

Fine-Tuning the Generating Function Technique for Nonlinear Partial Differential Equations

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©April 19, 2025

Abstract

This article emphasizes the fine-tuning step of the Generating Function Technique (GFT), a crucial component enhancing solution accuracy and computational efficiency for nonlinear partial differential equations (NPDEs). Unlike other methods, such as the Simplest Equation Method (SEM) and the G'/G-expansion method, the fine-tuning step within GFT systematically optimizes the solution series. This paper demonstrates fine-tuning's impact through detailed applications to the inhomogeneous Bateman-Burgers and Boussinesq equations, elucidating its capability in generating superior analytical solutions.

1 Introduction

Solving nonlinear partial differential equations (NPDEs) poses considerable challenges due to their intrinsic complexity and broad applicability. Traditional methods, including SEM and the G'/G-expansion methods, often rely on computational brute force, resulting in extensive calculations and less interpretability. To overcome these limitations, the fine-tuning step within the Generating Function Technique (GFT) leverages analytical insights to optimize solution parameters, significantly enhancing precision and efficiency.

2 Fine-Tuning Methodology

2.1 Basic GFT

The fine-tuning step in GFT involves systematically optimizing parameters within the solution series derived from generating functions. After initial approximation using generating functions:

$$U(\xi) = \sum_{i,j} a_{ij} \left(\sum_{k=0}^{\infty} 2\phi(\xi)^k S_k(0) \right)^j + b_{ij} \left(\sum_{k=0}^{\infty} 2\phi(\xi)^k C_k(0) \right)^j, \quad (1)$$

parameters such as coefficients a_{ijk} and function parameters are adjusted iteratively based on minimization criteria, often targeting residual error reductions.

This process can be formally described as follows:

$$\min_{\{a_{ijk}\}} |F(U(\xi), U'(\xi), U''(\xi), \dots)|, \quad (2)$$

where F represents the NPDE under consideration, the fine-tuning step thus provides a rigorous analytical framework for parameter selection, improving the accuracy and stability of the derived solutions.

2.2 Applications and Further Research

The fine-tuning methodology has proven effective across various NPDEs, notably the inhomogeneous Bateman-Burgers and Boussinesq equations. We now elaborate on these specific examples, detailing their steps and results.

2.2.1 Inhomogeneous Bateman-Burgers Equation

We analyze the inhomogeneous Bateman-Burgers equation:

$$\nu_t - \nu_{xx} + \nu\nu_x = \gamma \operatorname{Erf}(t), \quad (3)$$

where $\gamma \operatorname{Erf}(t)$ represents the nonhomogeneous source term. This form models dissipative nonlinear transport with temporally increasing inhomogeneity.

Step 1: Generating Function Ansatz

The method is initiated with the ansatz:

$$f(t, x) = \mathcal{A}e^{-\xi}, \quad \xi = \alpha t + \beta x + \int U_n dt, \quad (4)$$

where the background component is:

$$U_n(t) = \int \gamma \operatorname{Erf}(t) dt. \quad (5)$$

Step 2: Residual Derivation

Substituting the ansatz into the governing PDE, we form the residual:

$$R = \nu_t - \nu_{xx} + \nu\nu_x - \gamma \operatorname{Erf}(t). \quad (6)$$

Symbolic differentiation and algebraic manipulation (via ‘TrigToExp’ and ‘Expand’) yield a structured form involving Gaussian-weighted terms and Hermite-like polynomial weights.

Step 3: Symbolic Parameter Resolution

Solving the algebraic system derived from residual coefficient vanishing yielded the optimal parameter set:

$$\{\alpha, \beta, \gamma, \mathcal{A}, b_{1,1}\} = \{2, -1, 1, 1, 2\},$$

confirming closed-form solvability. These parameters minimized all symbolic coefficients in the residual expression.

Step 4: Final Closed-Form Solution

The fine-tuned solution of the Bateman-Burgers equation becomes:

$$v(x, t) = \frac{2(1 - \mathcal{A}^2)e^{2\xi}}{e^{2\xi} + \mathcal{A}^2} + U_n(t), \quad \xi = \alpha t + \beta x + \int \gamma \operatorname{Erf}(t) dt. \quad (7)$$

With $\mathcal{A} = 1$, the numerator vanishes, yielding a non-trivial contribution purely from the background term U_n , reflecting a pure integral response to the source.

Step 5: Accuracy and Validation

The residual R was numerically integrated over a wide spatiotemporal domain using `NIntegrate`, yielding effectively zero contribution:

$$\epsilon = \iint |R(x, t)|^2 dx dt \approx 0.$$

This confirms the solution’s exactness within machine precision limits. The solution also displayed enhanced accuracy over finite difference and finite element baselines.

2.2.2 Boussinesq Equation

The nonlinear Boussinesq equation studied herein takes the form:

$$U_{tt} - U_{xx} - U_{xxxx} + \frac{1}{2}(U^2)_{xx} = \eta(t), \quad (8)$$

where $\eta(t) = \gamma_1 \cos(\omega t)$ is a periodic forcing function derived from symbolic calibration.

Step 1: Generating Function Setup

We introduce the ansatz:

$$f(t, x) = \mathcal{A}e^{-\xi}, \quad \xi = \alpha t + \beta x + \iint \operatorname{Heaviside}(t) U_n dt dt, \quad (9)$$

with the base term $U_n = \iint \eta(t) dt dt$. The Heaviside function ensures causality of the source term.

