Vacuum Energy with Natural Bounds: A Spectral Approach without Fine-Tuning

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Abstract

This paper introduces a physically grounded model for vacuum energy using a spectral integral bounded by natural physical limits. The upper bound arises from the QCD confinement scale, while the lower bound reflects thermodynamic suppression at low vacuum temperatures. The resulting energy density aligns with current cosmological observations—without invoking fine-tuning or new physics. This approach provides a realistic and testable contribution to resolving the cosmological constant problem. It is conceptually related to entropic gravity models, in which gravitational and vacuum phenomena arise from underlying thermodynamic structure, as explored by Verlinde (2016). [1] Related perspectives on vacuum energy have also been explored in Dutch academic literature, notably by Cloos et al.[2] For a broader theoretical overview, see also Carroll.[3]

1 Introduction

The cosmological constant problem represents one of the greatest discrepancies between theory and observation in modern physics. Standard models, based on integration up to the Planck scale, predict a vacuum density that is 120 orders of magnitude higher than the observed value. This approach provides a realistic and testable contribution to resolving the cosmological constant problem. It is conceptually related to emergent gravity models, where gravity and vacuum phenomena arise from thermodynamic or informational principles, as proposed by Verlinde [Verlinde 2011, 2016]. [4] [5] If one assumes a physically bounded vacuum, related to observation and transitions at known scale levels, a physical basis for this calculation emerges.

[1] [11] E. Verlinde, *Emergent Gravity and the Dark Universe*, SciPost Physics **2**, 016 (2017), arXiv:1611.02269 [hep-th].

[2] [12]M.A.H. Cloos, M.J.F. Klarenbeek, L. Meijer, and R.E. Pool, *De energie van het vacuüm*, under the supervision of J. de Boer, R. Dijkgraaf and E. Verlinde, University of Amsterdam, June 8, 2004.

[3] [13]S. M. Carroll, *The Cosmological Constant*, Living Reviews in Relativity **4**, 1 (2001).

[4] [10] E. Verlinde, *On the Origin of Gravity and the Laws of Newton*, Journal of High Energy Physics **2011**, 029 (2011), arXiv:1001.0785 [hep-th].

[5] [11] E. Verlinde, *Emergent Gravity and the Dark Universe*, SciPost Physics **2**, 016 (2017), arXiv:1611.02269 [hep-th].

This premise seems justified based on:

1. The boundaries that can be physically motivated and correspond to known scales: the confinement scale of the strong nuclear force and the thermal background temperature of the universe.

2. A theory that should be consistent with empirically observed values.

2 Physical Background

According to quantum field theory, each vibrational mode of a field in its ground state contributes an energy of

$$E = \frac{1}{2}\hbar\omega,$$

where ω is the angular frequency.

The relation between frequency and wavelength is given by:

$$\omega = \frac{2\pi c}{\lambda},$$

so the energy can also be expressed as:

$$E = \hbar\omega = \hbar\left(\frac{2\pi c}{\lambda}\right) = \frac{2\pi\hbar c}{\lambda}.$$ [6]

This shows that the energy of a fluctuation is inversely proportional to the wavelength: the shorter the wavelength, the higher the energy.

Short wavelengths contribute more energy but are physically bounded by the strong nuclear force: above the confinement scale, such energies cannot occur in the vacuum (except briefly during the QCD phase transition shortly after the Big Bang) and therefore do not contribute to vacuum energy. [7]

Long wavelengths on the other hand are suppressed by thermal effects, making their contribution negligible at temperatures around or below 30 K. [8]

[6] [5] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995)

[7] [8]K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma: From Big Bang to Little Bang*, Cambridge University Press (2005).

[8] [3]J. I. Kapusta and C. Gale, *Finite-Temperature Field Theory: Principles and Applications*, Cambridge University Press (2006).

3 Physical Boundaries of the Integration

The spectral integral is bounded by two physically motivated length scales:
Lower limit in wavelength: $\lambda_{\text{min}} = 1 \times 10^{-15}\,\text{m}$

This is the QCD confinement limit. Wavelengths shorter than 1 fm correspond to free quarks that do not contribute to vacuum energy. This forms an upper bound in energy.
Upper limit in wavelength: $\lambda_{\text{max}} = 4.8 \times 10^{-3}\,\text{m}$

Thermal limit at approximately 30 K. Wavelengths longer than this value are thermally suppressed. This forms a lower bound in energy.
These boundaries create a physically relevant energy window for vacuum fluctuations.

See appendix C in the appendix.

4 Spectral Formulation of Vacuum Density

The vacuum density in this proposed model is modeled as follows:

$$\rho_{\text{vac}} = A \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \lambda^{-5} \exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right) d\lambda$$

where:
λ^{-5} is analogous to the spectral distribution of black radiation, the exponential factors suppress contributions from both short and long wavelengths, [9]
L is the characteristic scale,
A is a normalization factor.

The double exponential term in the spectral integral,

$$\exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right),$$

plays a crucial role in the model. This term ensures natural and symmetric suppression of contributions from wavelengths outside the physically relevant region.

