The Topologically Protected Singularity Qubit (TPSQ): A Cosmic Qubit for Trans-Aeonic Information Transfer

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Abstract

I propose a unified framework for the Topologically Protected Singularity Qubit (TPSQ), a quantum information carrier designed to resolve the trans-aeonic information paradox in Penrose's Conformal Cyclic Cosmology (CCC). By synthesizing four core axioms–topological protection, holographic embedding, topological evolution, and maximal entropy–the TPSQ provides a mathematically robust mechanism for information survival across conformal singularities. This work establishes the TPSQ as a bridge between quantum gravity, topology, and cosmology, with testable implications for the Cosmic Microwave Background (CMB) and quantum simulators.

1 Introduction: The CCC Information Paradox

Penrose's Conformal Cyclic Cosmology (CCC) posits an infinite sequence of universes (*aeons*), each terminating in a conformal singularity that births the next. The central challenge is the survival of information across these boundaries, where scale-dependent structures are erased by infinite conformal stretching ($\Omega \to \infty$).

The Topologically Protected Singularity Qubit (TPSQ) resolves this paradox by leveraging:

- **Topological quantum computation** (non-local encoding),
- **Loop Quantum Gravity** (discrete spacetime),
- **Holographic quantum error correction** (via AdS/CFT).

2 The Four Axioms: A Unified Framework

Axiom 2.1 (Topologically Protected State Space)**.** The TPSQ Hilbert space $\mathcal{H}_{\text{TPSQ}} \simeq \mathbb{C}^2$ is stabilized by a topological Hamiltonian:

$$H_{\text{eff}} = -K \sum_v A_v - J \sum_p B_p, \quad [A_v, B_p] = 0, \tag{1}$$

where $A_v = \prod_{j \in s(v)} \sigma_j^x$ and $B_p = \prod_{k \in \partial p} \sigma_k^z$. Key properties:

- Energy gap $\Delta > 0$ protects against decoherence,
- Ground state degeneracy $\dim(\mathcal{H}_0) = 2^g$ (genus g),
- Conformal invariance under $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$.

Axiom 2.2 (Holographic Embedding)**.** There exists an isometric embedding:

$$\iota : \mathcal{H}_{\text{TPSQ}} \hookrightarrow L^2(\overline{\mathcal{A}}, d_{\mu_{\text{AL}}})_{\text{LQG}} \otimes \bigoplus_{\Delta} \mathcal{V}_\Delta \otimes \overline{\mathcal{V}}_\Delta, \tag{2}$$

where:

- The LQG component encodes background-independent geometry,
- The CFT component captures conformal boundary states.

Axiom 2.3 (Topological Evolution)**.** The evolution of TPSQ states is governed by flat-connection holonomies:

$$U(\gamma) = \mathcal{P} \exp\left(-i \int_\gamma A\right) \in G, \quad F_A = 0, \quad G \subseteq E_8. \tag{3}$$

With:

- Path-independence: $U(\gamma_1) = U(\gamma_2)$ for homotopic $\gamma_1 \simeq \gamma_2$,
- Projective representations of $\pi_1(\Sigma)$,
- E_8 symmetry stabilizing $\mathcal{H}_0$ via non-Abelian anyons.

Axiom 2.4 (Maximal Entropy at Singularity)**.** At the conformal boundary:

$$\rho = \frac{1}{2} I \quad \Rightarrow \quad S_{\text{vN}}(\rho) = 1 \text{ bit}. \tag{4}$$

- Encodes irreducible trans-aeonic information,
- Enforces quantum no-cloning via entropy saturation.

3 Extended Mathematical Framework

3.1 Topological State Space Formalization

The protected Hilbert space is defined on a genus-g Riemann surface Σ_g:

$$\mathcal{H}_{\text{TPSQ}} = \text{span}\{|\psi_i\rangle\} \subset \mathcal{H}_{\text{phys}}, \quad \dim(\mathcal{H}_{\text{TPSQ}}) = 2^g,$$

where the physical space emerges from LQG spin networks:

$$\mathcal{H}_{\text{phys}} = \bigoplus_{\Gamma} \bigotimes_{e\in\Gamma} \mathcal{H}_e^{j_e}/\text{gauge}.$$

The spectral gap theorem ensures protection:

$$\Delta = \inf_{\phi\perp\mathcal{H}_0} \frac{\langle\phi|H_{\text{eff}}|\phi\rangle}{\|\phi\|^2} > 0,$$

with $\mathcal{H}_0 = \{|\psi\rangle : A_v|\psi\rangle = B_p|\psi\rangle = |\psi\rangle\}$.

