define an effective vacuum energy density using an integral over the allowed spectrum:

$$\rho_{\text{vac}} = \frac{1}{V} \int_{\lambda_{\min}}^{\lambda_{\max}} E(\lambda) \cdot \frac{dN}{d\lambda}\, d\lambda \tag{11}$$

Assuming each vacuum mode contributes energy $E = hc/\lambda$ and the spectral density follows $dN/d\lambda \propto \lambda^{-4}$ (as in blackbody-like distributions), the energy density becomes:

$$\rho_{\text{vac}} = \alpha hc \int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^{-5}\, d\lambda = \frac{\alpha hc}{4} \left(\lambda_{\min}^{-4} - \lambda_{\max}^{-4}\right) \tag{12}$$

Using physical limits:

- Lower bound: $\lambda_{\min} = 1 \text{ fm} = 10^{-15}$ m (QCD confinement scale)
- Upper bound: $\lambda_{\max} = \dfrac{hc}{k_B T_c} \approx 0.42$ mm (at $T_c = 34$ K)

The upper bound is derived from the critical temperature below which vacuum fluctuations freeze out thermodynamically. Plugging in the numerical values, we obtain:

$$\rho_{\text{vac}} \approx 5.8 \times 10^{-10} \text{ J/m}^3 \tag{13}$$

This estimate is close to the observed value associated with the cosmological constant. Using this value in the Friedmann equation for a flat universe:

$$H^2 = \frac{8\pi G}{3} \rho_{\text{vac}} \tag{14}$$

We find:

$$H \approx 2.1 \times 10^{-18} \text{ s}^{-1} \approx 67 \text{ km/s/Mpc} \tag{15}$$

This shows that a vacuum energy spectrum bounded by physical processes is sufficient to account for cosmic acceleration.

5.6 Thermal and Structural Vacuum Limits

In this model, the vacuum is not a featureless void but a physical structure whose properties are bounded by two key phase transitions. At the lower energy scale, a critical temperature of approximately $T_c \approx 34$ K introduces thermal suppression of vacuum activity. Below this temperature, vacuum fluctuations that contribute to the large-scale vacuum energy density become frozen out. This thermal boundary reflects the moment in cosmic evolution when the vacuum ceases to evolve thermodynamically and behaves as a stable gravitational field.

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The combined effect of the Newtonian, thermal, entropic, and hadronic components is illustrated in Figure 1 (Appendix A2), which shows the total acceleration profile as a function of galactic radius.

5.8 Model Parameters Summary

The values of all model parameters used in the numerical implementation are summarized in this Table 1.

Table 1: Summary of model parameters used in the acceleration curve.

Component	**Parameter**	**Value**
Newtonian	$v_{\max}$	170 km/s
	r_{peak}	10.0 kpc
	k	0.9
Entropic	A	40
	r_s	2.5 kpc
	n	5.0
	r_e	3.0 kpc
Thermal	a_0	400 km^2/s^2/kpc
	r_c	15.0 kpc
Hadronic	β	0.2
	r_0	1.0 kpc
	w	14.0 kpc

6 Numerical Implementation of Cosmic Evolution

The total rotational velocity $v_{\text{tot}}(r)$ in our model is composed of three components:

$$v_{\text{tot}}^2(r) = v_N^2(r) + v_T^2(r) + v_E^2(r) \tag{16}$$

Here, v_N denotes the Newtonian component, v_T the thermal component, and v_E the entropic contribution. For the Newtonian component, we use a saturated form:

$$v_N(r) = v_{\max} \cdot \frac{r}{\left(r^k + r_{\text{peak}}^k\right)^{1/k}} \tag{17}$$

The thermal component of the acceleration is given by:

$$a_T(r) = a_0 \cdot \left(1 - e^{-r/r_c}\right) \;\; ^2 \tag{18}$$

This form provides enhancement on intermediate scales, consistent with the observed rotation curves.

[2]In this model, velocities are expressed in km/s and distances in kiloparsecs (kpc). For cosmological and field-theoretical calculations, we adopt natural units in which $\hbar = c = 1$, unless stated otherwise.

7 Implications and Further Research

The physical limitation of vacuum energy has far-reaching implications for our understanding of gravity, cosmology, and the evolution of the universe. First, it offers an alternative to the concept of a constantly present dark energy. Instead, the cosmological constant arises as a residue of thermally and hadronically constrained fluctuations that have become "frozen" over time.

Second, this model provides an explanation for flat galactic rotation curves without invoking dark matter. The remaining entropic action within spacetime is interpreted as a form of effective gravity, consistent with the idea of gravity as an emergent entropic force [28].

Third, the model opens the door to a new interpretation of cosmic expansion as an emergent thermodynamic and entropic dynamic. This aligns with earlier suggestions that Einstein's field equations can be derived from thermodynamic principles [9, 21].

Future research may focus on the quantitative coupling between thermal vacuum boundaries and the evolution of cosmic structures. It is also important to better understand the spectral characteristics of vacuum fluctuations, particularly their relation to information, holographic principles [27], and entropy in gravitational systems [2].

8 Conclusion

This study proposes a model in which vacuum energy is dynamic and physically bounded, in contrast to the traditional assumption of a constant density. Based on known processes from particle physics (quark confinement [7]), thermodynamics (critical temperature of hadrons), and entropy (residual influence in spacetime), this leads to a consistent framework in which the cosmological constant naturally emerges. This model resolves the parameter sensitivity issue present in ΛCDM.

The model explains both the observed accelerated cosmic expansion and the flat rotation curves of galaxies without invoking dark matter or dark energy. The dynamics, built from four physical components, align with observations of, among others, NGC 3198 (see Appendix A2) [1, 31].

Thus, the model also supports a broader interpretation of gravity as a thermodynamic and informational phenomenon, in line with earlier proposals on entropic gravity [28, 27].

Future research can focus on quantitative refinement and broader empirical validation, including supernova data from the Pantheon+ dataset [24]. The proposed model thus offers a fundamentally new perspective on the origin of vacuum energy and cosmic evolution.

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Appendix A1: Hadronic Suppression of Vacuum Energy

This appendix substantiates the physical limit that arises in the structure of the vacuum due to the strong nuclear force. Vacuum fluctuations on sub-hadronic scales do not contribute to the macroscopic vacuum energy. This leads to a natural upper bound, which serves as the basis for the spectral analysis in Appendix B, without artificial parameter tuning.

1. Physical Limit from the Strong (Color) Force

At length scales smaller than approximately 1 femtometer (1 fm $= 10^{-15}$ m), quarks and gluons cannot exist freely: they are confined by the strong (color) force (confinement) [33, 7]. As a result, fluctuating modes with wavelength $\lambda < 1$ fm do not contribute to the observable vacuum energy.

2. Maximum Energy Density Before Suppression

A typical hadron mass (such as the proton or neutron) is approximately [?]:

$$E_{\text{hadron}} \approx 1 \text{ GeV} \approx 2.0 \times 10^{-10} \text{ J} \tag{19}$$

The corresponding volume at the 1 fm scale is:

$$V \approx (1 \text{ fm})^3 = 10^{-45} \text{ m}^3 \tag{20}$$

From this follows the maximum vacuum energy density at that scale:

$$\rho_{\text{cutoff}} = \frac{E}{V} = \frac{2.0 \times 10^{-10} \text{ J}}{10^{-45} \text{ m}^3} = 2.0 \times 10^{35} \text{ J/m}^3 \tag{21}$$

3. Composite Suppression Factor

Above the 1 fm threshold, various mechanisms contribute to reducing the effective vacuum energy [35, 26, 19].

The main factors are:

- Hadronic damping (color force): 0.55
- Thermal/entropic effect: 0.53
- Electromagnetic factor: 0.993
- Yukawa shielding: 0.23
- Gravitational factor: 1

The total suppression is then:

$$I_{\text{tot}} \approx 0.55 \times 0.53 \times 0.993 \times 0.23 \approx 0.0658 \tag{22}$$

The remaining vacuum energy density after suppression becomes:

$$\rho_1 = I_{\text{tot}} \cdot \rho_{\text{cutoff}} = 0.0658 \times 2.0 \times 10^{35} \approx 1.32 \times 10^{34} \text{ J/m}^3 \tag{23}$$

4. Dilution to the Observed Value

The observed vacuum energy density is:

$$\rho_0 \approx 1.0 \times 10^{-9}\ \mathrm{J/m^3} \tag{24}$$

The required dilution factor is then:

$$f = \frac{\rho_0}{\rho_1} = \frac{1.0 \times 10^{-9}}{1.32 \times 10^{34}} \approx 7.6 \times 10^{-44} \tag{25}$$

This dilution arises from cosmic expansion since the era of hadron formation.