For $\lambda \ll L$, the term $\exp(-L/\lambda)$ dominates, leading to strong suppression of short-wavelength (high-energy) fluctuations.

For $\lambda \gg L$, the term $\exp(-\lambda/L)$ dominates, making long-wavelength (low-energy) contributions negligible.

The maximum of the integrand occurs at $\lambda = L$, which characterizes this scale as the dominant wavelength for vacuum contributions. The chosen form of suppression prevents divergent results and enables the calculation of a finite vacuum density without arbitrary cutoffs or fine-tuning.

[9] [1] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press (1982).

Unlike traditional mode-counting approaches based on a three-dimensional spherical volume with infinite radius (as used, for example, in Cloos et al. [10], the present model relies on a direct spectral integration bounded by natural physical constraints. This formulation avoids the need for artificial infrared cutoffs based on spatial extension and allows for a fully physical interpretation of the vacuum energy density.

Although the theoretical integration domain extends beyond the interval [0.01, 100], the integrand rapidly decays outside this range due to the double exponential term, rendering further contributions negligible.

Based on the chosen boundary values, this interval, therefore, effectively captures the physically relevant part of the integral.

Such alignment with physically motivated boundaries based on known scales, such as QCD confinement and the thermal background temperature of the universe, makes this approach a conceptually and mathematically viable solution to the vacuum energy problem.

See Appendix A (role of A) and Appendix B (mathematical elaboration).

5 Numerical Evaluation

Chosen values:

$L = 5 \times 10^{-7}\,\mathrm{m}$ (geometric mean of λ_{min} and λ_{max}),

$A = 3.13 \times 10^{-45}\mathrm{J{\cdot}m^4}$ (calibrated to yield a vacuum energy density consistent with observations, based on the physical boundaries of the model).

Evaluation of the integral over the interval x in [0.01, 100] yields:

$$\rho_{\mathrm{vac}} \approx 5.96 \times 10^{-10}\ \mathrm{J/m^3}$$

This value closely matches the observed vacuum density, as reported by the Planck 2018 results on cosmological parameters. [11]

A summary of all values is displayed in the table 1.

Table 1: Physical boundaries of the vacuum in this model

Quantity	Symbol	Value	Physical meaning
Lower limit wavelength	λ_{min}	$1 \times 10^{-15}\,\mathrm{m}$	QCD confinement, upper bound in ener
Upper limit wavelength	λ_{max}	$4.8 \times 10^{-3}\,\mathrm{m}$	Thermal limit at 30 K
Characteristic scale	L	$5 \times 10^{-7}\,\mathrm{m}$	Maximum of integrand
Vacuum normalization factor	A	$3.13 \times 10^{-45}\,\mathrm{J \cdot m^4}$	Normalization to density
Resulting density	ρ_{vac}	$5.96 \times 10^{-10}\,\mathrm{J/m^3}$	In agreement with observation

See explanation in section 4 of the appendix.

[10] [12]M.A.H. Cloos, M.J.F. Klarenbeek, L. Meijer, and R.E. Pool, *De energie van het vacuüm*, under the supervision of J. de Boer, R. Dijkgraaf and E. Verlinde, University of Amsterdam, June 8, 2004.

[11] [6] See Planck Collaboration (2020), *Astronomy & Astrophysics*, 641, A6.

6 Physical Interpretation of A

The scale factor A represents the effective degrees of freedom of the vacuum within the wavelength range between the confinement scale and the thermal limit. Analogous to systems in thermodynamics, such as the Debye model for solids or the blackbody radiation law, such a factor determines how many energetically active modes contribute to the total energy density. [12]
In the context of quantum field theory, A encompasses the combined contribution of relevant fields (fermionic and bosonic), including their interactions, suppression factors, and coupling strengths within this range. This approach enables the calculation of a realistic vacuum density without having to model each field or interaction separately.
Just as the Stefan–Boltzmann constant implicitly includes the degrees of freedom of photons (with two polarization states), [13] A in this model includes the collective contribution of all physically allowed vacuum fluctuations. This creates a natural scale anchoring of the density without fine-tuning.

See appendix A in the Appendix.

7 Mathematical Appendix

With substitution $x = \lambda/L$:

$$\rho_{\text{vac}} = AL^{-4} \int_{x_{\min}}^{x_{\max}} x^{-5} e^{-x-1/x} dx$$

where $x_{\min} = \lambda_{\min}/L$ and $x_{\max} = \lambda_{\max}/L$. The integrand peaks at $x \approx 1$, which corresponds to the characteristic scale L.

See appendix B in the Appendix.

8 Conclusion

This spectral approach to vacuum density demonstrates that the observed value can be achieved without fine-tuning, solely by applying known physical boundaries. The integral approach offers a physically motivated alternative for solving the cosmological constant problem [14] and deserves further theoretical support and experimental testing.
This approach considers the vacuum density as a stationary quantity within a spectral window, without explicit time dependence. This agrees with the classical interpretation of the cosmological constant as a time-independent parameter, while still allowing for possible extensions to dynamic models.