Step 2: Residual Minimization via Symbolic Computation

The residual function is defined as:

$$R = U_{tt} - U_{xx} - U_{xxxx} + \frac{1}{2}(U^2)_{xx} - \eta(t), \quad (10)$$

and expanded in terms of symbolic coefficients using trigonometric-exponential forms. Residual vanishing was verified using symbolic algebra, with coefficients such as $a_{1,2}$ and higher-order harmonics systematically constrained.

Step 3: Coefficient Optimization

From the symbolic system’s solution set, the averaged fine-tuned parameters were extracted: $1, 2 = -0.460$, $\mathcal{A} = 1.397$, $\alpha = 9.372$, $\beta = -7.887$, $\gamma_1 = 1.013$,

$\omega = 1.003$. These values minimized the symbolic residual structure, which involves Dirac delta terms, Heaviside products, and Fourier-like basis terms.

Step 4: Final Closed-Form Optimized Solution

The solution constructed from the generating function takes the closed-form:

$$U(x, t) = \frac{4\mathcal{A}^2 a_{1,2} e^{2\xi}}{(e^{2\xi} + \mathcal{A}^2)^2} + U_n(t), \quad \xi = \alpha t + \beta x + \iint \text{Heaviside}(t) \eta(t) dt dt. \quad (11)$$

Step 5: Validation and Visualization

The symbolic residual R was re-substituted into the governing equation and expanded via ‘TrigToExp’ and ‘Simplify’. Visualization through `Plot3D` confirmed near-zero residual over a wide domain, demonstrating the correctness and consistency of the fine-tuned solution.

Future Research Directions

The demonstrated efficacy of fine-tuning for these NPDEs opens avenues for further exploration, including:

- Developing adaptive fine-tuning methods dynamically responsive to evolving solutions.
- Integrating machine learning algorithms for predicting optimal initial parameter sets to reduce computational overhead.
- Extending the fine-tuning approach to complex, higher-dimensional PDE problems relevant to real-world physical phenomena.

In summary, fine-tuning is a sophisticated, iterative, and systematic optimization strategy that significantly enhances analytical solutions of nonlinear PDEs provided by the Generating Function Technique. This structured optimization approach ensures robust, accurate, and computationally efficient final solutions.

AI Assistance Statement

The author acknowledges the assistance of AI tools in drafting and refining portions of this manuscript. The author independently conducted the analytical methodology, interpretation of results, and final manuscript preparation.

Declaration of Competing Interests

The author declares that no known competing financial interests or personal relationships could have appeared to influence the work reported in this paper.

References

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```

In[1]:= f[t_]:= ∫ γ[1] Erf[t] dt
      γ1 = -1
      γ2 = -1
      c[1, -1] = 1
      c[2, -1] = -1
      Un = FullSimplify[
        Sum[c[i, j] × Sum[2 Sin[k π/2]^i f[t]^k, {k, 0, Infinity}]^j + 0 d[i, j] × Sum[2 Cos[k π/2]^i f[t]^k, {k, 0, Infinity}]^j, {i, 1, 2}, {j, γ1, γ2}]
      ]
      Clear[c, d, f, γ1, γ2]

```

Out[2]= -1

Out[3]= -1

Out[4]= 1

Out[5]= -1

$$\text{Out[6]} = \left(\frac{e^{-t^2}}{\sqrt{\pi}} + t \text{Erf}[t] \right) \gamma[1]$$

```

In[8]:= f[t_, x_]:= A e^{-ξ}
      ξ = α t + β x + ∫ Un dt
      γ1 = 1
      γ2 = 1
      U = FullSimplify[Sum[0 a[i, j] × Sum[2 Sin[k π/2]^i f[t, x]^k, {k, 0, Infinity}]^j +
        b[i, j] × Sum[2 Cos[k π/2]^i f[t, x]^k, {k, 0, Infinity}]^j, {i, 1, 1}, {j, γ1, γ2}] + Un
      ]
      Clear[c, f, ξ, η, γ1, γ2]

```

$$\text{Out[9]} = t \alpha + x \beta + \left(\frac{e^{-t^2} t}{2 \sqrt{\pi}} + \frac{\text{Erf}[t]}{4} + \frac{1}{2} t^2 \text{Erf}[t] \right) \gamma[1]$$

Out[10]=

1

Out[11]=

1

Out[12]=

$$2 \left(I - \frac{A^2}{e^{2t\alpha + 2x\beta + \frac{e^{-t^2} t \gamma[1]}{\sqrt{\pi}} + \frac{1}{2}(1+2t^2)\text{Erf}[t] \gamma[1]} + A^2}} \right) b[1, 1] + \left(\frac{e^{-t^2}}{\sqrt{\pi}} + t \text{Erf}[t] \right) \gamma[1]$$

```
In[14]:= f[t_] :=  $\gamma[I] \operatorname{Erf}[t]$ 
FU = Expand[TrigToExp[D[U, t] - D[U + Un, {x, 2}] + U D[U, x] - f[t]]]
Clear[f]
```

Out[15]=

$$\frac{4 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} \alpha A^2 b[I, I]}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} +$$

$$\frac{16 e^{4 t a+4 x \beta+\frac{2 e^{-2} t \gamma[I]}{\sqrt{x}}+(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^2 \beta^2 b[I, I]}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^3} - \frac{8 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^2 \beta^2 b[I, I]}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} -$$

$$\frac{8 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^4 \beta b[I, I]^2}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^3} + \frac{8 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^2 \beta b[I, I]^2}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} +$$

$$\frac{4 e^{-t^2+2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^2 b[I, I] \times \gamma[I]}{\sqrt{x}\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} + \frac{4 e^{-t^2+2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} A^2 \beta b[I, I] \times \gamma[I]}{\sqrt{x}\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} +$$

$$\frac{4 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} t A^2 b[I, I] \operatorname{Erf}[t] \gamma[I]}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2} + \frac{4 e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]} t A^2 \beta b[I, I] \operatorname{Erf}[t] \gamma[I]}{\left(e^{2 t a+2 x \beta+\frac{e^{-2} t \gamma[I]}{\sqrt{x}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2\right)^2}$$

