# Path Integral Formulation of the Quantum Brain: Neurons as Quantum Mechanical Oscillator

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**Abstract**

We develop a path integral formulation of brain function by modeling neurons as quantum mechanical oscillators. Each neuron is represented as a mesoscopic quantum system with a firing frequency $\nu$, an amplitude $A$ corresponding to neurotransmitter release, and an associated Planck-like constant $\hbar^B$. This framework extends to coupled oscillators, assemblies, and layered cortical structures with excitatory and inhibitory dynamics. The quantum amplitude of brain states is derived from a Lagrangian formulation and propagated via Feynman path integrals. We explore resonance, inhibition, perceptual collapse, cerebellar timing, and group-level coherence in a unified quantum brain model.

## 1 Introduction

Recent advances in neuroscience and quantum theory motivate a reexamination of how cognitive processes such as perception, memory, and motor coordination might be interpreted within a quantum field framework. We propose a comprehensive model in which each neuron is treated as a quantum harmonic oscillator, with dynamics governed by a mesoscopic Planck-like constant $\hbar^B$. This model is rooted in the path integral formalism of Feynman, enabling us to calculate quantum amplitudes over networks of coupled neuronal oscillators.

In this work, we apply this perspective to construct a complete quantum brain model that includes cortical columns, Purkinje cells, inter-assembly coherence, and group-level cognition. The model incorporates excitatory and inhibitory coupling, oscillator synchrony, and collapse mechanisms. We introduce the concept of the cognitive Planck scale to align temporal resolution with neuronal energy exchange, and develop mathematical formulations for timing, interference, and perceptual Zeno effects.

Our approach bridges the microscopic structure of neurons with large-scale cognitive phenomena using the language of quantum mechanics, offering a framework to understand consciousness, motor learning, and information integration as field-theoretic phenomena.

# 2 Retinotopic Field as Quantum Configuration Space

We model the visual cortex as a 2-dimensional spatial field $\phi(x,t)$, where $x \in \mathbb{R}^2$ represents retinotopic coordinates, and $t$ denotes perceptual time. Each field configuration $\phi(x,t)$ encodes the neuronal excitation state of a cortical microcolumn.

The total quantum amplitude for perceptual evolution in V1 is defined by:

$$\Psi_{\mathrm{V1}} = \int \mathcal{D}[\phi(x,t)]\; e^{\frac{i}{h^B} S[\phi]} \tag{1}$$

where $h^B$ is the Planck-like constant of the brain, and $S[\phi]$ is the action functional defined over V1.

# 3 Cognitive Lagrangian for Visual Processing

We define a cognitive Lagrangian density $\mathcal{L}(\phi, \partial_t \phi, \nabla \phi)$ of the form:

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{v^2}{2}(\nabla \phi)^2 - V(\phi) \tag{2}$$

where $v$ represents the speed of signal propagation within cortical layers, and $V(\phi)$ denotes an effective potential encoding inhibitory and excitatory contributions.

# 4 Decoherence and Collapse in Visual Cortex

Perceptual collapse corresponds to the projection of the amplitude $\Psi_{\mathrm{V1}}$ onto a localized neuronal configuration $\phi^*(x)$:

$$\Psi_{\mathrm{collapsed}} = \delta[\phi(x,t_c) - \phi^*(x)] \cdot \Psi_{\mathrm{V1}} \tag{3}$$

This projection may occur due to environmental interaction or attentional feedback, serving as a quantum-to-classical transition.

# 5 NBQZE: Neuro-Biological Quantum Zeno Effect

Let $\tau_c$ be the collapse time scale and $\tau_f$ the visual frame duration (e.g., ∼25 ms). If perceptual monitoring occurs faster than decoherence, we get:

$$P(t) \approx \left| \prod_{n=1}^{N} \hat{P} e^{-iH\Delta t} \Psi_0 \right|^2, \quad \Delta t = \frac{t}{N},\ \Delta t < \tau_c \tag{4}$$

The repeated projections $\hat{P}$ freeze the perceptual evolution, leading to a neural analogue of the quantum Zeno effect.

# 6 Simulation Outlook

Initial modeling can be done using a 1D strip of V1 neurons with Gaussian connectivity kernel and external input mimicking stimulus onset. The path integral can be numerically approximated using time-slicing or lattice discretization.

# 7 From Schrödinger's Cat to the Visual Cortex: A Path Integral Route via Tangent Perceptual Space

We construct a path integral formulation that links the external quantum superposition of Schrödinger's cat to the internal neural processing within the primary visual cortex (V1), via the intermediary structure of the visual tangent space $T_{\text{Vision}}$, as defined in our earlier axiomatic framework.