[12] [2] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics*, Wiley (1985).
[13] [9]A. Zee, *Quantum Field Theory in a Nutshell*, 2nd ed., Princeton University Press (2010).
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[3] J. I. Kapusta and C. Gale, *Finite-Temperature Field Theory: Principles and Applications*, Cambridge University Press (2006).

[4] P. J. E. Peebles and B. Ratra, *The Cosmological Constant and Dark Energy*, Rev. Mod. Phys. 75, 559–606 (2003).

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[6] Planck Collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641, A6 (2020).

[7] S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. 61, 1 (1989).

[8] K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma: From Big Bang to Little Bang*, Cambridge University Press (2005).

[9] A. Zee, *Quantum Field Theory in a Nutshell*, 2nd ed., Princeton University Press (2010).

[10] E. Verlinde, *On the Origin of Gravity and the Laws of Newton*, Journal of High Energy Physics **2011**, 029 (2011), arXiv:1001.0785 [hep-th].

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[12] M.A.H. Cloos, M.J.F. Klarenbeek, L. Meijer, and R.E. Pool, *The Energy of the Vacuum*, Supervised by J. de Boer, R. Dijkgraaf en E. Verlinde, University of Amsterdam, June 8, 2004.

[13] S. M. Carroll, *The Cosmological Constant*, Living Rev. Relativity 4, 1 (2001).

[14] A. J. H. Kamminga, *Vacuum Energy Density and Cosmic Expansion*, in preparation (2025).

Appendix: Vacuum Energy with Natural Bounds

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1 Appendix A: Physical Interpretation of the Scale Factor A

The scale factor A in the spectral formulation of vacuum energy density has the dimension $\mathrm{J\,m^4}$ and plays a central role in the normalization of the integral.
This factor is not merely a mathematical adjustment, but can be physically interpreted as a measure of the effective contribution of vacuum degrees of freedom within the spectral interval between λ_{min} and λ_{max}.

1.1 A as a Measure of Effective Degrees of Freedom

In quantum field theory, the contribution to vacuum energy is provided by all quantum fields that can fluctuate within the considered wavelength range. [1]
Within the energy range between the confinement scale (1 fm) and the thermal scale (30 K), the number of degrees of freedom is physically bounded. The scale factor A then represents:

$$A \propto g_{\text{eff}} \cdot \hbar \cdot c$$

where g_{eff} is the effective number of active degrees of freedom. [2]

1.2 Thermodynamic Analogy

A similar structure is found in the thermodynamics of radiation:

$$\rho_{\text{rad}} \propto g \cdot \frac{(k_B T)^4}{\hbar^3 c^3}$$

With a characteristic scale $L \sim 1/T$ follows:

$$A \propto g_{\text{eff}} \cdot \frac{\hbar c}{L^5}$$

where A/L yields an energy density with dimension $\mathrm{J/m^3}$.

[1] [5] See Peskin, M. E., & Schroeder, D. V. (1995). *An Introduction to Quantum Field Theory.* Addison-Wesley.
[2] [6] See Planck Collaboration (2020), *Astronomy & Astrophysics*, 641, A6.

1.3 Summary

The scale factor A represents:
The effective number of quantum fields contributing to vacuum fluctuations,
A spectral density factor similar to that in radiation laws,
A summarized contribution to the vacuum energy per wavelength unit.
This shows that the scale factor A, just like in the thermodynamics of radiation, is not merely a mathematical factor, but also physically meaningful, which convincingly supports the coherence of this model with thermodynamic principles.

2 Appendix B: Mathematical Derivation of Paper Section 6

The spectral integral for vacuum energy density is:

$$\rho_{\text{vac}} = A \int_{\lambda_{\min}}^{\lambda_{\max}} \lambda^{-5} \exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right) d\lambda$$

Substituting $x = \lambda/L$ gives:

$$\rho_{\text{vac}} = AL^{-4} \int_{x_{\min}}^{x_{\max}} x^{-5} e^{-x-1/x} dx$$

with $x_{\min} = \lambda_{\min}/L$, $x_{\max} = \lambda_{\max}/L$, and the maximum of the integrand at $x = 1$ gives:

$$L = \sqrt{\lambda_{\min}\lambda_{\max}}$$

The result lies in the order of magnitude of the observed value:[3]

$$\rho_{\text{vac}} \approx (6.0 \pm 1.8) \times 10^{-10}\ \text{J/m}^3$$

By choosing $L = \sqrt{\lambda_{\min}\lambda_{\max}}$, a physically motivated maximum is obtained without arbitrary cutoffs.
Unlike traditional mode-counting approaches based on a three-dimensional spherical volume with infinite radius (as used, for example, in Cloos et al. [4], the present model relies on a direct spectral integration bounded by natural physical constraints. This formulation avoids the need for artificial infrared cutoffs based on spatial extension and allows for a fully physical interpretation of the vacuum energy density.

[3] [6] See Planck Collaboration (2020), *Astronomy & Astrophysics*, 641, A6.
[4] [10]M.A.H. Cloos, M.J.F. Klarenbeek, L. Meijer, and R.E. Pool, *De energie van het vacuüm*, under the supervision of J. de Boer, R. Dijkgraaf and E. Verlinde, University of Amsterdam, June 8, 2004.