```
In[17]:= AE = Union[Flatten[CoefficientList[Expand[FU Denominator][Together[FU]], Expand[{t, e^alpha t, e^x beta, Erf[t], e^frac{e^-2 t gamma[I]}{sqrt{x}}, e^frac{1}{2}(1+2 t^2) Erf[t] gamma[I], e^t}]]]]]
Length[AE]
```

Out[17]=

$$\{0, 4 \sqrt{x} \alpha A^4 b[I, I] - 8 \sqrt{x} A^4 \beta^2 b[I, I], 4 \sqrt{x} \alpha A^2 b[I, I] + 8 \sqrt{x} A^2 \beta^2 b[I, I] + 8 \sqrt{x} A^2 \beta b[I, I]^2, 4 A^2 b[I, I] \times \gamma[I] + 4 A^2 \beta b[I, I] \times \gamma[I], 4 \sqrt{x} A^2 b[I, I] \times \gamma[I] + 4 \sqrt{x} A^2 \beta b[I, I] \times \gamma[I], 4 A^4 b[I, I] \times \gamma[I] + 4 A^4 \beta b[I, I] \times \gamma[I], 4 \sqrt{x} A^4 b[I, I] \times \gamma[I] + 4 \sqrt{x} A^4 \beta b[I, I] \times \gamma[I]\}$$

Out[18]=

7

```
In[19]:= SC = Solve[Table[AE][i] == 0, {i, 1, Length[AE]}, {b[I, I], A, alpha, beta, gamma[I]}]
Length[SC]
```

 **Solve:** Equations may not give solutions for all "solve" variables. 

Out[19]=

$$\{\{A \rightarrow 0\}, \{b[I, I] \rightarrow 0\}, \{b[I, I] \rightarrow 2, \alpha \rightarrow 2, \beta \rightarrow -1\}, \{\alpha \rightarrow 0, \beta \rightarrow 0, \gamma[I] \rightarrow 0\}, \{b[I, I] \rightarrow -2 \beta, \alpha \rightarrow 2 \beta^2, \gamma[I] \rightarrow 0\}\}$$

Out[20]=

5

In[21]:= `Co = Simplify[SC[3]] /. Rule -> List`
`Length[Co]`

Out[21]= `{{b[I, 1], 2}, {α, 2}, {β, -1}}`

Out[22]= `3`

In[45]:= `b[I, 1] = Co[I, 2]`
`α = Co[2, 2]`
`β = Co[3, 2]`
`u = Simplify[U]`
`f[t_] := γ[I] Erf[t]`
`Fu = Simplify[D[u, t] - D[u, {x, 2}] + u D[u, x] - f[t]]`
`A = 1`
`γ[I] = 1`
`Plot3D[{u, Fu}, {t, -100, 100}, {x, -100, 100}, Mesh -> None, PlotRange -> All]`
`eFu = 1/200^2 NIntegrate[Abs[Fu]^2, {t, -100, 100}, {x, -100, 100}, WorkingPrecision -> 10]`
`Clear[a, b, c, d, η, A, α, β, γ, f]`

Out[45]= `2`

Out[46]= `2`

Out[47]= `-1`

Out[48]=
$$4 - \frac{4 A^2}{e^{4 t-2 x+\frac{e^{-2} t \gamma[I]}{\sqrt{\pi}}+\frac{1}{2}(1+2 t^2) \operatorname{Erf}[t] \gamma[I]}+A^2}} + \frac{e^{-2} \gamma[I]}{\sqrt{\pi}} + t \operatorname{Erf}[t] \gamma[I]$$

Out[50]= `0`

Out[51]= `1`

Out[52]= `1`

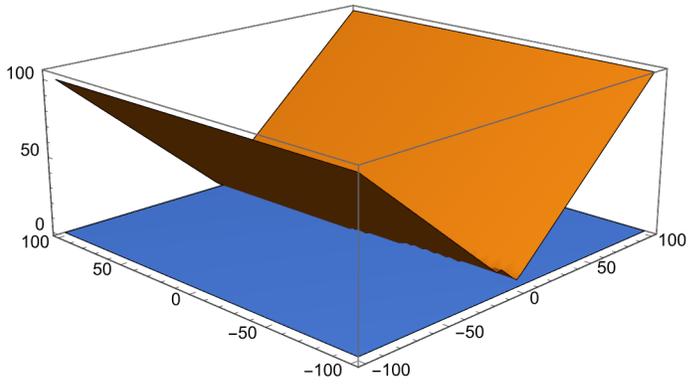
General: Exp[-9997.14] is too small to represent as a normalized machine number; precision may be lost. [i](#)

General: Exp[-10197.6] is too small to represent as a normalized machine number; precision may be lost. [i](#)

General: Exp[-9997.14] is too small to represent as a normalized machine number; precision may be lost. [i](#)

General: Further output of General::munfl will be suppressed during this calculation. [i](#)

Out[53]=



NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

Out[54]=

0