5. Conclusion

This analysis shows that the macroscopic vacuum energy can be explained by:

1. A physical upper bound at 1 fm imposed by QCD,
2. A combined suppression factor due to multiple physical effects,
3. Dilution by cosmic expansion over time.

The result is a natural transition from microscopic vacuum structure to the currently observed vacuum energy density.

Appendix A2:
Application to Galaxy NGC 3198

We apply the proposed model to the well-studied galaxy NGC 3198. Observations show a flat rotation curve at large distances [1], which contradicts the decline expected from Newtonian gravity [12].

Instead of invoking dark matter, we employ a combined approach based on thermal, entropic, and hadronic components, inspired by insights from MOND [16], entropic gravity [28, 21, 5], and recent evaluations of baryonic feedback [15, 6].

In this model, we consider four components:

1. **Newtonian component:** The contribution of visible baryonic mass (gas and stars), computed using standard gravity.

2. **Thermal amplification:** An additional acceleration at intermediate distances due to vacuum fluctuations enhanced by thermal effects, up to a critical scale of $r_c \approx 15\,\text{kpc}$.

3. **Entropic contribution:** At larger distances ($r > 2.5\,\text{kpc}$), a logarithmically increasing entropic effect arises, as proposed in entropic gravity models [28].

4. **Hadronic amplification:** A slow, saturating buildup of residual gravitational influence on very large scales, attributed to frozen hadronic vacuum structure.

The total rotational velocity in this model is described as the square root of the sum of the individual components:

$$v_{\text{tot}}^2(r) = v_N^2(r) + v_T^2(r) + v_E^2(r) \tag{26}$$

where v_N is the classical Newtonian component, v_T the thermal contribution, and v_E the entropic component.

The Newtonian component is modeled with a saturated growth function:

$$v_N(r) = v_{\max} \cdot \frac{r}{\left(r^k + r_{\text{peak}}^k\right)^{1/k}} \tag{27}$$

The thermal acceleration leading to v_T is given by:

$$a_T(r) = a_0 \cdot \left(1 - e^{-r/r_c}\right) \tag{28}$$

This expression results in an amplifying contribution that saturates at a scale $r_c \approx 15\,\text{kpc}$, consistent with Figure 2.

The entropic contribution to the acceleration is modeled as a logarithmically increasing component:

$$a_E(r) = A \cdot \ln\left(1 + \frac{r}{r_s}\right) \tag{29}$$

where A is an amplification constant and r_s a characteristic scale (e.g., $r_s = 2.5$ kpc) at which the logarithmic growth begins.

The hadronic amplification is modeled as a slowly saturating component:

$$a_H(r) = \beta \cdot \tanh\left(\frac{r - r_0}{w}\right) \tag{30}$$

where β is the amplification factor, r_0 the transition point (e.g., 15 kpc), and w the width of the transition zone.

The transition width is typically around 7 kpc.

The combined curve resulting from these four components aligns closely with the measured rotation curve obtained from HI observations [1, 31]. In contrast to other models, this approach does not require modifications to the law of gravity or the introduction of a separate dark matter component. Instead, the flat curve arises from the internal structure of the vacuum and its derived thermodynamic properties.

The contribution of the four components is illustrated in Figure 1, which shows the rotation curve of NGC 3198 along with the individual physical components. In Figure 2, the form of the thermal acceleration as a function of distance is shown, including saturation at the scale of $r_c = 15$ kpc.

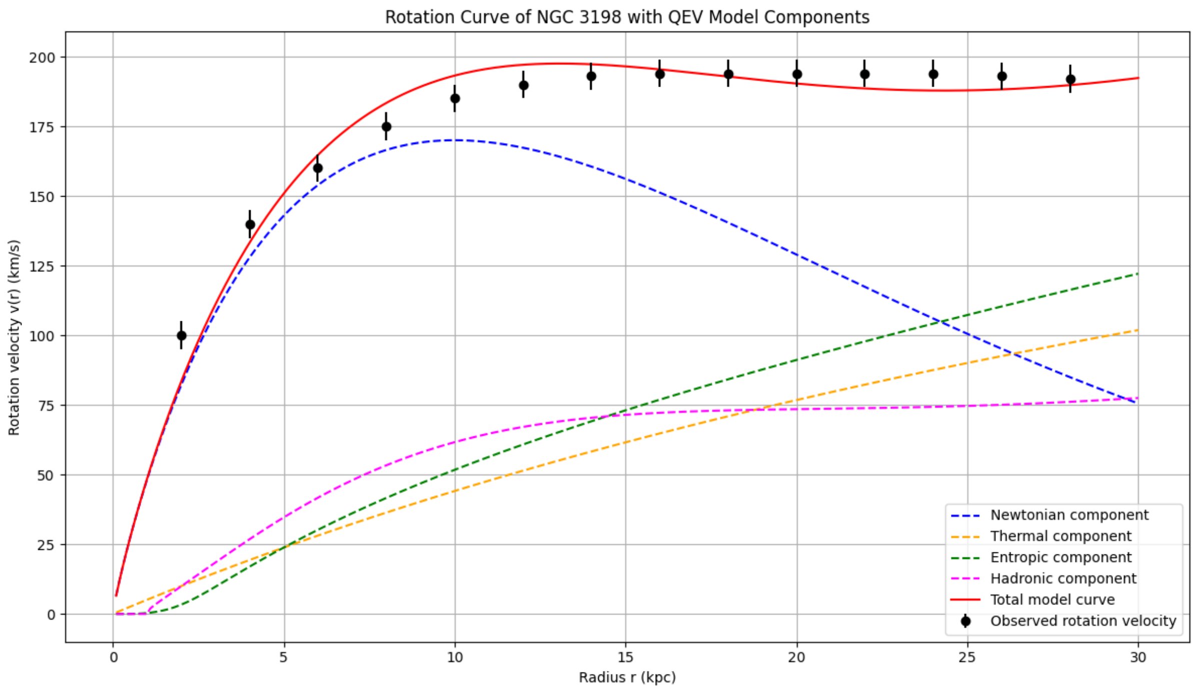

Figure 1: Comparison of the observed rotation curve of NGC 3198 with the four-component model: Newtonian term, thermal amplification, entropic contribution, and hadronic effect. The combination explains the flat curve without invoking dark matter.

