Basic formulas for number theory

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Abstract

Below I found certain formulas about number theory. The proofs are very complex and I'll submit them soon. For example there are formulas that require advanced mathematical tools like L'hôpital's rule, the Residue theorem, Jordan's lemma, the Cauchy Principal Value, the Dirichlet series expansions, the Wallis product.

$$3+\frac{1}{7+\frac{1}{16+\frac{1}{100}}}\approx 3.14159$$

$$\prod_{k=1}^{+\infty}\frac{4k^2}{4k^2-1}=\frac{\pi}{2}$$

$$\sum_{n=1}^{+\infty}(-1)^{n+1}\frac{\frac{(2n-3)!!}{(2n-2)!!})^{2*(\frac{\pi}{2})}}{n}=\frac{\Gamma(\frac{1}{4})^2}{2\sqrt{2\pi}}-\frac{2\sqrt{2}*\pi^{\frac{3}{2}}}{\Gamma(\frac{1}{4})^2}$$

$$-\left(\frac{1}{x}\left(\int_0^{\infty} t\,e^{-t}\log(t)\,dt\right)x-1\right)=\gamma$$

$$\sum_{k=0}^{\infty}\frac{2^k\,k!}{(2\,k+1)!}=\sqrt{\frac{e\,\pi}{2}}\ \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)$$

$$6\times\frac{\frac{\log^2(2)}{\sqrt{3}}\sum_{n=0}^{\infty}\frac{(-1)^n}{3^n\,(2\,n+1)}}{\log^2(2)}=\pi$$

$$\lim_{n\to+\infty}\sum_{k=1}^{n}\left(\frac{1}{k}-ln(1+\frac{1}{k})+\frac{Cte}{n}\right)=\gamma+Cte$$

$$\int_0^{\infty}\frac{\cos(x)}{1+x^2}\,dx=\frac{\pi}{2\,e}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$$

$$\sum_{n=1}^{\infty}\frac{\mu(n)}{n^s}=\frac{1}{\zeta(s)}$$

$$e^{\frac{1}{2}\sum_{k=1}^{\infty}\log\left(\frac{4k^2}{4k^2-1}\right)}=\sqrt{\frac{\pi}{2}}$$