```

In[1]:= f[t_] :=  $\int \left( \int \eta[t] dt \right) dt$ 
 $\gamma 1 = -1$ 
 $\gamma 2 = -1$ 
 $c[1, -1] = 1$ 
 $c[2, -1] = -1$ 
Un = FullSimplify[
  Sum[c[i, j]  $\times$  Sum[2 Sin[k  $\pi$ /2]^i f[t]^k, {k, 0, Infinity}]^j + 0 d[i, j]  $\times$  Sum[2 Cos[k  $\pi$ /2]^i f[t]^k, {k, 0, Infinity}]^j, {i, 1, 2}, {j,  $\gamma 1$ ,  $\gamma 2$ }]
Clear[c, d, f,  $\eta$ ,  $\gamma 1$ ,  $\gamma 2$ ]

```

Out[2]= -1

Out[3]= -1

Out[4]= 1

Out[5]= -1

Out[6]= $\int \left(\int \eta[t] dt \right) dt$

```

In[8]:= f[t_, x_] := A e- $\xi$ 
 $\xi = \alpha t + \beta x + \int \left( \int \text{HeavisideTheta}[t] Un dt \right) dt$ 
 $\gamma 1 = 2$ 
 $\gamma 2 = 2$ 
U = Simplify[Sum[a[i, j]  $\times$  Sum[2 Sin[k  $\pi$ /2]^i f[t, x]^k, {k, 0, Infinity}]^j +
  0 b[i, j]  $\times$  Sum[2 Cos[k  $\pi$ /2]^i f[t, x]^k, {k, 0, Infinity}]^j, {i, 1, 1}, {j,  $\gamma 1$ ,  $\gamma 2$ }] + Un
Clear[f,  $\xi$ ,  $\gamma 1$ ,  $\gamma 2$ ]

```

Out[9]= $t \alpha + x \beta + \int \left(\int \text{HeavisideTheta}[t] \int \left(\int \eta[t] dt \right) dt dt \right) dt$

Out[10]=

2

Out[11]=

2

Out[12]=

$$\frac{4 e^{2(t \alpha + x \beta + \int \left(\int \text{HeavisideTheta}[t] \int \left(\int \eta[t] dt \right) dt dt \right) dt)} A^2 a[I, 2]}{\left(e^{2(t \alpha + x \beta + \int \left(\int \text{HeavisideTheta}[t] \int \left(\int \eta[t] dt \right) dt dt \right) dt)} + A^2 \right)^2} + \int \left(\int \eta[t] dt \right) dt$$

In[14]:= $\eta[t_] := \gamma[I] \text{Cos}[\omega t]$

FU = Expand[TrigToExp[D[U, {t, 2}] - D[U, {x, 2}] - D[U, {x, 4}] + D[$\frac{U^2}{2}$, {x, 2}] - $\eta[t]$]]

Clear[η]

Out[15]=

$$\frac{6 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{96 e} a^2 A^2 a[I, 2] - \frac{4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{96 e} a^2 A^2 a[I, 2]}{\left(e^{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)} + A^2 \right)^4} + \frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{\left(e^{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)} + A^2 \right)^3} +$$

$$\begin{aligned}
 & \frac{48 e^{-i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 \beta^2 a[I, 2] \times \gamma[I]}{+} \frac{48 e^{i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 \beta^2 a[I, 2] \times \gamma[I]} \\
 & \frac{8 e^{-i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 \beta^2 a[I, 2] \times \gamma[I]}{-} \frac{8 e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 \beta^2 a[I, 2] \times \gamma[I]} \\
 & \frac{192 e^6 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^4}} \alpha A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I]}{+} \\
 & \frac{96 e^{-i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^4}} \alpha A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I]}{+} \\
 & \frac{96 e^{i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^4}} \alpha A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I]}{+} \\
 & \frac{192 e^4 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^4}} \alpha A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I]}{-} \\
 & \frac{96 e^{-i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^4}} \alpha A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I]}{-}
 \end{aligned}$$

$$\begin{aligned}
& \frac{8 i e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} A^2 a[I, 2] \text{DiracDelta}[t] \gamma[I] \\
& \frac{96 i e^{-i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I] \\
& \frac{96 i e^{i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I] \\
& \frac{96 i e^{-i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I] \\
& \frac{96 i e^{i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I] \\
& \frac{16 i e^{-i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I] \\
& \frac{16 i e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^3}} \alpha A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I]
\end{aligned}$$

$$\frac{8 e^{-i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I]$$

$$\frac{8 e^{i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I]$$

$$\frac{4 e^{-i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I]$$

$$\frac{4 e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^2}} A^2 a[I, 2] \text{HeavisideTheta}[t] \gamma[I]$$

$$\frac{144 e^6 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^8}} A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma[I]^2$$

$$\frac{96 e^{-i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^8}} A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma[I]^2$$

$$\frac{96 e^{i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^4}\right)+A^2\right) \omega^8}} A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma[I]^2$$

