A Gdel–FLRW Fusion: Rotating-Expanding Spacetime with Observable Dark Energy Behavior

Ivan Berdichevsky

May 31, 2025

Abstract

We present a novel solution to Einstein's field equations that fuses the rotating structure of the Gdel universe with the expansion dynamics of the FLRW cosmology. This physically structured and testable metric yields a stress-energy tensor with components exhibiting dark energy-like behavior through negative pressure sourced purely by rotation-expansion coupling. The model bridges metaphysical and observable domains while remaining grounded in general relativity.

1 Introduction

Cosmological models typically rely on either spatial homogeneity and isotropic expansion (FLRW) or allow for rotation without expansion (Gdel). Here, we propose a hybrid metric that combines the expansion of FLRW with Gdel's rotation, resulting in a spacetime that is both dynamically evolving and rotationally structured. This framework allows for a new interpretation of dark energy as a geometric consequence of rotation-expansion coupling.

2 Metric and Coordinates

We define the spacetime using cylindrical coordinates $\{t, r, \phi, z\}$ with rotation parameter Ω and Hubble expansion rate H:

$$ds^2 = \left[dt + \Omega r^2 d\phi\right]^2 - e^{2Ht}\left(dr^2 + r^2 d\phi^2 + dz^2\right). \tag{1}$$

3 Ricci Scalar

The Ricci scalar curvature derived from this metric is:

$$R = 12H^2 - 2\Omega^2 e^{-4Ht}. \tag{2}$$

The first term corresponds to FLRW-like expansion curvature, while the second encodes Gdel-type rotation.

4 Lagrangian Density

We construct the gravitational Lagrangian density (setting $\kappa = 8\pi G$):

$$\mathcal{L} = \frac{1}{2\kappa}(R - 2\Lambda) + \mathcal{L}_{\text{matter}} = \frac{1}{2\kappa}\left(12H^2 - 2\Omega^2 e^{-4Ht} - 2\Lambda\right) + \mathcal{L}_{\text{matter}}. \tag{3}$$

5 Einstein Tensor Components $G_{\mu\nu}$

Energy Density G_{00}

$$G_{00} = 3H^2 + 3\Omega^2 e^{-4Ht} - H^2\Omega^2 r^2 e^{-2Ht}. \tag{4}$$

Radial Energy Flux G_{01}

$$G_{01} = -H\Omega^2 r e^{-2Ht}. \tag{5}$$

Axial Pressure G_{33}

$$G_{33} = -\left(3H^2 e^{2Ht} + \Omega^2\right) e^{-2Ht}. \tag{6}$$

This component represents pressure along the z-direction and exhibits *negative pressure*, a hallmark of dark energy.

6 Stress-Energy Tensor

From Einstein's field equations $G_{\mu\nu} = \kappa T_{\mu\nu}$, we obtain:

$$T_{\mu\nu} = \frac{1}{\kappa} G_{\mu\nu}. \tag{7}$$

This tensor encodes:

- Energy density of a rotating-expanding spacetime
- Momentum flow arising from geometry alone
- Negative pressure behavior sourced by rotation+expansion

7 Physical Interpretation

The hybrid geometry leads to unique physical insights:

- G_{00} encodes the total energy content, mixing expansion and rotation
- G_{01} shows direct momentum flux from spacetime curvature
- G_{33} acts like a dynamic cosmological tension, supporting a geometric origin of dark energy

8 Conclusion

We have constructed a viable cosmological model that unifies rotating and expanding geometries within general relativity. The resulting stress-energy tensor reveals physical implications such as energy flux from curvature and residual negative pressure. These effects suggest that dark energy may emerge from purely geometric interactions rather than exotic matter.