A Two-Index Approach to Linear Term Generation in Arithmetic Structures

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Abstract

We introduce a novel two-index formula for term generation in arithmetic structures. Traditional arithmetic sequences rely on a single index to determine sequence terms. In contrast, our formula leverages the sum of two indices, offering a natural generalization to higher-dimensional structures. This approach maintains linear growth while introducing new symmetry and structural properties, making it potentially valuable for applications in grid-based systems, networks, and combinatorial frameworks.

1 Introduction

Arithmetic sequences are sequences of numbers with a constant difference between consecutive terms. The traditional formula for the n-th term is:

$$T_n = a + (n-1)d$$

where a is the initial term and d is the common difference.

Mathematical generalization often involves introducing additional variables, as seen in multivariable calculus and two-variable functions. Inspired by this, we propose a new structure where two indices generate the term through their sum, enriching the theory of arithmetic sequences.

2 Definition and Formalization

We define the two-index arithmetic structure by the formula:

$$T_{n_1,n_2} = a + (n_1 + n_2 - 2)d$$

where:

- a is the initial term,
- d is the common difference,
- $n_1, n_2 \in N^+$ are positive integers.

Notation Table

Symbol	Meaning
a	Initial term
d	Common difference
n_1, n_2	Positive integer indices
T_{n_1,n_2}	Term at position (n_1, n_2)

3 Properties

3.1 Linearity

The term grows linearly with respect to $n_1 + n_2$.

3.2 Symmetry

$$T_{n_1,n_2} = T_{n_2,n_1}$$

Addition is commutative, ensuring symmetric structure.

3.3 Special Cases

When $n_1 = 1$ or $n_2 = 1$, the formula simplifies to resemble the traditional sequence.

3.4 Constant Sum Lines

Terms with constant $n_1 + n_2$ lie along diagonals in a 2D grid, showing uniform term increments.

3.5 Comparison to Pascal's Triangle

While Pascal's triangle sums indices for combinatorial values, our structure maintains linearity, differentiating it.

4 Examples

Consider $a = 0$, $d = 1$. The table below shows T_{n_1,n_2} values:

$n_1 \backslash n_2$	1	2	3	4
1	0	1	2	3
2	1	2	3	4
3	2	3	4	5
4	3	4	5	6

Notice the diagonals are lines of constant sum $n_1 + n_2$.

5 Possible Applications

- **Grids and matrices**: Efficient indexing in 2D data structures.
- **Number theory**: Investigating properties related to sums of integers.
- **Combinatorics**: Analysis of structures where combined indices matter.
- **Algorithms**: Dynamic programming tables where two indices drive state changes.

6 Future Work

- Extension to more than two indices, e.g., $T_{n_1,n_2,n_3} = a + (n_1 + n_2 + n_3 - 3)d$.
- Allowing different step sizes for each index.
- Exploring non-linear growth models (quadratic, exponential generalizations).

7 Conclusion

This two-index structure provides a simple yet profound generalization of arithmetic sequences. It offers new perspectives for understanding multi-dimensional sequences and presents promising directions for mathematical exploration.

References

- Stewart, J., *Calculus: Early Transcendentals.* (for inspiration from multivariable functions)
- Basic textbooks on arithmetic sequences and series.
- General references on Pascal's triangle and number theory.