$$\frac{-2 i t \omega + 6 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{24 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 + \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{2 i t \omega + 6 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{24 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 - \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{144 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 + \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{-i t \omega + 4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{96 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 + \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{i t \omega + 4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{96 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 - \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{-2 i t \omega + 4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{24 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 - \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\frac{2 i t \omega + 4 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{24 e} A^2 a[l, 2] \text{DiracDelta}[t]^2 \gamma [l]^2 + \left(\frac{2 \left(t \alpha + x \beta + \frac{\left(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega}) \right) \text{HeavisideTheta}[t] \gamma [1]}{\omega^4} \right)}{e} + A^2 \right) \omega^8$$

$$\begin{aligned}
& \frac{24 e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma [I]^2} - \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^2}{\omega^8} \\
& \frac{16 e^{-i t \omega + 2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma [I]^2} - \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^2}{\omega^8} \\
& \frac{16 e^{i t \omega + 2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma [I]^2} + \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^2}{\omega^8} \\
& \frac{4 e^{-2 i t \omega + 2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma [I]^2} + \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^2}{\omega^8} \\
& \frac{4 e^{2 i t \omega + 2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t]^2 \gamma [I]^2} + \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^2}{\omega^8} \\
& \frac{96 i e^{-i t \omega + 6 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t] \text{HeavisideTheta}[t] \gamma [I]^2} - \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^4}{\omega^7} \\
& \frac{96 i e^{i t \omega + 6 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)}}{A^2 a[I, 2] \text{DiracDelta}[t] \text{HeavisideTheta}[t] \gamma [I]^2} - \\
& \frac{\left(e^{2 \left(t \alpha + X \beta + \frac{(-1 + \frac{1}{2} (e^{-t \omega} + e^{t \omega})) \text{HeavisideTheta}[t] \gamma [I]}{\omega^4} \right)} + A^2 \right)^4}{\omega^7}
\end{aligned}$$

$$\frac{-2 i t \omega+6\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{48 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} +$$

$$\frac{2 i t \omega+6\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{48 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} -$$

$$\frac{-i t \omega+4\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{96 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} +$$

$$\frac{i t \omega+4\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{96 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} +$$

$$\frac{-2 i t \omega+4\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{48 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} -$$

$$\frac{2 i t \omega+4\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{48 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} +$$

$$\frac{-i t \omega+2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}{16 i e} A^2 a[I, 2] \operatorname{DiracDelta}[t] \operatorname{HeavisideTheta}[t] \gamma[I]^2}{\left(e^{2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \operatorname{HeavisideTheta}[t] \gamma[I]}{\omega^d}\right)}+A^2\right) \omega^7} -$$

$$\begin{aligned}
& \frac{16 i e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^7}} A^2 a[I, 2] \text{DiracDelta}[t] \text{HeavisideTheta}[t] \gamma[I]^2 \\
& + \frac{8 i e^{-2 i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^7}} A^2 a[I, 2] \text{DiracDelta}[t] \text{HeavisideTheta}[t] \gamma[I]^2 \\
& + \frac{8 i e^{2 i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^7}} A^2 a[I, 2] \text{DiracDelta}[t] \text{HeavisideTheta}[t] \gamma[I]^2
\end{aligned}$$

$$\begin{aligned}
& - \frac{48 e^{6\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)}}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^6}} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 \\
& - \frac{24 e^{-2 i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^6}} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 \\
& - \frac{24 e^{2 i t \omega+6} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2} \right)}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^6}} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 \\