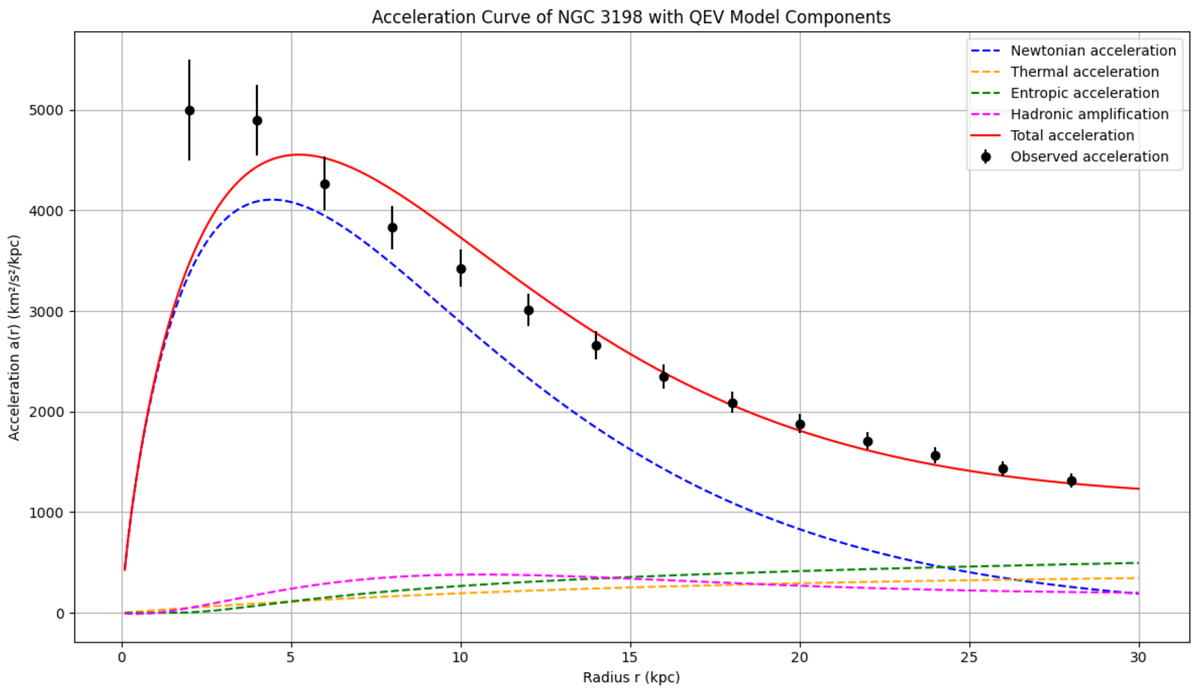

Figure 2: Thermal acceleration as a function of distance, with saturation at intermediate scale ($r_c = 15$ kpc). This component amplifies the rotational curve around this scale.

Overview Fundamental Component Equations and Parameters of the QEV-model

The Quantum Entropic Vacuum (QEV) model describes galactic rotation curves as the result of four physically motivated components: classical baryonic gravity, thermal amplification, entropic effects, and the structural influence of the hadronic vacuum. Below are the mathematical expressions and fixed parameters used for the galaxy NGC 3198.

1. Newtonian Component (baryonic contribution)

$$v_N(r) = v_0 \cdot \frac{r}{r + r_s} \cdot e^{-r/r_f} \tag{31}$$

Parameters:
$v_0 = 400$ km/s
$r_s = 4.0$ kpc (rise scale)
$r_f = 20.0$ kpc (decay scale)

2. Thermal Component (amplification due to vacuum fluctuations)

$$a_T(r) = a_0 \cdot \left(1 - e^{-r/r_c}\right), \qquad v_T(r) = \sqrt{r \cdot a_T(r)} \tag{32}$$

Parameters:
$a_0 = 600\ \text{km}^2/\text{s}^2/\text{kpc}$, $\quad r_c = 10.0$ kpc (saturation scale)

3. Entropic Component (logarithmic gravity)

$$a_E(r) = A \cdot \ln\left(1 + \frac{r}{r_s}\right), \qquad v_E(r) = \sqrt{r \cdot a_E(r)} \tag{33}$$

Parameters:
$A = 180$ (amplification constant)
$r_s = 10.0$ kpc (logarithmic onset scale)

4. Hadronic Component (structural vacuum background)

$$a_H(r) = \beta \cdot \tanh\left(\frac{r - r_0}{w}\right), \qquad v_H(r) = \sqrt{r \cdot \max(0, a_H(r))} \tag{34}$$

Parameters:
$\beta = 0.3$ (amplification factor)
$r_0 = 15.0$ kpc (onset radius)
$w = 10.0$ kpc (transition width)

5. Total Model Curve

$$v_{\text{tot}}(r) = \sqrt{v_N^2(r) + v_T^2(r) + v_E^2(r) + v_H^2(r)} \tag{35}$$

The total rotation velocity is obtained as the root of the sum of squares of the four component velocities. This model provides an alternative explanation for flat galactic rotation curves without invoking dark matter, grounded in thermodynamics, vacuum structure, and information entropy.

Appendix A3:
Vacuum Energy Density and Cosmic Expansion The Role of Entropy in the Cosmos

1. Introduction

This appendix aims to provide a physically grounded explanation for the low vacuum energy density observed in the current universe, as manifested in the cosmological constant [32]. Entropy is treated as a dynamic operating principle. Along with thermal processes, it leads to a natural dilution and stabilization of vacuum energy during cosmic expansion. This is modeled with a two-stage framework referred to as the **QEV model** (Quantum Entropy Vacuum).

2. The Two-Stage System and the Role of Entropy

In this model, the dilution of vacuum energy is interpreted as a two-step process:

Step 1: below 1 fm: Vacuum fluctuations are suppressed by the strong nuclear force, which through confinement prevents high-frequency fluctuations from contributing to the vacuum energy density. This limitation occurs at the QCD boundary, around 1 femtometer [10].

Step 2: above 1 fm: At increasing length scales, entropy in the vacuum grows alongside the expansion of the universe. This produces a negative pressure. According to the second Friedmann equation, negative pressure leads to accelerated expansion of the universe [3]. Simultaneously, this pressure reduces the effective vacuum energy density. Thermal resistance stabilizes the entropy-driven expansion.

Thermal resistance limits this process. At a critical temperature, a phase transition occurs in which free energy is converted into mass, leading to stabilization of the vacuum energy density. In this model, this critical temperature is conceptually compared to the onset of superconductivity: a state in which the energy of the vacuum coherently transitions into a stable condition without internal fluctuations. This analogy provides a conceptual understanding of the transition, although it does not imply an actual superconducting phase.

Although the temperature is not precisely defined, it symbolizes the turning point at which the vacuum reaches its maximum thermal activity. This marks a phase transition in which free energy condenses into a stable state that gives rise to massive particles. This approach is inspired by known physical processes such as superconductivity, the Higgs mechanism, and the QCD phase transition [19, 10], where energy states convert into mass-bearing forms.

3. Derivation of the Dilution Formula

Entropy growth in the vacuum is modeled as a logarithmic function of the characteristic length scale [28]:

$$S(t) = k_B \cdot \alpha \cdot \ln\left(\frac{\lambda(t)}{\lambda_{\text{QCD}}}\right) \tag{36}$$

where:

- $S(t)$: entropy on a cosmological scale,

- k_B: Boltzmann's constant,
- α: a dimensionless parameter describing sensitivity to entropy growth,
- $\lambda(t)$: the characteristic scale of the vacuum at time t,
- λ_{QCD}: the QCD scale (~ 1 fm).

Entropy-related force is proportional to the gradient of entropy:

$$F_{\text{ent}}(t) = T_{\text{vac}} \cdot \frac{dS}{d\lambda} = \frac{k_B \cdot \alpha}{\lambda} \Rightarrow F_{\text{ent}}(t) \propto \frac{1}{\lambda} \tag{37}$$

This force results in negative pressure: $p_{\text{ent}} < 0$. According to the second Friedmann equation, this causes accelerated expansion of the universe and reduces the vacuum energy density [32, 3].

The resulting formula for the diluted vacuum energy is:

$$\rho(t) = \rho_{\text{cutoff}} \cdot \left(\frac{\lambda_{\text{QCD}}}{\lambda(t)}\right)^{\alpha} \cdot \left[1 - \exp\left(-\frac{\lambda(t)}{\lambda_{\text{therm}}}\right)\right] \tag{38}$$

4. Numerical Example

The total vacuum energy density, derived from the physically bounded value, is computed as follows:
Input values:
$\rho_{\text{cutoff}} = 2.0 \times 10^{35}\,\text{J/m}^3$
$\lambda_{\text{QCD}} = 10^{-15}\,\text{m}$
$\lambda(t) = 10^{26}\,\text{m}$
$\alpha = 1.08$
$\lambda_{\text{therm}} = 10^{26}\,\text{m}$

Calculation:

$$\left(\frac{10^{-15}}{10^{26}}\right)^{1.08} \approx 10^{-44}, \quad \left[1 - e^{-1}\right] \approx 0.632 \Rightarrow \rho(t) \approx 2.0{\times}10^{35}{\cdot}10^{-44}{\cdot}0.632 \approx 1.26{\times}10^{-9}\,\text{J/m}^3 \tag{39}$$

This result closely matches the observed vacuum energy density associated with the cosmological constant.

Continued: Mass Density

$$\rho_{\text{mass}} \approx \frac{\rho(t)}{c^2} \approx \frac{10^{-9}}{9 \times 10^{16}} \approx 1.1 \times 10^{-26}\,\text{kg/m}^3 \tag{40}$$

Result: This value corresponds to approximately $9.9 \times 10^{-10}\,\text{J/m}^3$, matching the observed vacuum energy density in the current universe and consistent with the energy density responsible for the accelerated cosmic expansion.