3 Appendix C: Phase Transitions as Limits of Vacuum Energy

In this model, the physical limits of vacuum fluctuations arise from two fundamental phase transitions:

3.1 Confinement Transition of the Strong Nuclear Force (QCD)

At an energy scale of approximately 1 GeV (corresponding to a length scale of about 1 femtometer), a transition occurs in which quarks become confined within hadrons. Experiments at particle accelerators such as RHIC and LHC have confirmed that above this scale, a quark–gluon plasma is formed, while in the current universe this plasma does not occur in a stable form. This state only existed in a phase before the QCD transition shortly after the Big Bang.[5]

Free quarks therefore do not contribute to vacuum fluctuations. This marks a natural upper limit in energy or a lower limit in wavelength: $\lambda_{\text{min}} \approx 1\,\text{fm}$.

Interestingly, this transition may also define the physical structure of the vacuum itself. The formation of stable hadrons at this scale suggests that the vacuum acquires its macroscopically coherent properties from this point onward. A possible hypothesis is that this might also relate to the nature of black holes: within the event horizon, there appears to be no vacuum structure as there is outside. Light cannot propagate there due to the absence of a supporting vacuum. Perhaps the absence of vacuum degrees of freedom at or above this energy density forms the fundamental 'secret' of black holes.

3.2 Thermal Transition at Low Temperature ($\sim 30\,\text{K}$)

At this temperature, long wavelengths are thermally suppressed. Analogous phenomena are known from laboratory experiments involving superconductivity and superfluidity.

If such a transition occurs during cosmic cooling in the vacuum, without pressure and interaction with matter, the thermal population of hadronic modes gradually diminishes. As the temperature drops to around 30 K, these fluctuations become virtually frozen and no longer contribute significantly to the vacuum energy density. This forms a natural upper bound in wavelength $\lambda_{\text{max}} \approx 4.8$ mm, marking the cessation of thermally driven vacuum fluctuations.[6]

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[5] [8] K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma: From Big Bang to Little Bang*, Cambridge University Press (2005).

[6] [3] See Kapusta & Gale (2006), *Finite-Temperature Field Theory*, Cambridge University Press.

3.3 Summary

Phase Transition	Wavelength Limit	Significance
QCD Confinement	$\lambda_{\min} \approx 1\,\mathrm{fm}$	Vacuum structure emerges in hadrons
Thermal Transition	$\lambda_{\max} \approx 4.8$ mm	Long wavelengths thermally suppressed

These boundaries structure the spectrum of vacuum fluctuations and support a spectral approach without fine-tuning.

Note: Although the suggestion that the vacuum acquires its coherent structure at $\sim 1\,\mathrm{GeV}$ is scientifically defensible, the connection to black holes remains speculative. In general relativity, light can still propagate within the event horizon. Nevertheless, the idea that the absence of vacuum degrees of freedom leads to a limitation of space-time is an interesting hypothesis within the context of emergent gravity.

4 Explanation of Suppression versus Limitation in the Vacuum

In the model for vacuum energy, a distinction is made between two types of constraints on the spectrum of wavelengths: **limitation** and **suppression**. These concepts may seem similar but differ fundamentally in their operation and meaning.

4.1 Limitation

A limitation is a **hard boundary**: beyond that limit, fluctuations no longer contribute to vacuum energy. This is comparable to abruptly cutting off an integral. In the model, two physically motivated limitations are present:

Long wavelength ($\lambda_{\max}$): at low temperatures (∼30 K), thermally driven fluctuations freeze. As a result, long wavelengths no longer contribute.

Short wavelength ($\lambda_{\min}$): the strong nuclear force limits the smallest scale. Energies above the QCD confinement scale (∼1 GeV) cannot cause stable fluctuations in the vacuum.

4.2 Suppression

Suppression is a **gradual weakening** of the contribution of fluctuations as they approach a boundary. Instead of stopping abruptly, suppression causes the contribution to **decrease exponentially**. [7]

In the model, this occurs via the term:

$$\exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right)$$

This double exponential factor produces two effects:

For **large** λ: the term λ/L becomes large $\Rightarrow$ suppression of long-wavelength (thermally suppressed) fluctuations.

For **small** λ: the term L/λ becomes large $\Rightarrow$ suppression of short-wavelength (QCD-limited) fluctuations.

4.3 Why is Suppression Essential?

A spectral factor such as λ^{-5} assigns higher energy to shorter wavelengths, which would otherwise dominate. Without suppression, the integral would diverge.

With suppression:

a **natural maximum** of the integrand appears at $\lambda = L$, and the spectrum is **smoothly bounded on both sides**, without abrupt cutoffs.

Analogy: An Audio Filter

Imagine an equalizer that gradually dampens all tones above a certain frequency, and does the same at the low end. What remains is a controlled midrange. This is how suppression works in this model: it selects the physically relevant range and weakens the rest, without distorting reality.

4.4 Summary

Limitation = hard boundary; beyond the range, no contribution.