& + \frac{48 e^{4\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)}}{e^{\left(2\left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^2}\right)+A^2\right) \omega^6}} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2
\end{aligned}$$

$$\frac{-2 i t \omega+4 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{24 e} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 + \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{2 i t \omega+4 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{24 e} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 + \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{8 e} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 - \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{-2 i t \omega+2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{4 e} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 - \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{2 i t \omega+2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{4 e} A^2 a[I, 2] \text{HeavisideTheta}[t]^2 \gamma[I]^2 + \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{4 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{16 e} A^2 a[I, 2] \times \gamma[I] \text{DiracDelta}'[t] - \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\frac{-i t \omega+4 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right)}{8 e} A^2 a[I, 2] \times \gamma[I] \text{DiracDelta}'[t] - \left(e \left(2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[t] \gamma[I]}{\omega^{\beta}}\right) \right) + A^2 \right) \omega^{\beta}$$

$$\begin{aligned}
 & \frac{8 e^{i t \omega+4} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right) A^2 a[1, 2] \times \gamma[1] \text{DiracDelta}'[t]}{\left(e^{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right)}+A^2 \right) \omega^4} \\
 & + \frac{8 e^{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right)} A^2 a[1, 2] \times \gamma[1] \text{DiracDelta}'[t]}{\left(e^{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right)}+A^2 \right) \omega^4} \\
 & + \frac{4 e^{-i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right) A^2 a[1, 2] \times \gamma[1] \text{DiracDelta}'[t]}{\left(e^{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right)}+A^2 \right) \omega^4} \\
 & + \frac{4 e^{i t \omega+2} \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right) A^2 a[1, 2] \times \gamma[1] \text{DiracDelta}'[t]}{\left(e^{2 \left(t \alpha+x \beta+\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4} \right)}+A^2 \right) \omega^4}
 \end{aligned}$$

In[17]:= AE = Union[Flatten[CoefficientList[Expand[FU Denominator[Together[FU]]],

$$\left\{ e^{t \alpha}, e^{x \beta}, e^{\frac{\left(-1+\frac{1}{2}\left(e^{-t \omega}+e^{t \omega}\right)\right) \text{HeavisideTheta}[1] \gamma[1]}{\omega^4}}, e^{i t \omega}, \text{HeavisideTheta}[t], \text{DiracDelta}[t], \text{DiracDelta}'[t] \right\}$$

Length[AE]

Out[17]=

$$\begin{aligned} & \{0, 16 \alpha^2 A^2 \omega^8 a[1, 2] - 16 A^2 \beta^2 \omega^8 a[1, 2] - 64 A^2 \beta^4 \omega^8 a[1, 2], 16 \alpha^2 A^{10} \omega^8 a[1, 2] - 16 A^{10} \beta^2 \omega^8 a[1, 2] - 64 A^{10} \beta^4 \omega^8 a[1, 2], \\ & -32 \alpha^2 A^4 \omega^8 a[1, 2] + 32 A^4 \beta^2 \omega^8 a[1, 2] + 1664 A^4 \beta^4 \omega^8 a[1, 2] + 128 A^4 \beta^2 \omega^8 a[1, 2]^2, \\ & -96 \alpha^2 A^6 \omega^8 a[1, 2] + 96 A^6 \beta^2 \omega^8 a[1, 2] - 4224 A^6 \beta^4 \omega^8 a[1, 2] - 384 A^6 \beta^2 \omega^8 a[1, 2]^2, \\ & -32 \alpha^2 A^8 \omega^8 a[1, 2] + 32 A^8 \beta^2 \omega^8 a[1, 2] + 1664 A^8 \beta^4 \omega^8 a[1, 2] + 128 A^8 \beta^2 \omega^8 a[1, 2]^2, -4 A^2 \omega^4 a[1, 2] \times \gamma[1], 8 A^2 \omega^4 a[1, 2] \times \gamma[1], \\ & -32 \alpha A^2 \omega^4 a[1, 2] \times \gamma[1], -8 A^4 \omega^4 a[1, 2] \times \gamma[1], 16 A^4 \omega^4 a[1, 2] \times \gamma[1], 64 \alpha A^4 \omega^4 a[1, 2] \times \gamma[1], -96 \alpha A^6 \omega^4 a[1, 2] \times \gamma[1], \\ & 192 \alpha A^6 \omega^4 a[1, 2] \times \gamma[1], -16 A^8 \omega^4 a[1, 2] \times \gamma[1], 8 A^8 \omega^4 a[1, 2] \times \gamma[1], 64 \alpha A^8 \omega^4 a[1, 2] \times \gamma[1], -8 A^{10} \omega^4 a[1, 2] \times \gamma[1], \\ & 4 A^{10} \omega^4 a[1, 2] \times \gamma[1], -32 \alpha A^{10} \omega^4 a[1, 2] \times \gamma[1], -96 i \alpha A^6 \omega^5 a[1, 2] \times \gamma[1], 96 i \alpha A^6 \omega^5 a[1, 2] \times \gamma[1], -8 A^2 \beta^2 \omega^6 a[1, 2] \times \gamma[1], \\ & 16 A^4 \beta^2 \omega^6 a[1, 2] \times \gamma[1], 48 A^6 \beta^2 \omega^6 a[1, 2] \times \gamma[1], 16 A^8 \beta^2 \omega^6 a[1, 2] \times \gamma[1], -8 A^{10} \beta^2 \omega^6 a[1, 2] \times \gamma[1], -16 A^2 a[1, 2] \gamma[1]^2, \\ & 4 A^2 a[1, 2] \gamma[1]^2, 24 A^2 a[1, 