## 7.1 Quantum Superposition and External Projection

We begin with the external quantum state:

$$|\Psi_{\text{cat}}\rangle = \frac{1}{\sqrt{2}} \left( |\text{Alive}\rangle + |\text{Dead}\rangle \right) \tag{5}$$

This state resides in an external Hilbert space $\mathcal{H}_{\text{ext}}$, inaccessible to the observer's neural apparatus except via perceptual projection.

### 7.2 Projection onto Visual Tangent Space

Let $\hat{P}_{\text{Vision}}$ denote the projection operator from $\mathcal{H}_{\text{ext}}$ onto the visual tangent space $T_{\text{Vision}} \subset T_p(\mathcal{M})$ of the observer:

$$\phi_{\text{vis}}(x,t) = \hat{P}_{\text{Vision}} |\Psi_{\text{cat}}\rangle \tag{6}$$

Here, $\phi_{\text{vis}}$ is a perceptual field representing visual information encoded in the geometric interface of the observer.

### 7.3 Path Integral over $T_{\text{Vision}}$

We construct the cognitive amplitude in the perceptual manifold via:

$$\Psi_{T_{\text{Vision}}} = \int \mathcal{D}[\phi] \; e^{\frac{i}{\hbar^B} S_{T_{\text{Vision}}}[\phi]} \tag{7}$$

with the action functional:

$$S_{T_{\text{Vision}}}[\phi] = \int dt \int_{T_{\text{Vision}}} d^2x \; \mathcal{L}(\phi, \partial_t \phi, \nabla \phi) \tag{8}$$

### 7.4 Mapping into Visual Cortex (V1)

Let $\hat{\Pi}_{\text{V1}}$ denote the physiological mapping from $T_{\text{Vision}}$ into the neuronal activation field of the primary visual cortex:

$$\phi_{\text{V1}}(x,t) = \hat{\Pi}_{\text{V1}} \circ \hat{P}_{\text{Vision}} |\Psi_{\text{cat}}\rangle \tag{9}$$

This mapping aligns the retinotopic layout of V1 with the observer's perceptual field.

### 7.5 Neural Path Integral in V1

The neural amplitude is then:

$$\Psi_{\text{V1}} = \int \mathcal{D}[\phi_{\text{V1}}] \; e^{\frac{i}{\hbar^B} S_{\text{V1}}[\phi]} \tag{10}$$

where $S_{\text{V1}}$ is a Lagrangian constructed using mesoscopic neural dynamics, signal propagation speed $v$, and local excitation potential $V(\phi)$.

### 7.6 Summary of Transformations

We may summarize the total transformation as:

$$|\Psi_{\text{cat}}\rangle \xrightarrow{\hat{P}_{\text{Vision}}} \phi_{\text{vis}} \xrightarrow{\text{Path Integral}} \Psi_{T_{\text{Vision}}} \xrightarrow{\hat{\Pi}_{\text{V1}}} \phi_{\text{V1}} \xrightarrow{\text{Neural Path Integral}} \Psi_{\text{V1}}(t) \tag{11}$$

This formalism offers a bridge between external quantum events and internal perceptual collapse, interpreted geometrically within the framework of von Neumann's quantum measurement theory and the observer's differential perceptual space.

# 8 Multi-Feature Quantum Perception via Subspaces of the Visual Tangent Space

We extend the path integral formulation to account for additional perceptual features beyond shape and orientation. The visual tangent space $T_{\text{Vision}}$ is decomposed into orthogonal feature-specific subspaces:

$$T_{\text{Vision}} = T_{\text{Color}} \oplus T_{\text{Motion}} \oplus T_{\text{Form}} \oplus \cdots \tag{12}$$

Each subspace encodes a specific perceptual modality corresponding to cortical feature maps observed in V1 and V2.

## 8.1 Color Subspace $T_{\text{Color}}$

A perceptual field in $T_{\text{Color}}$ is modeled as:

$$\phi_{\text{Color}}(x, y, t, \lambda) = A(t) \cdot \delta(x - x_0)\delta(y - y_0) \cdot \chi(\lambda) \tag{13}$$

where $\lambda$ denotes wavelength, and $\chi(\lambda)$ is a spectral function (e.g., red, green, blue components). The delta terms localize the color percept spatially.

## 8.2 Motion Subspace $T_{\text{Motion}}$

Motion is encoded via a directional flow field:

$$\phi_{\text{Motion}}(x, y, t) = A(t) \cdot \delta(x - v_x t - x_0) \cdot \delta(y - v_y t - y_0) \tag{14}$$

with $(v_x, v_y)$ the perceived velocity vector. This formulation models the trajectory of motion features over perceptual time.