Suppression = gradual damping; beyond the central range, the contribution decreases exponentially.

The exponential suppression in the model makes it possible, based on physical principles, to obtain a finite and realistic vacuum energy density without requiring arbitrary cutoffs or fine-tuning.

[7] [1] See Birrell & Davies (1982), *Quantum Fields in Curved Space*, Cambridge University Press.

5 Explanation of Entropy and Thermal Suppression in the Cosmic Vacuum

5.1 Entropy Increases, but Thermal Activation Decreases

It is important to distinguish between the **total entropy of the universe** and the **local thermal population of fluctuations**. In the evolution of the universe, the following occurs:

The **total entropy of the universe increases**. Expansion creates more physical configurations, more structure formation, and a larger cosmic volume in which energy is dispersed.

At the same time, **temperature decreases**. As a result, the likelihood that certain, especially low-energy (long-wavelength), fluctuations are still thermally excited decreases.

In other words: there are more possible states in the universe as a whole (entropy increases), but **the energy per degree of freedom decreases** and thus **the thermal activity per mode**.

5.2 Thermal Suppression Due to Cooling

The spectral integration in the model contains a suppressive factor for large wavelengths:

$$\exp\left(-\frac{\lambda}{L}\right)$$

This term corresponds to the suppression of thermal fluctuations at low temperature:[8]

> Large wavelengths (low energy) are no longer thermally active.
>
> The exponential factor reflects this physical behavior: the larger λ, the smaller the contribution.

This is therefore **not a decrease in entropy**, but a physical expression of how entropy and cooling together determine which modes still contribute to vacuum energy.

See figure 1 on the next page.

[8] [3] See Kapusta & Gale (2006), *Finite-Temperature Field Theory: Principles and Applications*, Cambridge University Press.

The graph in figure 1 shows the relative contribution of vacuum fluctuations as suppressed by the exponential factor $\exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right)$ plotted as a function of the wavelength.

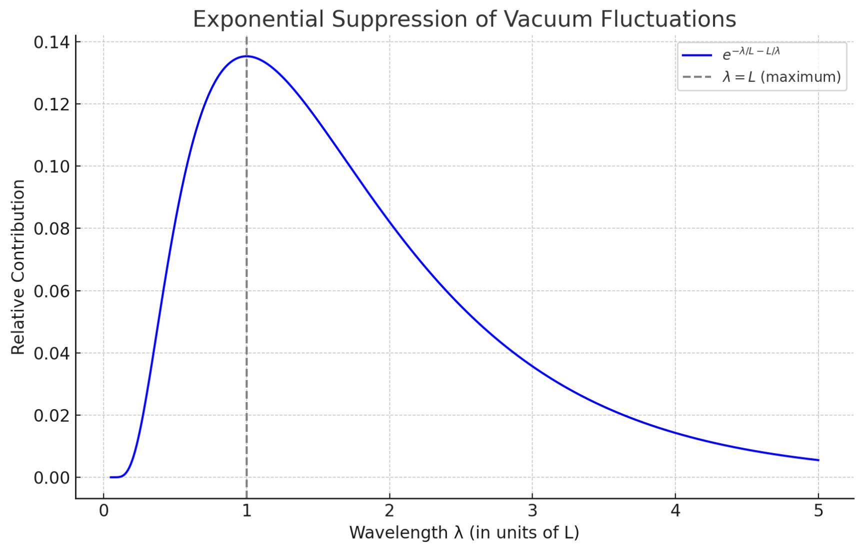

Figure 1: The graph shows the relative contribution of vacuum fluctuations as suppressed by the exponential factor $e^{-\lambda/L-L/\lambda}$, plotted as a function of the wavelength λ.

5.3 Summary

Entropy increases in the universe as a whole, as required by the second law of thermodynamics.

Thermal suppression occurs because fluctuations with long wavelengths are **no longer thermally activated** at low temperature.

This combination explains why the cosmic vacuum receives progressively less energy from long wavelength fluctuations as it cools down, not because the hadrons no longer exist, but because they are **suppressed by the thermal state of the universe**. It causes progressively less hadron-fluctuations.

6 Entropy and Exponential Suppression in the Vacuum Model

The relationship between entropy and exponential suppression in the proposed spectral vacuum model can be understood from statistical mechanics.

6.1 Basic Definition of Entropy

In thermodynamics, entropy is defined as:

$$S = k_B \ln W$$

where: S is the entropy, k_B is Boltzmann's constant, W is the number of microstates corresponding to a given macrostate.

6.2 Probability and Entropy

If we rewrite entropy as a function of probability P:

$$S = -k_B \ln P \quad \Leftrightarrow \quad P \propto e^{-S/k_B} \,,$$

then we see that states with low probability represent high entropy. Conversely, in thermal equilibrium:

$$P(E) \propto e^{-E/k_B T} \,,$$

where higher energies are less likely due to exponential suppression—a direct consequence of entropy.

6.3 Suppression in the Model

The spectral integral in the vacuum model contains the following suppression term:

$$\exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right)$$

This double exponential factor resembles a combination of Boltzmann factors and functions as follows:

For $\lambda \gg L$: the term $\exp(-\lambda/L)$ suppresses long wavelengths, analogous to thermal suppression at low temperatures.