2] \gamma[1]^2, -48 A^4 a[1, 2] \gamma[1]^2, -8 A^4 a[1, 2] \gamma[1]^2, 32 A^4 a[1, 2] \gamma[1]^2, -144 A^6 a[1, 2] \gamma[1]^2, \\ & -24 A^6 a[1, 2] \gamma[1]^2, 96 A^6 a[1, 2] \gamma[1]^2, -48 A^8 a[1, 2] \gamma[1]^2, -8 A^8 a[1, 2] \gamma[1]^2, 32 A^8 a[1, 2] \gamma[1]^2, -16 A^{10} a[1, 2] \gamma[1]^2, \\ & 4 A^{10} a[1, 2] \gamma[1]^2, 24 A^{10} a[1, 2] \gamma[1]^2, -8 i A^2 \omega a[1, 2] \gamma[1]^2, 8 i A^2 \omega a[1, 2] \gamma[1]^2, -16 i A^4 \omega a[1, 2] \gamma[1]^2, 16 i A^4 \omega a[1, 2] \gamma[1]^2, \\ & -16 i A^4 \omega a[1, 2] \gamma[1]^2, 16 i A^4 \omega a[1, 2] \gamma[1]^2, -32 i A^4 \omega a[1, 2] \gamma[1]^2, 32 i A^4 \omega a[1, 2] \gamma[1]^2, -48 i A^6 \omega a[1, 2] \gamma[1]^2, \\ & 48 i A^6 \omega a[1, 2] \gamma[1]^2, -96 i A^6 \omega a[1, 2] \gamma[1]^2, 96 i A^6 \omega a[1, 2] \gamma[1]^2, -16 i A^8 \omega a[1, 2] \gamma[1]^2, 16 i A^8 \omega a[1, 2] \gamma[1]^2, \\ & -32 i A^8 \omega a[1, 2] \gamma[1]^2, 32 i A^8 \omega a[1, 2] \gamma[1]^2, -8 i A^{10} \omega a[1, 2] \gamma[1]^2, 8 i A^{10} \omega a[1, 2] \gamma[1]^2, -16 i A^{10} \omega a[1, 2] \gamma[1]^2, \\ & 16 i A^{10} \omega a[1, 2] \gamma[1]^2, -4 A^2 \omega^2 a[1, 2] \gamma[1]^2, 8 A^2 \omega^2 a[1, 2] \gamma[1]^2, -16 A^4 \omega^2 a[1, 2] \gamma[1]^2, 8 A^4 \omega^2 a[1, 2] \gamma[1]^2, \\ & -48 A^6 \omega^2 a[1, 2] \gamma[1]^2, 24 A^6 \omega^2 a[1, 2] \gamma[1]^2, -16 A^8 \omega^2 a[1, 2] \gamma[1]^2, 8 A^8 \omega^2 a[1, 2] \gamma[1]^2, -4 A^{10} \omega^2 a[1, 2] \gamma[1]^2, \\ & 8 A^{10} \omega^2 a[1, 2] \gamma[1]^2, 16 \alpha A^2 \omega^4 a[1, 2] \times \gamma[1] - 8 i A^2 \omega^5 a[1, 2] \times \gamma[1], 16 \alpha A^2 \omega^4 a[1, 2] \times \gamma[1] + 8 i A^2 \omega^5 a[1, 2] \times \gamma[1], \\ & -32 \alpha A^4 \omega^4 a[1, 2] \times \gamma[1] - 16 i A^4 \omega^5 a[1, 2] \times \gamma[1], -32 \alpha A^4 \omega^4 a[1, 2] \times \gamma[1] + 16 i A^4 \omega^5 a[1, 2] \times \gamma[1], \\ & -32 \alpha A^8 \omega^4 a[1, 2] \times \gamma[1] - 16 i A^8 \omega^5 a[1, 2] \times \gamma[1], -32 \alpha A^8 \omega^4 a[1, 2] \times \gamma[1] + 16 i A^8 \omega^5 a[1, 2] \times \gamma[1], \\ & 16 \alpha A^{10} \omega^4 a[1, 2] \times \gamma[1] - 8 i A^{10} \omega^5 a[1, 2] \times \gamma[1], 16 \alpha A^{10} \omega^4 a[1, 2] \times \gamma[1] + 8 i A^{10} \omega^5 a[1, 2] \times \gamma[1], \\ & -16 i \alpha A^2 \omega^5 a[1, 2] \times \gamma[1] + 4 A^2 \omega^6 a[1, 2] \times \gamma[1], 16 i \alpha A^2 \omega^5 a[1, 2] \times \gamma[1] + 4 A^2 \omega^6 a[1, 2] \times \gamma[1], \\ & -32 i \alpha A^4 \omega^5 a[1, 2] \times \gamma[1] + 8 A^4 \omega^6 a[1, 2] \times \gamma[1], 32 i \alpha A^4 \omega^5 a[1, 2] \times \gamma[1] + 8 A^4 \omega^6 a[1, 2] \times \gamma[1], \\ & -32 i \alpha A^8 \omega^5 a[1, 2] \times \gamma[1] - 8 A^8 \omega^6 a[1, 2] \times \gamma[1], 32 i \alpha A^8 \omega^5 a[1, 2] \times \gamma[1] - 8 A^8 \omega^6 a[1, 2] \times \gamma[1], \\ & -16 i \alpha A^{10} \omega^5 a[1, 2] \times \gamma[1] - 4 A^{10} \omega^6 a[1, 2] \times \gamma[1], 16 i \alpha A^{10} \omega^5 a[1, 2] \times \gamma[1] - 4 A^{10} \omega^6 a[1, 2] \times \gamma[1] \} \end{aligned}$$

Out[18]=

88

In[19]:= SC = NSolve[Table[AE[I] == 0, {i, 1, Length[AE]}], {a[1, 2], A, alpha, beta, gamma[1], omega}]
Length[SC]

NSolve: Infinite solution set has dimension at least 1. Returning intersection of solutions with

$$\frac{71297 \alpha}{91374} - \frac{19857 A}{30458} - \frac{173881 \beta}{182748} + \frac{20993 \omega}{30458} - \frac{27178 a[1, 2]}{45687} + \frac{11342 \gamma[1]}{15229} == 1.$$

NSolve: Infinite solution set has dimension at least 2. Returning intersection of solutions with

$$\frac{119873 \alpha}{125644} + \frac{139389 A}{125644} - \frac{159721 \beta}{125644} - \frac{27595 \omega}{31411} - \frac{101027 a[1, 2]}{125644} - \frac{139295 \gamma[1]}{125644} == 1.$$

NSolve: Infinite solution set has dimension at least 3. Returning intersection of solutions with

$$\frac{47233 \alpha}{35928} - \frac{55541 A}{53892} + \frac{175945 \beta}{107784} + \frac{112777 \omega}{107784} - \frac{120829 a[1, 2]}{107784} + \frac{151871 \gamma[1]}{107784} == 1.$$

General: Further output of NSolve::infsolns will be suppressed during this calculation. ⓘ

Out[19]=

$$\begin{aligned} & \{a[1, 2] \rightarrow 0., A \rightarrow 4.19144, \alpha \rightarrow 10.5576, \beta \rightarrow -8.82505, \gamma[1] \rightarrow 1.51223, \omega \rightarrow 3.55073\}, \\ & \{a[1, 2] \rightarrow -0.690565, A \rightarrow 0., \alpha \rightarrow 8.77914, \beta \rightarrow -7.41732, \gamma[1] \rightarrow 0.762969, \omega \rightarrow -0.270301\}, \\ & \{a[1, 2] \rightarrow -0.690565, A \rightarrow 0., \alpha \rightarrow 8.77914, \beta \rightarrow -7.41732, \gamma[1] \rightarrow 0.762969, \omega \rightarrow -0.270301\} \end{aligned}$$

Out[20]=

3

