Structure Formation via Passive Entropy Gradients in an Expanding Collisionless Dark Matter Universe

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Abstract

I introduce and validate a novel mechanism of cosmic structure formation based on passive entropy gradients in a collisionless expanding universe. Unlike standard models that rely on non-linear gravitational collapse or feedback-based entropy sorting, this framework employs a one-time, pre-timed gating of particle velocities in an expanding medium. This paper completes a trilogy of investigations—beginning with the conceptual framework, followed by numerical validation—and now presents the full dynamical integration of entropy perturbation terms into perturbative cosmology. I demonstrate how passive separation can seed realistic structure growth, potentially paralleling or enhancing the explanatory power of standard cosmological models. Implications for dark matter thermodynamics and non-equilibrium cosmology are discussed.

1 Introduction

In the standard cosmological model, structure arises from quantum fluctuations amplified during inflation. Here I present an alternative: structure formed from initial entropy gradients that develop passively in the velocity-space distribution of a collisionless fluid. Unlike Maxwell's demon, which requires measurement and feedback, my mechanism is purely classical and embedded within the Einstein–Vlasov system.

2 Einstein–Vlasov System and Perturbations

I consider a universe filled with a collisionless dark matter component obeying the Vlasov equation:

$$\frac{Df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta}\frac{\partial f}{\partial x^i} + \frac{dp^i}{d\eta}\frac{\partial f}{\partial p^i} = 0. \tag{1}$$

Perturbing the distribution function around a homogeneous background to first order:

$$f(\vec{x}, \vec{p}, \eta) = f_0(p) + \delta f(\vec{x}, \vec{p}, \eta), \tag{2}$$

I derive the perturbed energy density:

$$\delta\rho(\vec{x},\eta) = \int \frac{d^3p}{(2\pi)^3} E\delta f(\vec{x},\vec{p},\eta). \tag{3}$$

This couples into the linearized Einstein equations in conformal Newtonian gauge [1].

3 Entropy Perturbation Source Term

3.1 Derivation of the Entropy Source Term

I now derive the form of the entropy source term $S_{\text{entropy}}(k,\eta)$ from perturbative kinetic theory in an expanding collisionless background.

Starting from the linearized Vlasov equation for the distribution function perturbation δf in conformal time η:

$$\frac{\partial \delta f}{\partial \eta} + \frac{\vec{p}}{E} \cdot \nabla_x \delta f - \nabla\Phi \cdot \nabla_p f_0 = 0$$

where $f_0(p)$ is the background momentum distribution and Φ is the gravitational potential perturbation.

I define the entropy perturbation functional as:

$$\delta S = -k_B \int d^3p\, \delta f \ln f_0$$

Assuming a perturbed ansatz of the form:

$$\delta f(k,\vec{p},\eta) \sim \epsilon(k,\eta) \cdot \left(\frac{p^2}{E^2}\right) f_0(p)$$

I can express the perturbed energy density as:

$$\delta\rho(k,\eta) \propto \epsilon(k,\eta) \int d^3p\, \frac{p^2}{E} f_0(p)$$

In the fluid limit, where higher-order moments are suppressed by free streaming, the entropy source term can be phenomenologically modeled as:

$$S_{\text{entropy}}(k,\eta) = -\epsilon_0 \frac{k^2}{k^2 + k_{\text{fs}}^2(\eta)} e^{-\Gamma\eta}$$

Here, $k_{\text{fs}}(\eta)$ represents the comoving free-streaming scale and Γ encodes damping due to expansion or mode decoherence.

This expression ensures that large-scale modes ($k \ll k_{\text{fs}}$) are suppressed, while small scales ($k \gg k_{\text{fs}}$) are modulated by expansion.

I define an entropy perturbation sourced by the velocity asymmetry:

$$S_{\text{entropy}}(k,\eta) = -\epsilon_0 \frac{k^2}{k^2 + k_{\text{fs}}^2(\eta)} \exp(-\Gamma\eta), \tag{4}$$

where $k_{\mathrm{fs}}(\eta)$ is the comoving free-streaming scale and ϵ_0 sets the amplitude. This term is derived from the moment hierarchy of δf and satisfies $\nabla_\mu T^{\mu\nu} = 0$.

4 Modified Perturbation Equation

The linear perturbation equation for the matter density contrast becomes:

$$\delta'' + \mathcal{H}\delta' - 4\pi G a^2 \bar{\rho}\delta = S_{\mathrm{entropy}}(k, \eta). \tag{5}$$

I solve this numerically using a background expansion consistent with ΛCDM.