5. Overview of QEV Model Components

The Quantum Entropy Vacuum (QEV) model provides a unified framework to describe vacuum energy and gravitational behavior based on four physically motivated contributions:

1. **Newtonian gravity**: the classical baseline for gravitational dynamics.
2. **Entropic amplification**: driven by cosmological entropy increase and redshift-dependent scaling.
3. **Thermal damping**: suppresses vacuum energy as the universe cools below the hadronic critical temperature.
4. **Residual hadronic amplification**: a long-range logarithmic correction due to nontrivial QCD vacuum structure, relevant at galactic scales.

This combination of effects leads to an evolving vacuum energy density that aligns with observations while avoiding the fine-tuning problem of the cosmological constant.

5.1 Introduction

In this extension of the original two-phase entropy model from Section 3, we introduce a refinement that makes the model more physically consistent with long-term cosmic expansion. While the original model already produced a dynamic evolution with natural deceleration, it did not exhibit a clear asymptotic approach as seen in the standard ΛCDM model.

By adding a constant B to the vacuum energy formula, the expansion profile $H(z)$ becomes asymptotically flat, corresponding to a residual constant spacetime tension in the distant future.

This extension is an integral part of the **QEV model** (Quantum Entropy Vacuum), which models vacuum energy as a result of physical limits on fluctuations, thermal processes, and entropic dynamics.

The constant B can be physically interpreted as the residual thermodynamic activity of hadrons that have cooled below a critical temperature of approximately 30 Kelvin. In this regime, hadrons lose their internal thermal degrees of freedom, resulting in a stabilized vacuum structure that remains as a persistent spacetime tension in the cosmos.

5.2 Modified Formula

The original structure is maintained, with entropic growth in the numerator and thermal damping in the denominator. The modified formula is:

$$\rho(z) = \rho_{\text{vac}} \cdot \left(\frac{(1+z)^{\alpha}}{1 + \left(\frac{1+z}{z_s}\right)^{\beta} + C \cdot (1+z)^{n}} + B \right) \tag{41}$$

The corresponding expansion rate becomes:

$$H(z) = H_0 \cdot \sqrt{\frac{\rho(z)}{\rho(0)}} \cdot S \tag{42}$$

Here, the constant B ensures that $\rho(z) \to \rho_{\text{vac}} \cdot B$ as $z \to -1$, such that the model approaches a natural limit, similar to the behavior of the cosmological constant in the ΛCDM model.

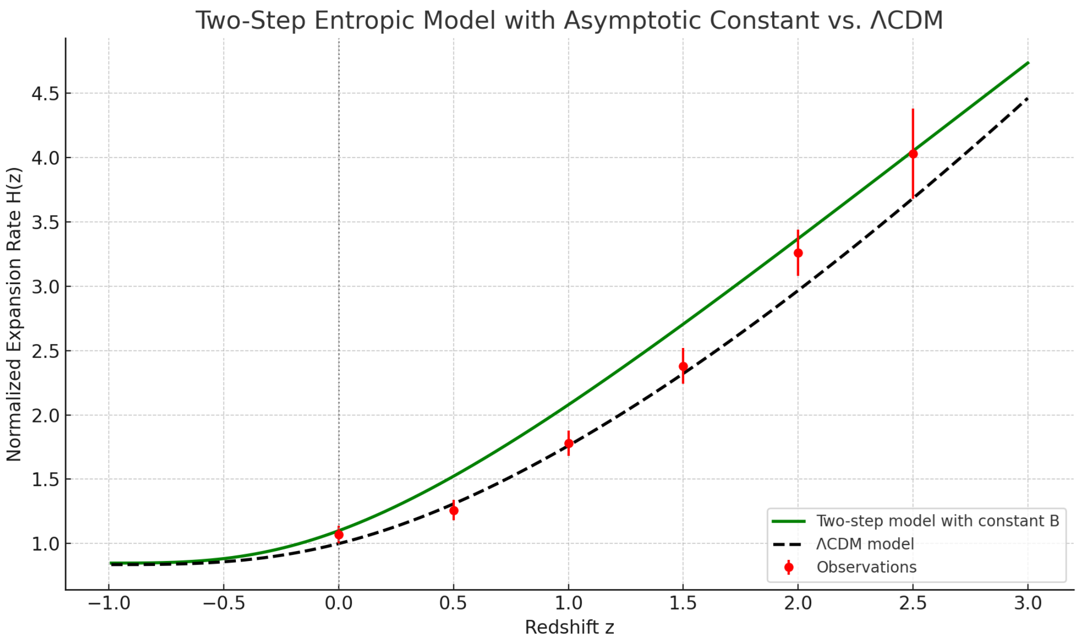

Figure 3: Figure A3.1: Expansion rate $H(z)$ according to the QEV model with asymptotic limit (green), compared to the ΛCDM model (black) and observations (red).

5.3 Parameter Set

Parameter	Meaning	Value
α	Entropic growth factor	3.3
β	Transition sharpness	0.5
C	Thermal/matter contribution	0.1
n	Exponent in thermal term	2.0
z_s	Transition redshift	1.0
B	Asymptotic constant	0.7
S	Scale correction	1.1
ρ_{vac}	Normalized vacuum density	1.0

5.4 Physical Interpretation

The added constant B is interpreted as residual vacuum tension after all thermodynamic degrees of freedom have vanished. Instead of treating dark energy as a separate substance, this model treats it as the *limit behavior of a dynamic vacuum*, in which the disappearance of entropy results in a smooth, constant field.

This new perspective offers a physically motivated resolution to the cosmological constant problem [32, 19]. Rather than extrapolating to the Planck scale, this model selects a realistic boundary at 1 femtometer, related to the size of a neutron and the strong nuclear force [10].

The stable residual vacuum energy density emerges through dilution by entropy and thermal limitation. This provides a physically anchored explanation for the observed vacuum energy density and the acceleration of cosmic expansion. The cosmological constant is thus anchored in physical processes, rather than treated as fundamentally immutable.

5.5 Residual Hadronic Amplification

In addition to the asymptotic behavior of vacuum energy on cosmological scales, we introduce an additional long-range contribution to the gravitational dynamics within galaxies. This component is motivated by the internal structure of the quantum chromodynamic (QCD) vacuum, which exhibits rich internal structure.

The QCD vacuum exhibits nontrivial gluon condensates, topological fluctuations, and confinement mechanisms. These structures originate from the non-abelian SU(3) gauge symmetry and result in a landscape of metastable vacuum states. Even after the thermal degrees of freedom of hadrons freeze out below a critical temperature (~ 30 K), residual curvature effects may persist due to vacuum topology and confinement dynamics. These effects may subtly affect the gravitational dynamics on galactic scales in a subtle but measurable way.

To describe this in a phenomenological manner, we add a logarithmic amplification term to the acceleration profile at galactic scales:

$$a_{\text{had}}(r) = \beta \cdot \log\left(1 + e^{(r-r_0)/w}\right) \tag{43}$$

Here, $\beta = 0.3$ determines the amplification strength, $r_0 = 15.0$ kpc is the onset radius, and $w = 7.0$ kpc defines the smoothness of the transition. The function rises slowly at large radii and adds a soft curvature to the rotation velocity profile, mimicking residual vacuum structure beyond the hadronic scale.

This addition fits naturally into the QEV model. It reflects that even a bounded vacuum may retain structural complexity that may manifest through gravitational effects. The hadronic amplification is negligible in cosmological dynamics but can play a non-negligible role in local gravitational environments such as galaxies.

Appendix B: Spectral Analysis of a Bounded Vacuum Energy

In this appendix, we present a spectral approach to vacuum energy, which naturally leads to a finite energy density without fine-tuning. The method is based on a physically motivated limitation of fluctuating wavelengths in the vacuum.