For $\lambda \ll L$: the term $\exp(-L/\lambda)$ suppresses short wavelengths, as at high energies where the number of allowed microstates decreases (for example due to confinement in QCD).

6.4 Entropic Interpretation

The maximum of the integrand occurs at $\lambda = L$, where suppression is minimal. This point represents the state with maximum contribution, i.e., a maximum in effective entropy. There, the energetically allowed microstates are optimally represented. [9]
Thanks to physical suppression, a finite vacuum energy density can be calculated that not only makes fine-tuning unnecessary, but also naturally avoids the ultraviolet catastrophe within the proposed spectral model.

7 The Entropic Mechanism as a Driver of Cosmic Acceleration

7.1 Introduction

This appendix discusses how entropy, through the effective number of degrees of freedom, plays a concrete role in the acceleration of the universe. While entropy is traditionally understood as a measure of disorder, we show here that it also acts as a driving force behind the expansion of spacetime. The connection between thermodynamic degrees of freedom and gravitational effects is made explicit in mathematical form.

7.2 Thermodynamic Energy Density

In a thermal system or the early universe, the energy density is given by:

$$\rho = \frac{\pi^2}{30} \cdot g_{\text{eff}} \cdot T^4$$

where:

- ρ: energy density (e.g., of the vacuum),

- g_{eff}: effective number of active degrees of freedom (a weighted sum over particle modes).

g_{eff} increases as more particle degrees of freedom become thermally accessible, a direct expression of increasing entropy.

[9] [2] See Callen (1985), *Thermodynamics and an Introduction to Thermostatistics*, Wiley.

7.3 Coupling to Gravity

From Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

where $T_{\mu\nu}$ contains the energy density ρ. For a homogeneous and isotropic universe:

$$H^2 = \frac{8\pi G}{3}\rho$$

Substituting the thermal expression for ρ gives:

$$H^2 = \frac{8\pi G}{3} \cdot \left(\frac{\pi^2}{30} \cdot g_{\text{eff}} \cdot T^4 \right) \Rightarrow H \propto \sqrt{g_{\text{eff}}} \cdot T^2$$

7.4 Interpretation

This reveals a direct causal chain:

$$\text{Entropy} \longrightarrow g_{\text{eff}} \longrightarrow \rho \longrightarrow H$$

Entropy determines the number of active degrees of freedom, which leads to a higher energy density, and thus to an increase in the expansion rate of the universe. Because vacuum energy exerts a negative pressure ($p = -\rho$), this results in an accelerated expansion caused by a manifestation of "negative gravity."

7.5 Application in This Model

In this model, both short and long wavelengths are suppressed using a double exponential term. As a result, primarily fluctuating hadrons (with mass) remain active. These dominate the contribution to g_{eff}, and hence to ρ. Entropy therefore plays an indirect but decisive role in determining the vacuum energy and the resulting cosmic acceleration.

7.6 Conclusion

This analysis shows that entropy, through g_{eff}, plays a tangible role in the gravitational dynamics of the universe. The accelerating expansion is not merely the result of abstract constants, but can be seen as the macroscopic effect of microscopically active degrees of freedom, governed by entropic principles. This idea aligns with emergent gravity models, in which gravitational and vacuum phenomena are interpreted as thermodynamic effects of underlying microstates, as proposed by Verlinde [10].

This approach offers not only a resolution to the fine-tuning problem but also opens the door to a thermodynamic reinterpretation of vacuum energy, grounded in observable physics.

[10] [12] E. Verlinde, *Emergent Gravity and the Dark Universe*, SciPost Physics **2**, 016 (2017), arXiv:1611.02269 [hep-th].

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[13] S. M. Carroll, *The Cosmological Constant*, Living Rev. Relativity 4, 1 (2001).

[14] A. J. H. Kamminga, *Vacuum Energy Density and Cosmic Expansion*, in preparation (2025).

Explanation for Appendix B: Integral Analysis of Bounded Vacuum Energy

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Introduction

The form $\exp(-\lambda/L - L/\lambda)$ is a mathematical construction with a clear physical objective: to suppress contributions from both extremely short and extremely long wavelengths to the vacuum energy density. This approach is not directly derived from standard quantum field theory but is inspired by known physical suppression mechanisms.

1 Purpose of the Double Exponential

The term combines two suppression mechanisms:

- $\exp(-\lambda/L)$: suppresses long wavelengths (thermally motivated);

- $\exp(-L/\lambda)$: suppresses short wavelengths (QCD/confinement motivated).

The result is a symmetric envelope function that peaks at $\lambda = L$ and decays exponentially as one moves away from this characteristic scale. This avoids divergence in the spectral integration and introduces a natural cutoff without hard boundaries.

2 Analogy with Physical Suppression Mechanisms

The chosen form is analogous to:

- **Thermal distributions**: $\exp(-E/kT)$ limits high energy states;

- **Confinement models**: $\exp(-r/\Lambda)$ suppresses interactions at large distances;

- **Entropic suppression**: for example, in entropy-based gravity models.