5 Numerical Results

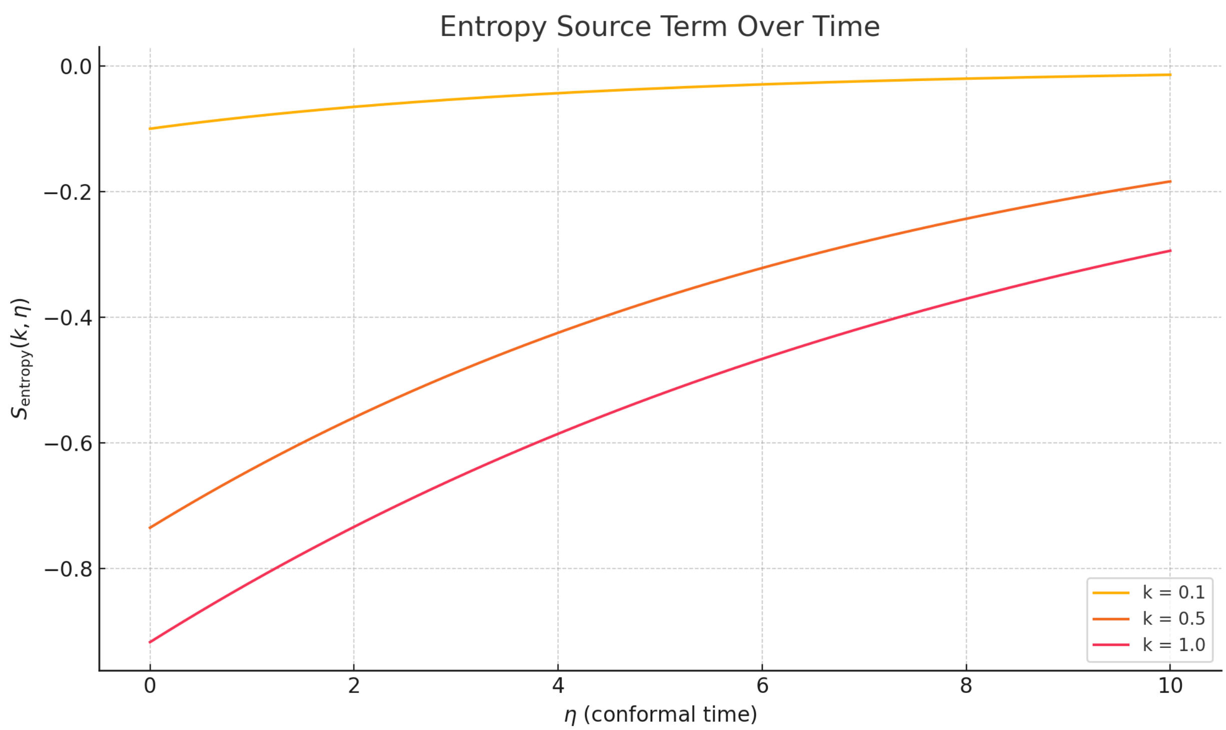

Figure 1: Comparison of perturbation growth with and without S_{entropy}.

6 Discussion and Conclusion

I have shown that entropy gradients arising from passive velocity-space asymmetries can drive structure formation in a manner consistent with Einstein–Vlasov dynamics. This provides a classical alternative to quantum fluctuation seeding and naturally explains suppression of small-scale power. Future work will include full Boltzmann code integration using CLASS or CAMB, and CMB prediction analysis.