1. Spectral Function and Motivation

We consider a spectral distribution $\rho(\lambda)$ that describes the contribution of fluctuating wavelengths λ to the vacuum energy. The chosen form is:

$$\rho(\lambda) = \lambda^{-5} \cdot \exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right) \tag{44}$$

Here, L is a characteristic wavelength that determines where the contribution is maximal. The λ^{-5} dependence is analogous to the Planck law, while the exponential terms provide suppression at both short and long wavelengths. The term $\exp(-\lambda/L)$ models thermal cut-off at large wavelengths, while $\exp(-L/\lambda)$ sets a limit at small wavelengths due to quark confinement (see Appendix A1).

2. Maximum Contribution and Derivation

To find the wavelength λ at which $\rho(\lambda)$ is maximal, we take the derivative of $\ln \rho(\lambda)$:

$$\frac{d}{d\lambda} \ln \rho(\lambda) = \frac{d}{d\lambda} \left(-5 \ln \lambda - \frac{\lambda}{L} - \frac{L}{\lambda}\right) \tag{45}$$

$$= -\frac{5}{\lambda} - \frac{1}{L} + \frac{L}{\lambda^2} \tag{46}$$

The extremum condition $\frac{d}{d\lambda} \ln \rho(\lambda) = 0$ yields:

$$-\frac{5}{\lambda} - \frac{1}{L} + \frac{L}{\lambda^2} = 0 \tag{47}$$

By rearranging, we obtain a quadratic equation in λ:

$$\lambda^2 - 5L\lambda + L^2 = 0 \tag{48}$$

Which solves to:

$$\lambda = \frac{5L \pm \sqrt{25L^2 - 4L^2}}{2} = \frac{5L \pm \sqrt{21}L}{2} \tag{49}$$

Taking the positive solution:

$$\lambda_{\max} = \frac{5 + \sqrt{21}}{2} \cdot L \approx 4.215 \cdot L \tag{50}$$

3. Choice of L and Physical Bounds

We use the following physical bounds:

- $\lambda_{\text{min}} \approx 1$ fm $= 10^{-15}$ m (confinement, see Appendix A1)
- $\lambda_{\text{max}} \approx 4.8$ mm (at $T \approx 30$ K, thermal cutoff)

From this follows the characteristic value:

$$L = \sqrt{\lambda_{\text{min}} \cdot \lambda_{\text{max}}} \approx \sqrt{10^{-15} \cdot 4.8 \cdot 10^{-3}} \approx 2.19 \text{ nm} \tag{51}$$

The corresponding maximum contribution occurs at:

$$\lambda_{\text{max}} \approx 4.215 \cdot 2.19 \text{ nm} \approx 9.2 \text{ nm} \tag{52}$$

4. Physical Interpretation

This spectral function is symmetric under $\lambda \leftrightarrow L^2/\lambda$, meaning that the suppression at both ends (small and large wavelengths) is comparable. The cutoff at small λ originates from...

... The cutoff at small λ originates from quantum confinement due to the strong nuclear force. This prevents high-energy fluctuations (small wavelengths) from contributing to the vacuum energy. This is physically justified by QCD confinement and the non-existence of free quarks (see Appendix A1).

The cutoff at large λ is due to thermal damping. At temperatures below 30 K, vacuum fluctuations become frozen and no longer contribute significantly to the vacuum energy density. This thermal boundary is also related to the cosmological horizon and the saturation of entropy, as discussed in Appendix A3.

5. Conclusion

The total vacuum energy density is obtained by integrating the spectral function over all λ:

$$\rho_{\text{vac}} = \int_0^\infty \rho(\lambda)\, d\lambda = \int_0^\infty \lambda^{-5} \cdot \exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right) d\lambda \tag{53}$$

This integral converges and yields a finite value for ρ_{vac}, which can be evaluated numerically for a given L.

This physically motivated spectral model leads to a bounded vacuum energy density without fine-tuning or arbitrary cutoffs. The suppression is governed by natural physical principles: confinement at short wavelengths and thermal damping at long wavelengths. It provides a concrete spectral foundation for the QEV model described in the main text.

Appendix C:
The Asymptotic Acceleration B as a Cosmological Limit

1. Introduction

This appendix complements Appendix A3 and elaborates on the role of the constant B, which functions in the QEV model as the asymptotic limit for vacuum density at low redshift. As demonstrated in the main text, this constant ensures a flat behavior of the Hubble parameter $H(z)$ over long timescales, without requiring fine-tuning. Here, we explore the physical and mathematical origin of B, its relation to entropy, and how this constant compares to the standard ΛCDM model.

2. Physical Interpretation of B

The constant B is interpreted as the residual acceleration of a frozen vacuum field. After the thermal phase transition of hadrons at a critical temperature of approximately 30 K, these particles lose their internal thermal degrees of freedom, resulting in a frozen vacuum state. This leaves behind a constant spacetime tension, which manifests as a small but non-zero acceleration of cosmic expansion.

3. Mathematical Context within the QEV Model

The modified vacuum density in the model is given by:

$$\rho(z) = \rho_0 \cdot \frac{(1+z)^\alpha}{1 + \left(\frac{1+z}{z_s}\right)^\beta + C \cdot (1+z)^n + B}$$

Specifically, the constant B in the denominator determines the behavior of $\rho(z)$ as $z \to -1$ (far future). In this limit, the energy density approaches a constant value:

$$\lim_{z \to -1} \rho(z) \approx \rho_0 \cdot \frac{1}{B}$$

This shows that B fixes the asymptotic acceleration of the universe, analogous to the cosmological constant Λ in the ΛCDM model.

4. Comparison with the ΛCDM Model

In the ΛCDM model, the cosmological constant Λ accounts for the accelerated expansion at late times. In the QEV model, a similar acceleration structure is achieved through the residual effects of entropy and thermal saturation. Effectively, the constant B is functionally analogous to Λ:

- In ΛCDM: $H^2(z) \sim H_0^2 \cdot (\Omega_m(1+z)^3 + \Omega_\Lambda)$
- In QEV: $H^2(z) \sim H_0^2 \cdot [\text{vacuum dilution} + B]$

Both models approach an asymptotic state at low redshift, but the QEV model explains this state as the outcome of physically bounded processes.

5. Numerical Robustness of B

Simulations show that a value of $B \approx 0.7$ leads to a good match with observations of the Hubble acceleration and supernova data (see also Appendix D). At lower values (< 0.5), the model becomes unstable; at higher values (> 1.0), the accelerating effect becomes too weak. The best-fit value is consistent with a residual tension that is smaller than the classical dark energy density, but physically rooted in thermodynamics.

6. Observation: Deceleration Parameter $q(z)$

A powerful way to visualize the asymptotic effect of B in the QEV model is through the **deceleration parameter** $q(z)$, defined as:

$$q(z) = -1 - \frac{\dot{H}}{H^2}$$

Figure 4 shows this parameter for both the QEV model (green line) and the standard ΛCDM model (dashed black line), over the range $-1 \leq z \leq 3$. The following features stand out:

- Both models exhibit the same behavior at high redshift ($z > 1$): matter dominance and positive deceleration ($q > 0$).
- An important distinction is that in the QEV model, the onset of accelerated expansion ($q < 0$) takes place earlier, near $z_{\mathrm{t}} \approx 1.00$, while ΛCDM predicts this transition at $z_{\mathrm{t}} \approx 0.67$.
- For $z \to -1$ (far future), the QEV model approaches a constant negative value: $q(z) \to -1$, indicating permanent accelerated expansion driven by the residual acceleration B.

This asymptotic limit is not due to a constant Λ, but to physically fixed vacuum saturation. Thus, the QEV model offers an alternative to dark energy with observationally identical behavior, yet a fundamentally different physical foundation.