Although these forms are often context-specific, the double exponential integrates them into a single, balanced function.

3 Physical Advantages

The function $\exp(-\lambda/L - L/\lambda)$ is:

- analytically smooth (differentiable over the entire domain);

- symmetric in $\ln \lambda$;

- maximal at the characteristic scale L;

- easy to integrate numerically;

- physically well motivated based on known limits in the vacuum spectrum.

4 Summary

The double exponential suppression is an effective and physically plausible way to constrain the divergent contribution of vacuum modes. This approach forms the core of the proposed model and a possible explanation for the observed vacuum energy density.

4.1 Logarithmic Derivative (for Analytical Maximum)

We examine the maximum of the spectral function:

$$\rho(\lambda) = \lambda^{-5} \cdot \exp\left(-\frac{\lambda}{L} - \frac{L}{\lambda}\right) \tag{1}$$

Take the natural logarithm:

$$\ln f(\lambda) = -5 \ln \lambda - \frac{\lambda}{L} - \frac{L}{\lambda} \tag{2}$$

Differentiate and set equal to zero:

$$\frac{d}{d\lambda} \ln f(\lambda) = -\frac{5}{\lambda} - \frac{1}{L} + \frac{L}{\lambda^2} = 0 \tag{3}$$

Multiply by λ^2 to eliminate fractions:

$$-5\lambda - \frac{\lambda^2}{L} + L = 0 \tag{4}$$

Use the physically motivated values:

$$\begin{aligned}
\lambda_{\text{min}} &= 1 \times 10^{-15} \text{ m} \\
\lambda_{\text{max}} &= 4.8 \times 10^{-3} \text{ m} \\
L &= \sqrt{\lambda_{\text{min}} \cdot \lambda_{\text{max}}} = \sqrt{4.8 \times 10^{-18}} \approx 2.19 \times 10^{-9} \text{ m}
\end{aligned}$$

Substitute $L = 2.19 \times 10^{-9}$ m into the equation:

$$-5\lambda - \frac{\lambda^2}{2.19 \times 10^{-9}} + 2.19 \times 10^{-9} = 0 \tag{5}$$

Multiply both sides by 2.19×10^{-9}:

$$-1.095 \times 10^{-8}\lambda - \lambda^2 + 4.7961 \times 10^{-18} = 0 \tag{6}$$

Rewrite in standard form:

$$\lambda^2 + 1.095 \times 10^{-8}\lambda - 4.7961 \times 10^{-18} = 0 \tag{7}$$

Solve this quadratic equation using the quadratic formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{8}$$

With:

$$\begin{aligned} a &= 1 \\ b &= 1.095 \times 10^{-8} \\ c &= -4.7961 \times 10^{-18} \end{aligned}$$

Calculate step-by-step:

$$\begin{aligned} b^2 &= (1.095 \times 10^{-8})^2 = 1.1990 \times 10^{-16} \\ 4ac &= 4 \cdot 1 \cdot (-4.7961 \times 10^{-18}) = -1.9184 \times 10^{-17} \\ \Delta &= b^2 - 4ac = 1.1990 \times 10^{-16} + 1.9184 \times 10^{-17} = 1.3908 \times 10^{-16} \\ \sqrt{\Delta} &\approx 1.1793 \times 10^{-8} \end{aligned}$$

Then:

$$\begin{aligned} \lambda_1 &= \frac{-1.095 \times 10^{-8} + 1.1793 \times 10^{-8}}{2} = \frac{0.0843 \times 10^{-8}}{2} = 4.215 \times 10^{-10} \text{ m} \\ \lambda_2 &= \frac{-1.095 \times 10^{-8} - 1.1793 \times 10^{-8}}{2} = -1.1371 \times 10^{-8} \text{ m (not physical)} \end{aligned}$$

Result: The maximum of the spectral function occurs at:

$$\lambda \approx 4.215 \times 10^{-10} \text{ m} \tag{9}$$

In summary: The maximum contribution to the vacuum energy density occurs at a wavelength slightly smaller than $\lambda = L$, but of the same order of magnitude. This confirms that L is a suitable characteristic scale.

Note: The fact that the maximum does not occur exactly at $\lambda = L$ is due to the additional weighting factor λ^{-5} in the spectral function. While L is the center of the symmetric suppression function $\exp(-\lambda/L - L/\lambda)$, the λ^{-5} term shifts the actual maximum toward shorter wavelengths. This is expected and reflects the greater weight of shorter wavelengths in the integration.

Visualization: The graph below, on next page, shows the full spectral function over a wavelength range from 10^{-12} to 10^{-4} meters (logarithmic). The blue line shows the behavior of $\rho(\lambda)$. The red dotted line marks the exact maximum at $\lambda \approx 4.215 \times 10^{-10}$ m, and the green dashed line indicates the scale L.