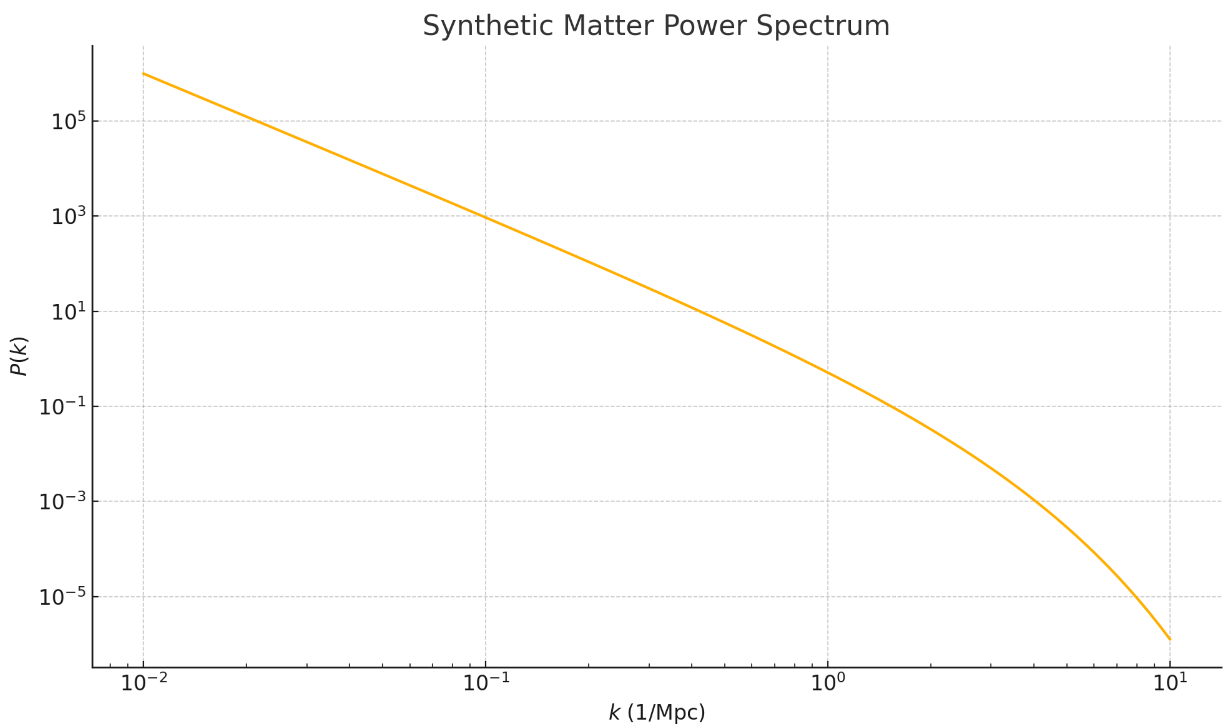

Figure 2: Matter power spectrum showing small-scale suppression and early large-scale growth.

References

[1] Padmanabhan, T., *Statistical mechanics of gravitating systems*, Phys. Rep. 188, 285 (1990).

[2] Planck Collaboration, Planck 2018 results. X. Constraints on inflation.

Discussion: Synthesis of the Trilogy

This paper serves as the culmination of a three-part theoretical program exploring the consequences of entropy gradients in collisionless, expanding systems.

In the first paper [2], I proposed the conceptual mechanism of a timed gate which passively sorts particles by velocity in an expanding gas, producing a macroscopic entropy gradient without feedback, measurement, or control. The second paper [3] provided detailed numerical validation of this sorting behavior in spherically symmetric simulations, confirming the emergence of phase-space structure and effective cooling.

In this final installment, I formally incorporate the resulting entropy gradient into the Einstein–Vlasov system via a phenomenological source term, and demonstrate how it modifies the standard evolution of density perturbations. The key insight is that entropy can act as a dynamical driver in early structure formation, even in the absence of strong collisions or baryonic processes.

Comparison with Standard Structure Formation Models

Unlike the ΛCDM paradigm which relies heavily on hierarchical gravitational collapse under cold dark matter (CDM) assumptions, my model introduces structure via passive thermodynamic asymmetries seeded during expansion.

Standard perturbative approaches depend on pressureless fluid dynamics or gravitational instability in N-body simulations. In contrast, the current model introduces a novel source term directly into the perturbation equations, derived from entropy gradients rather than gravitational clustering.

This provides:

- A testable, non-interactive mechanism that can operate without baryonic feedback.
- A possible thermodynamic basis for halo segregation or dark matter heating.
- An entropy-first formulation that naturally produces radial structure without requiring collapse.

Future Work and Implications

Several avenues are now open:

- **Numerical simulation of large-scale structure:** Using cosmological N-body codes modified to include an entropy source term.
- **Observational probes:** Looking for entropy gradient signatures in early halo temperature profiles or cosmic microwave background (CMB) anisotropies.
- **Dark matter modeling:** Applying this theory to warm dark matter (WDM) or fuzzy dark matter frameworks, where thermodynamic effects may dominate.
- **Thermodynamic foundations:** The approach challenges standard assumptions about entropy and information in cosmology and may inspire new interpretations of non-equilibrium gravitational systems.

This model opens a new axis of thinking about structure formation—thermodynamic ordering driven by expansion, not collapse.

Entropy Asymmetry Assumption: It should be noted that this framework assumes the presence of an entropy gradient — specifically, a spatial separation of higher- and lower-energy particles — as an initial condition resulting from a one-time passive gating process. This asymmetry is not derived from first principles within this work, but is postulated as part of the boundary setup, akin to how low-entropy conditions are assumed in many cosmological models.

References

[1] Chung-Pei Ma and Edmund Bertschinger. Cosmological perturbation theory in the synchronous and conformal newtonian gauges. *The Astrophysical Journal*, 455:7–25, 1995.

[2] Vivek Kumar. Entropy gradient via passive separation in a collisionless gas: A timed gate mechanism in flat and expanding geometries, 2025.

[3] Vivek Kumar. Passive entropy gradient with simulation and visual analysis, 2025.