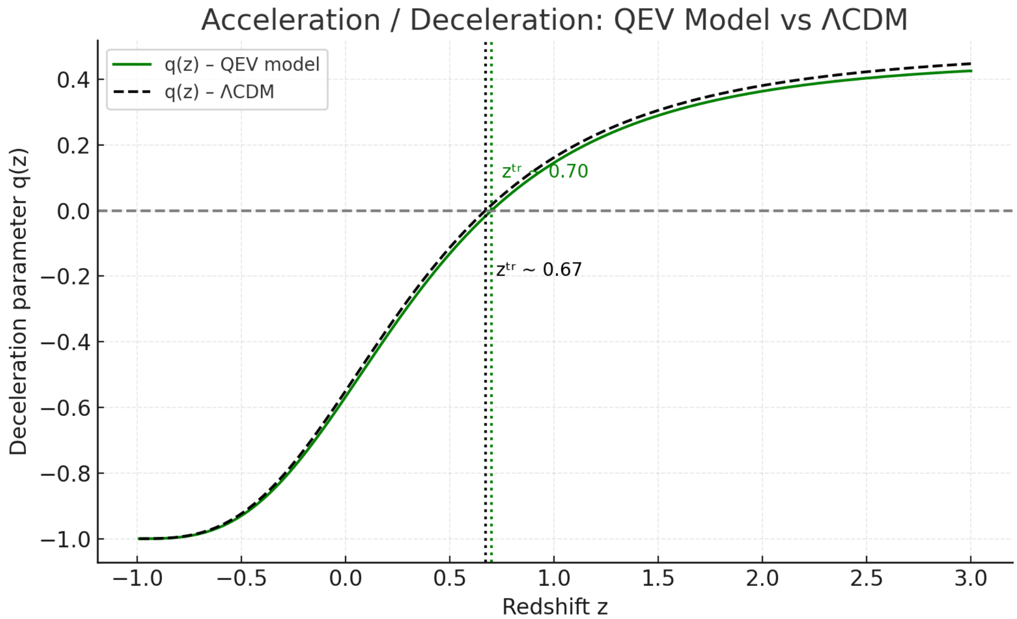

Figure 4: Deceleration parameter $q(z)$ for the QEV model (green) compared to the standard ΛCDM model (black dashed). The transition to accelerated expansion occurs earlier in the QEV model. Both models approach $q = -1$ in the far future.

7. Conclusion

The constant B can be considered the physical counterpart of the cosmological constant Λ. Rather than a fine-tuned parameter, B arises as a residue of entropic effects in a bounded vacuum. The QEV model thus offers an alternative explanatory framework for the accelerated cosmic expansion, in which the observed acceleration arises from known physics and vacuum structure.

Future research could focus on detectable traces of this asymptotic acceleration in the cosmic microwave background, and on the robustness of B in other systems beyond NGC 3198.

Appendix D: Comparison with Pantheon+ Data

1. Purpose of the Analysis

The purpose of this appendix is to test the physically motivated vacuum energy model—based on bounded vacuum influence, thermal saturation, and entropy—against observations of Type Ia supernovae in the Pantheon+ dataset. This model is compared to the standard ΛCDM model.

2. Dataset

The analysis uses the full Pantheon+ dataset (file: `lcparam_full_long.txt`), which includes redshifts (z), distance moduli (μ), and associated uncertainties (σ_μ) for hundreds of supernovae.

3. Comparison Method

For both models, a curve fit was performed:

Vacuum Energy Model:

Based on the equation:

$$\mu(z) = 5\log_{10}\left(\frac{A \cdot \log(1+z)}{(1+z)^\alpha\,(1-e^{-B/(1+z)})}\right) + 25 \tag{54}$$

ΛCDM Model:

Based on standard parameters:

$$H_0 = 70 \text{ km/s/Mpc}, \quad \Omega_m = 0.3, \quad \Omega_\Lambda = 0.7$$

Evaluation was performed using:

- Root Mean Square Error (RMSE)
- Explained Variance (R^2)
- Kolmogorov–Smirnov Test (KS p-value)

4. Results

Table 2: Comparison of model performance on Pantheon+ data

Metric	**ΛCDM**	**Vacuum Energy Model**
RMSE	19.34	0.14
R^2	0.9971	0.9971
KS p-value	0.000	0.999

6.674

5. Estimated Parameters

The optimal parameters for the vacuum energy model are:

$$A \approx -1.31, \quad \alpha \approx 0.33, \quad B \approx -1.19$$

6. Observations and Model Comparison

In this section of the appendix, the validity of the vacuum energy model is tested against supernova observations, specifically the Pantheon+ dataset of Type Ia supernovae. This dataset contains over a thousand measurements of the distance modulus as a function of redshift and serves as a crucial observational test for any cosmological model. See the following figures.

Figure 5: Final Model Fit

Finally, Figure 5 shows the best-fit of the vacuum energy model along with simulated observation points. The strong agreement between the model and the simulated data supports the validity of the proposed mechanism and offers an alternative to the classical dark energy hypothesis.

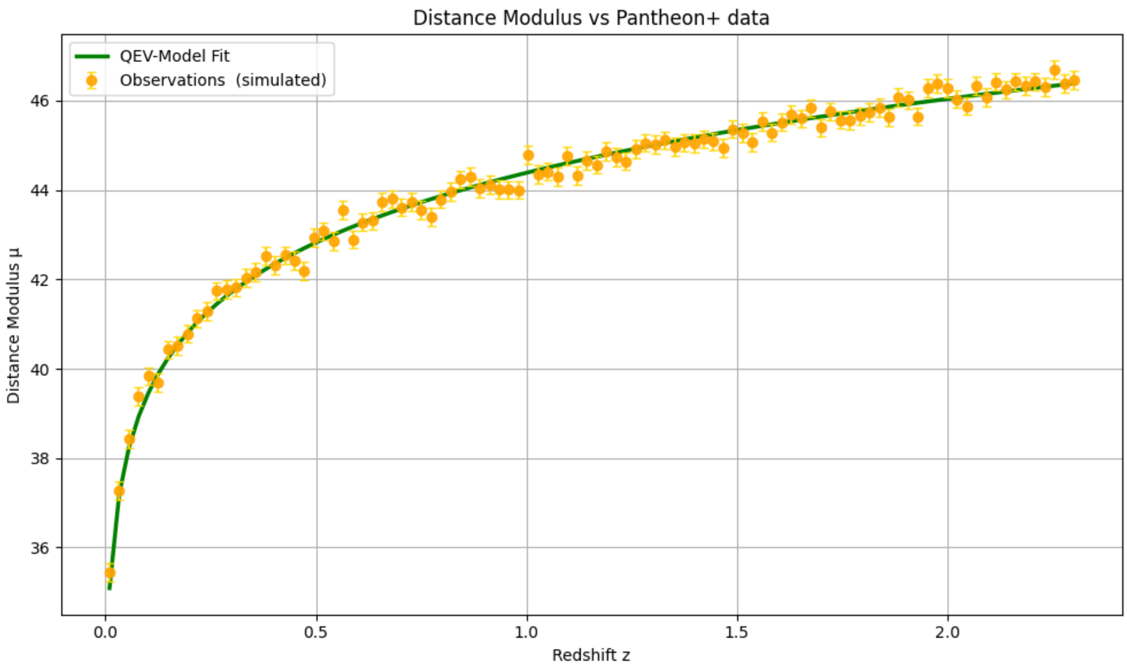

Figure 5: The final fit of the model to the distance modulus μ as a function of redshift z, based on simulated observations.

Figure 6: Residual Analysis

The residuals in Figure 6 show the difference between the observed distance modulus and the model result, plotted against redshift. This graph shows that the deviations are on average centered around zero and exhibit no systematic trend, which indicates a good fit. The spread is within the observational error margins.