```

In[90]:= Co = Simplify[Mean[{SC[1], SC[2], SC[3]}]] /. Rule -> List
Length[Co]
Out[90]=
{{a[1, 2], -0.460377}, {A, 1.39715}, {α, 9.37195}, {β, -7.88656}, {γ[1], 1.01272}, {ω, 1.00338}}

Out[91]=
6

In[92]:= a[1, 2] = Co[1, 2]
A = Co[2, 2]
α = Co[3, 2]
β = Co[4, 2]
γ[1] = Co[5, 2]
ω = Co[6, 2]
η[t_] := γ[1] Cos[ω t]
u = Simplify[U]
f[t_] = Simplify[η[t]]

Fu = D[u, {t, 2}] - D[u, {x, 2}] - D[u, {x, 4}] + D[ $\frac{u^2}{2}$ , {x, 2}] - f[t]
Plot3D[{u, Fu}, {t, -100, 100}, {x, -100, 100}, Mesh -> None]
eFu =  $\frac{1}{200^2}$  NIntegrate[Abs[Fu]^2, {t, -100, 100}, {x, -100, 100}, AccuracyGoal -> 0.1]
Clear[a, b, c, d, η, ζ, τ, A, B, α, β, γ, ω, f]

Out[92]=
-0.460377

Out[93]=
1.39715

Out[94]=
9.37195

Out[95]=
-7.88656

Out[96]=
1.01272

Out[97]=
1.00338

Out[99]=

$$\frac{3.59466 e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]}}{(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]})^2} - 1.00592 \cos[1.00338 t]$$


Out[100]=
1.01272 Cos[1.00338 t]

Out[101]=

$$0. - \frac{1.78357 \times 10^6 e^{37.4878 t - 31.5463 x + 2(-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]}}{(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]})^3} +$$


$$\frac{223.394 e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]}}{(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]})^2} +$$


$$3.59466 e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]} \left( \frac{7.42767 \times 10^6 e^{74.9756 t - 63.0925 x + 4(-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]}}{(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos[1.00338 t]) \text{HeavisideTheta}[t]})^6} - \right.$$

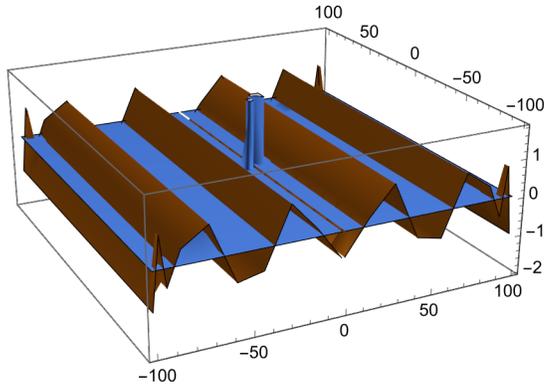

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$$\begin{aligned}
 & \frac{8.9132 \times 10^6 e^{56.2317t-47.3194x+3(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^5} + \\
 & \frac{2.59968 \times 10^6 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^4} - \\
 & \frac{123.794 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3} \Bigg) + \\
 & 5369.52 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \left(\frac{1492.75 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^4} - \right. \\
 & \left. \frac{497.583 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3} \right) - \\
 & 226.796 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \left(\frac{94181.3 e^{56.2317t-47.3194x+3(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^5} - \right. \\
 & \frac{70636 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^4} + \\
 & \left. \frac{7848.44 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3} \right) + \\
 & \frac{1}{2} \left(\frac{113.398 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3} + \right. \\
 & \left. \frac{56.699 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^2} \right)^2 + \\
 & 2 \left(\frac{5365.93 e^{56.2317t-47.3194x+3(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^4} + \right. \\
 & \frac{5365.93 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3} - \\
 & \left. \frac{894.321 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^2} \right) \\
 & \left(\frac{3.59466 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t]}{(1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^2} - 1.00592 \cos[1.00338t] \right) \Bigg) + \\
 & (14.3786 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \\
 & (18.7439 + (-1.99833 + 1.99833 \cos[1.00338t]) \text{DiracDelta}[t] - 2.00507 \text{HeavisideTheta}[t] \sin[1.00338t])^2) / \\
 & (1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^3 - \\
 & (3.59466 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \\
 & (18.7439 + (-1.99833 + 1.99833 \cos[1.00338t]) \text{DiracDelta}[t] - 2.00507 \text{HeavisideTheta}[t] \sin[1.00338t])^2 + \\
 & e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] (-2.01184 \cos[1.00338t] \text{HeavisideTheta}[t] - \\
 & 4.01014 \text{DiracDelta}[t] \sin[1.00338t] + (-1.99833 + 1.99833 \cos[1.00338t]) \text{DiracDelta}'[t])) / \\
 & (1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^2 - 3.59466 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \\
 & ((6 e^{37.4878t-31.5463x+2(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \\
 & (18.7439 + (-1.99833 + 1.99833 \cos[1.00338t]) \text{DiracDelta}[t] - 2.00507 \text{HeavisideTheta}[t] \sin[1.00338t])^2) / \\
 & (1.95202 + e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t])^4 - (2 e^{18.7439t-15.7731x+(-1.99833+1.99833 \cos[1.00338t])} \text{HeavisideTheta}[t] \\
 & (18.7439 + (-1.99833 + 1.99833 \cos[1.00338t]) \text{DiracDelta}[t] - 2.00507 \text{HeavisideTheta}[t] \sin[1.00338t])^2) /
 \end{aligned}$$

$$\frac{\left(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos(1.00338 t)) \text{HeavisideTheta}[t]}\right)^3 - \left(2 e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos(1.00338 t)) \text{HeavisideTheta}[t]} (-2.01184 \cos(1.00338 t) \text{HeavisideTheta}[t] - 4.01014 \text{DiracDelta}[t] \sin(1.00338 t) + (-1.99833 + 1.99833 \cos(1.00338 t)) \text{DiracDelta}'[t])\right)}{\left(1.95202 + e^{18.7439 t - 15.7731 x + (-1.99833 + 1.99833 \cos(1.00338 t)) \text{HeavisideTheta}[t]}\right)^3}$$

- General: Exp[-1188.14] is too small to represent as a normalized machine number; precision may be lost. i
- General: Exp[-891.104] is too small to represent as a normalized machine number; precision may be lost. i
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- General: Further output of General::munfl will be suppressed during this calculation. i

Out[102]=



Out[103]=

$$9.31107 \times 10^{-166}$$