3.2 Holographic Embedding Formalization

The isometric embedding satisfies:

$$\langle\iota(\psi)|\iota(\phi)\rangle_{\mathcal{H}_{\text{LQG}}\otimes\mathcal{H}_{\text{CFT}}} = \langle\psi|\phi\rangle_{\mathcal{H}_{\text{TPSQ}}}.$$

Realized through radial quantization:

$$\iota(|\psi\rangle) = \lim_{r\to\infty} r^{-\Delta_\psi} \mathcal{O}_\psi(\vec{x})|0\rangle,$$

where primary operators satisfy $[L_n, \mathcal{O}_\psi(0)] = 0$ $(n > 0)$. Entanglement entropy follows:

$$S(\rho_{\text{LQG}}) = S(\rho_{\text{CFT}}) = \frac{\text{Area}(\partial\mathcal{M})}{4G_N}.$$

3.3 Topological Evolution Formalization

The E_8-connection satisfies $F_A = dA + A \wedge A = 0$. Holonomies form representations:

$$\text{Hol}_A : \pi_1(\Sigma_g) \to E_8.$$

Unitary evolution via Wilson loops:

$$U(\gamma) = \text{tr}\left(\mathcal{P}\exp\oint_\gamma A\right).$$

Path independence from Stokes' theorem:

$$U(\gamma_1\gamma_2) - U(\gamma_2\gamma_1) = \iint_S F_A = 0.$$

3.4 Singularity Entropy Formalization

At conformal boundary $(t \to \infty)$:

$$\rho = \lim_{T\to\infty} \frac{1}{T}\int_0^T U(t)\rho_0 U^\dagger(t)dt = \frac{I}{2}.$$

Holevo bound saturation:

$$\chi(\rho) = S\left(\frac{1}{2}I\right) - \frac{1}{2}S(I) = 1 \text{ bit}.$$

No-cloning enforcement:

$$\text{Tr}(C^\dagger C\rho^{\otimes 2}) = \frac{1}{4} < 1 \quad \text{(unitarity violation)}.$$

3.5 Conformal Invariance Framework

Under Weyl transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$:

$$U(\gamma) \mapsto e^{-\Delta_\psi \sigma} U(\gamma) e^{\Delta_\psi \sigma} = U(\gamma).$$

Primary field correlations transform as:

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{\Omega^2 g} = \prod_i \Omega(x_i)^{\Delta_i} \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_g.$$

3.6 Extended Theoretical Framework

Non-Abelian anyon braiding:

$$B_{ij} = \exp\left(\frac{2\pi i}{k} h_{ij}\right),$$

with Cartan generators h_{ij}. Quantum error correction via $[[n, k, d]]$ code:

$$k/n = 1 - \frac{2}{g} \log_2 \epsilon, \quad d \sim e^{-cL/\xi}.$$

Conformal cyclic mapping:

$$\mathcal{M}_{\text{prev}} \xrightarrow{\Phi} I^+ \xrightarrow{\mathcal{C}} I^- \xrightarrow{\Psi} \mathcal{M}_{\text{next}},$$

where $\mathcal{C} : x^\mu \mapsto x^\mu / x^2$.

4 Experimental Signatures

4.1 Cosmological Probes

- **CMB anomalies:**
  - Non-Gaussian correlations in temperature,
  - B-mode vortices from projected holonomies.
- **Gamma-ray bursts:** Planck-scale time delays ($\sim 10^{-44}$ s) from LQG discreteness.

4.2 Laboratory Analogues

- **Superconducting qubits:** Emulate topological codes under rescaling,
- **Topological insulators:** Edge states mimic $\mathcal{H}_{\text{CFT}}$ behavior.

5 Open Challenges

1. **Background independence:** Merging $\mathcal{H}_{\text{LQG}}$ (diffeomorphism-invariant) with $\mathcal{H}_{\text{CFT}}$ (conformal-fixed)
2. **Decoherence:** Generalizing TPSQ to mixed states via $[[n, k, d]]$ codes
3. **Group selection:** Physical justification for $G = E_8$ symmetry
4. **Observational tests:** Quantitative modeling of T-B correlations in the CMB

Table 1: Interdependencies between TPSQ axioms

Interaction	**Mathematical Mechanism**	**Physical Significance**
A1 $\to$ A3	Ground state degeneracy enables path-independent $U(\gamma)$	Topological protection $\Rightarrow$ conformal invariance
A2 $\to$ A4	ι preserves $\text{Tr}(\rho^2) = \frac{1}{2}$	Holography ensures entropy conservation
A3 $\to$ A1	E_8 holonomies stabilize $\mathcal{H}_0$	Fault-tolerant computation with anyons

6 Conclusion

The TPSQ offers a novel solution to CCC's information paradox by encoding quantum bits across aeons. Its robustness stems from:

- Topological stability under conformal rescaling,
- Holographic embedding at singular boundaries,
- Holonomic evolution via E_8 symmetry,
- Entropic consistency (1 bit/*aeon*).

Further research will address background compatibility and seek empirical confirmation through cosmological and quantum simulation platforms.

References

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