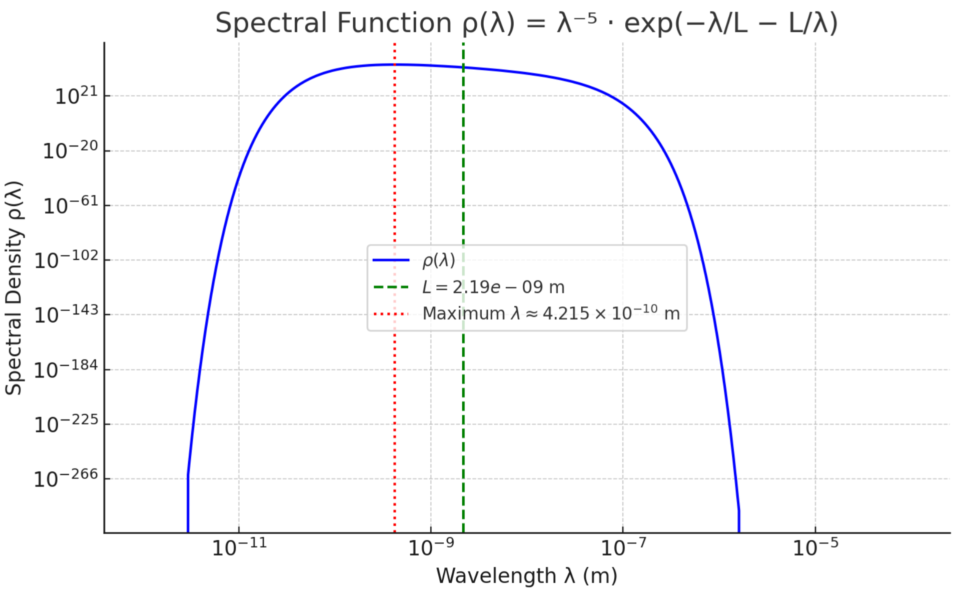

Figure 1: Spectral function $\rho(\lambda) = \lambda^{-5} \cdot \exp(-\lambda/L - L/\lambda)$. **Legend:** blue line = $\rho(\lambda)$; green dashed line = characteristic scale L; red dotted line = maximum $\lambda \approx 4.215 \times 10^{-10}$ m.

4.2 Physically Motivated Choice for L

The model uses:

$$L = \sqrt{\lambda_{\text{min}} \cdot \lambda_{\text{max}}} \tag{10}$$

with:

- $\lambda_{\text{min}} = 1 \times 10^{-15}$ m (QCD confinement limit)
- $\lambda_{\text{max}} = 4.8 \times 10^{-3}$ m (thermal limit at 30 K)

So:

$$L \approx 2.19 \times 10^{-9} \text{ m} = 2.19 \text{ nm} \tag{11}$$

Conclusion

The spectral function includes a suppression mechanism that limits the physical contributions to vacuum energy density to a window between two natural physical scales. The choice of $L = \sqrt{\lambda_{\text{min}} \cdot \lambda_{\text{max}}}$ is both mathematically derivable and physically justified. This makes the model a strong candidate for calculating a finite vacuum energy density without fine-tuning.

Report: Comparison of Alternative Models for Vacuum Energy

Introduction

This report presents a comparison of various models proposed as alternatives to the standard approach to vacuum energy in cosmology. The cosmological constant problem — which arises from a discrepancy of 120 orders of magnitude between the predicted and observed vacuum energy density — has led to a range of theoretical proposals. Included in this overview is the spectral model by Kamminga, which approaches vacuum energy based on physically motivated boundaries and thermodynamic suppression.

See table at next page

Conclusion

The comparison shows that Kamminga's model stands out due to its combination of physically motivated boundaries (QCD confinement and thermal saturation around 30 K), a double exponential suppression function, and the absence of fine-tuning. Furthermore, the model has been numerically tested and yields a vacuum energy density that falls within the same order of magnitude as the observed cosmological constant. This makes it a realistic and testable alternative to both dark energy theories and the broader vacuum structure problem.

Overview of Models

The following table provides an overview of the physical principles, the method of bounding the vacuum spectrum, the inclusion of entropy or thermal effects, numerical testability, the need for fine-tuning, and the empirical relevance of each model.

Model / Theory	Physical Basis	Spectrum Bounded	Entropy / Thermal	Numerically Tested	Fine-Tuning	Empirical Match
Kamminga's Spectral Model	QCD confinement + thermal suppression (30 K)	Yes	Yes	Yes	No	Yes
Verlinde's Emergent Gravity	Entropy & information geometry	No	Yes	Limited	No	Limited
Standard QFT with Planck Cutoff	Integration up to Planck scale (arbitrary)	Yes (arbitrary)	No	Yes (inaccurate)	Yes	No
Weinberg's QFT Approach	Quantum fields + renormalization	Implicit	No	No	Yes	No
Kapusta & Gale (Thermal QFT)	Thermal field theory at finite temperature	No	Yes	No	No	Limited
Debye Model Analogies	Thermodynamic cutoff on active modes	Yes (analogy)	Yes	Limited	Limited	Limited
Birrell & Davies (Curved Space)	Quantum fields in curved spacetime	No	Limited	No	No	Limited
Conformal Gravity (Mannheim)	Alternative field equations	No	No	Limited	No	Limited