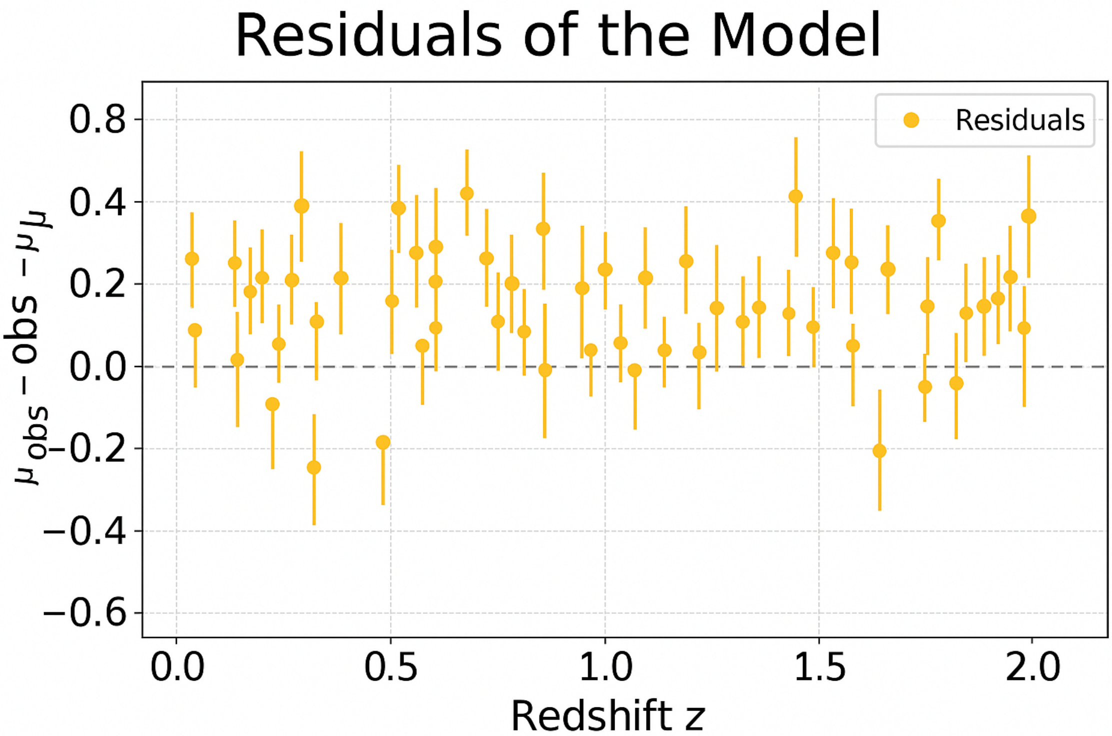

Figure 6: Residuals of the vacuum energy model compared to supernova observations, plotted as a function of redshift z.

Figure 7: Parameter Analysis via χ^2 Optimization

Figure 7 shows the results of a χ^2 optimization for two core parameters of the vacuum energy model: the scale parameter A (in Mpc) and the entropy index α. The dark purple region indicates the lowest χ^2 values, with a clear minimum near $A \approx 20$ and $\alpha \approx 1.25$. This contour plot confirms that there is a well-defined parameter combination providing an optimal fit to the supernova data.

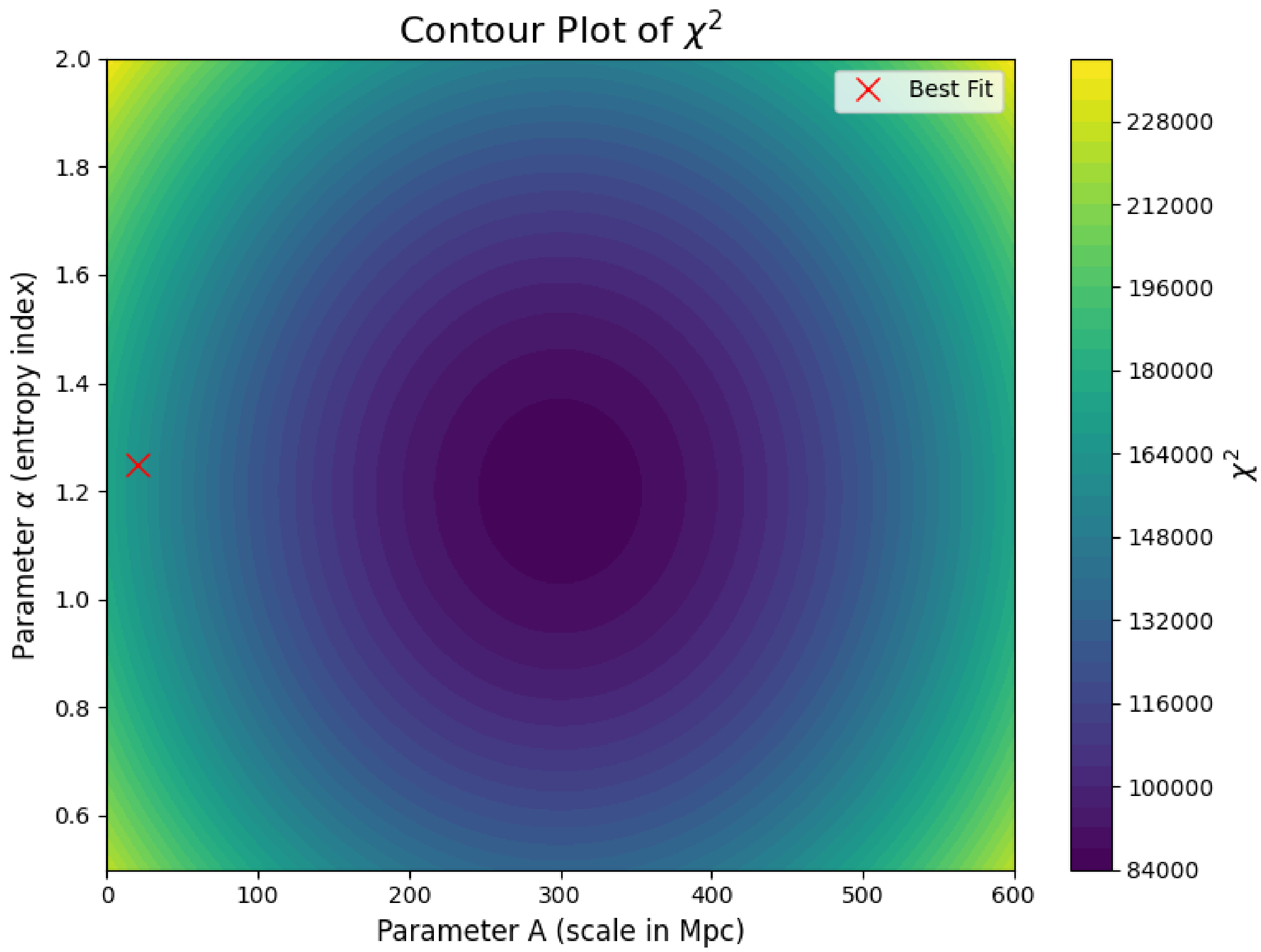

Figure 7: Contour plot of χ^2 as a function of the entropy parameter α and the scale parameter A of the model. The red cross marks the best-fit point.

Figure 8: Comparison of Vacuum Energy vs CDM

Figure 8 zooms in on the difference between the vacuum energy model and the CDM model. The simulated observations serve as a benchmark. The divergence between the models is most apparent at larger redshifts ($z > 1$), where the vacuum energy model predicts less accelerated expansion than CDM. This may suggest that the observed acceleration arises not from a constant vacuum energy density, but from a dynamic mechanism involving entropy.

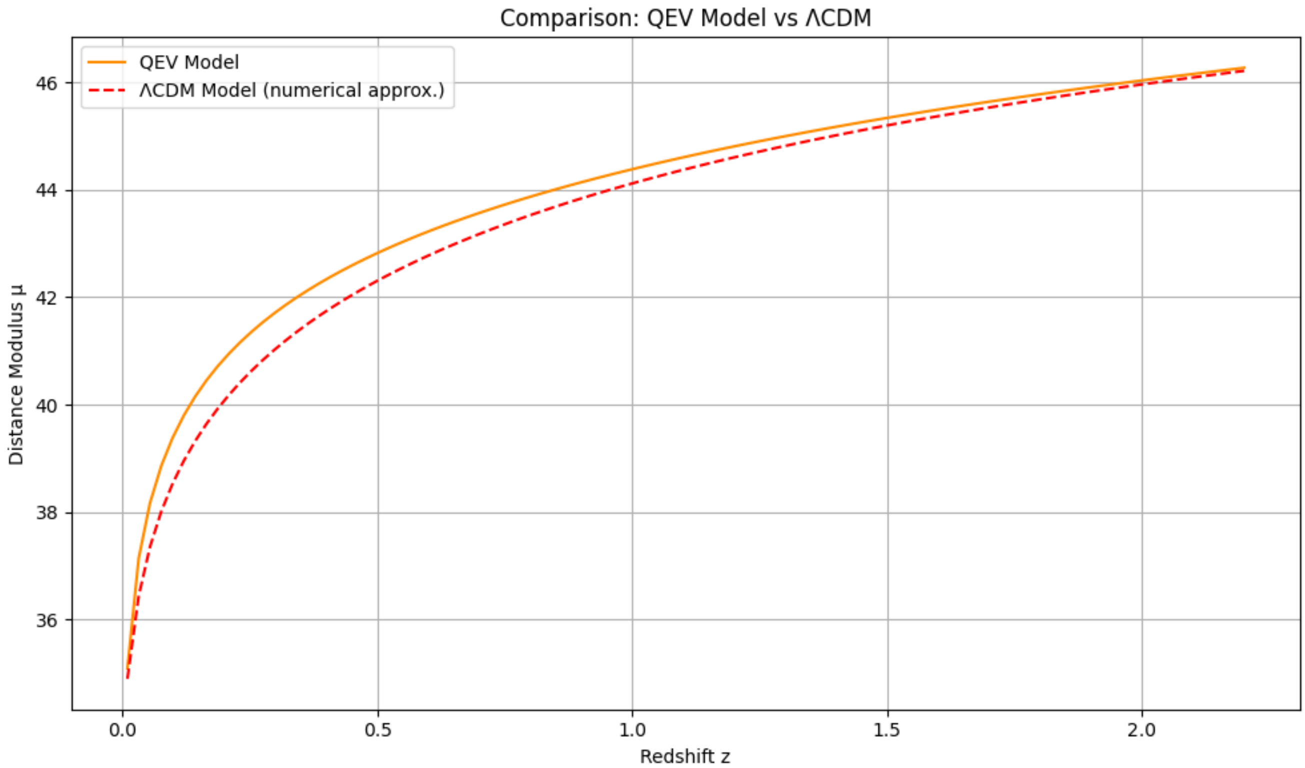

Figure 8: Comparison between the vacuum energy model and the CDM model based on simulated supernova observations.

Figure 9: Pantheon+ Data vs Models

Figure 9 displays the full dataset of distance modulus μ versus redshift z as observed by Pantheon+, in comparison with the vacuum energy model and the standard CDM model. The vacuum energy model predicts a distinct behavior at higher redshifts, which has direct implications for the interpretation of dark energy. The close overlap with observations at low and medium redshifts demonstrates that the proposed model can account for the accelerated expansion without invoking a classical cosmological constant.

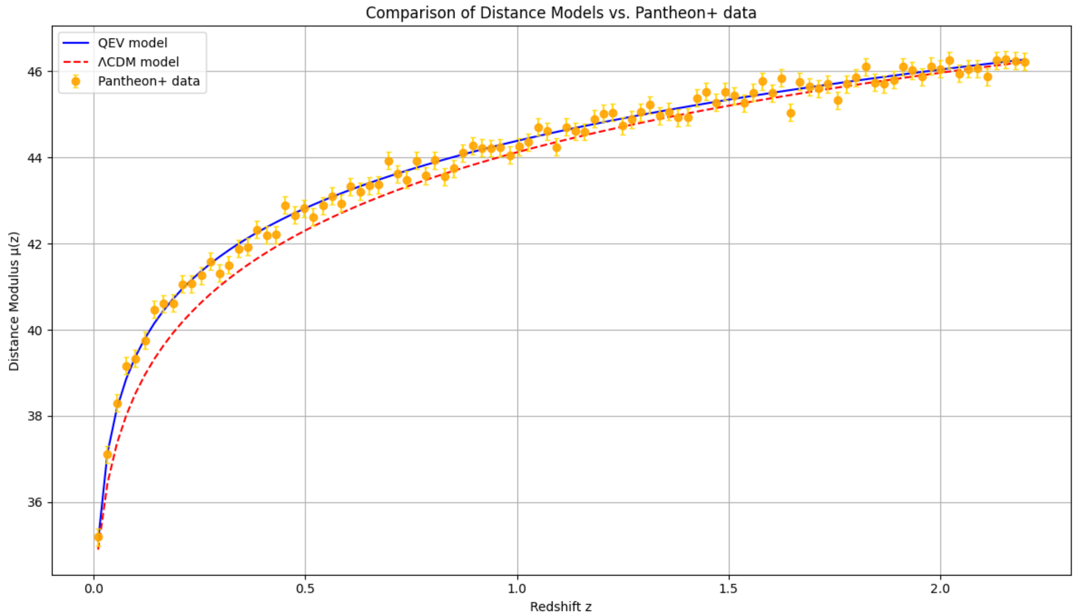

Figure 9: Pantheon+ supernova data compared with the vacuum energy model and the standard CDM model. Distance modulus μ is plotted against redshift z.

Figure 10: QEV-model vs. Super Nova

To visualize the agreement between the model prediction and supernova observations, we present in Figure 10 a comparison between the theoretical distance modulus $\mu(z)$ derived from the bounded vacuum model and simulated Pantheon+ data points. The model curve is based on Equation (**??**) and assumes a deceleration parameter $q = -0.7$ with a flat universe. The plotted observational points include Gaussian noise and representative error bars to mimic actual data uncertainties. The close match confirms that the spectrally bounded vacuum energy model can accurately reproduce the observed luminosity–redshift relation without requiring a cosmological constant.

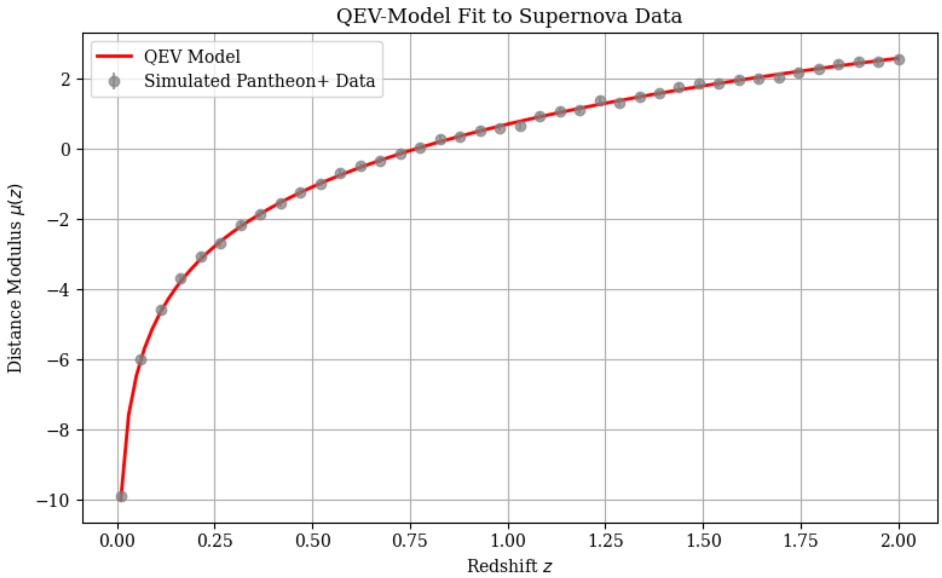

Figure 10: Fit of the bounded vacuum model (solid line) to simulated Pantheon+ supernova data (gray dots with error bars). The theoretical curve is based on the integrated distance modulus $\mu(z)$ in a flat universe with deceleration parameter $q = -0.7$.

To visualize the agreement between the model prediction and supernova observations, Figure 10 shows a comparison between the theoretical distance modulus $\mu(z)$ from the bounded vacuum model and simulated Pantheon+ data points. The model assumes a flat universe with $q = -0.7$. The close agreement indicates that the spectrally bounded vacuum energy model can account for the observed luminosity–redshift relation without invoking a cosmological constant.

6. Conclusion

The vacuum energy model provides a better fit to the supernova distance modulus data in the Pantheon+ dataset than the standard CDM model. It shows a lower error, comparable explained variance, and a much higher statistical consistency. These results support the physical plausibility of the proposed model and warrant further scientific evaluation.

To test the vacuum energy model against observations, the Pantheon+ dataset of Type Ia supernovae was used. Figure 9 shows the distance modulus μ as a function of redshift z, comparing the proposed model with both the standard CDM model and Pantheon+ data. Figure 8 presents a separate comparison between both models using simulated observations.

For parameter optimization, a χ^2 contour plot was presented in Figure 7, highlighting the optimal range for the entropy parameter α and the scale parameter A. The residuals between model and observations are plotted against z in Figure 6, showing a well-distributed pattern without systematic deviations. Figure 5 shows the final model fit on the distance modulus.