## 8.3 Form Subspace $T_{\text{Form}}$

Previously modeled as geometric structures (e.g., vertical or horizontal lines):

$$\phi_{\text{Form}}(x, y, t) = A(t) \cdot \delta(L(x, y)) \tag{15}$$

where $L(x, y) = 0$ defines the geometric constraint (e.g., $x - x_0 = 0$ for vertical, $y - y_0 = 0$ for horizontal).

### 8.4 Composite Quantum Percept

A complete perceptual field is a tensor product over all subspaces:

$$\phi_{\text{vis}}(x, y, t) = \phi_{\text{Color}} \otimes \phi_{\text{Motion}} \otimes \phi_{\text{Form}} \otimes \cdots \tag{16}$$

Each feature contributes to the total perceptual amplitude:

$$\Psi_{T_{\text{Vision}}} = \int \mathcal{D}[\phi]\, e^{\frac{i}{\hbar B} S[\phi]}, \quad S[\phi] = \sum_k S_k[\phi_k] \tag{17}$$

where $S_k$ is the action over feature-specific field $\phi_k$ in subspace $T_k$.

### 8.5 Collapse to Dominant Feature

Upon perceptual measurement or attention, the superposed percept collapses onto a specific subspace state:

$$\phi_{\text{vis}} \to \phi_k^*(x, y, t), \quad \text{with maximal activation in } T_k \tag{18}$$

For example, attention to color causes $\phi_{\text{vis}}$ to project strongly into $T_{\text{Color}}$.

### 8.6 Implications for Visual Consciousness

This decomposition suggests that visual consciousness is a multi-field amplitude distribution over orthogonal perceptual subspaces. Interference, decoherence, and perceptual collapse occur not just over spatial trajectories, but over perceptual features. Future modeling can simulate entangled perceptual states and their dynamics across $T_{\text{Vision}}$.

## 9 Multisensory Path Integral Framework for Interpersonal Interaction

We now generalize the path integral framework to model a real-time interpersonal interaction involving multiple sensory modalities. Consider two individuals engaged in conversation while also making physical contact (e.g., holding hands). This scenario activates:

- The **Visual Tangent Space** $T_{\text{Vision}}^{(i)}$

- The **Auditory Tangent Space** $T^{(i)}_{\text{Audio}}$
- The **Motor Speech Tangent Space** $T^{(i)}_{\text{Speech}}$
- The **Cerebellar Timing Tangent Space** $T^{(i)}_{\text{Cereb}}$
- The **Tactile Tangent Space** $T^{(i)}_{\text{Touch}}$

for each individual $i = A, B$.

## 9.1 Composite Tangent Bundle per Individual

Each individual operates with a total perceptual-motor tangent bundle:

$$T^{(i)}_{\text{Total}} = T^{(i)}_{\text{Vision}} \oplus T^{(i)}_{\text{Audio}} \oplus T^{(i)}_{\text{Speech}} \oplus T^{(i)}_{\text{Cereb}} \oplus T^{(i)}_{\text{Touch}} \tag{19}$$

These are defined over the individual's perceptual manifold $\mathcal{M}^{(i)}$.

## 9.2 Path Integrals over Multisensory Fields

Let $\phi^{(i)}_k(x,t)$ denote the perceptual or motor field in subspace $T^{(i)}_k$. The total action for individual $i$ becomes:

$$S^{(i)} = \sum_k S^{(i)}_k[\phi^{(i)}_k] = \sum_k \int dt \int_{T^{(i)}_k} d^n x \, \mathcal{L}^{(i)}_k(\phi_k, \partial_t \phi_k, \nabla \phi_k) \tag{20}$$

Then the total perceptual-motor amplitude is:

$$\Psi^{(i)} = \int \mathcal{D}[\{\phi^{(i)}_k\}] \; \exp\left( \frac{i}{\hbar^B} S^{(i)}[\{\phi_k\}] \right) \tag{21}$$

## 9.3 Cross-Coupling Between Individuals

Social and sensory interaction induces entanglement between subspaces:

- **Auditory–Speech coupling:** $T^{(A)}_{\text{Speech}} \leftrightarrow T^{(B)}_{\text{Audio}}$
- **Visual–Visual coupling:** $T^{(A)}_{\text{Vision}} \leftrightarrow T^{(B)}_{\text{Vision}}$
- **Touch–Touch coupling:** $T^{(A)}_{\text{Touch}} \leftrightarrow T^{(B)}_{\text{Touch}}$

This results in a cross-action term:

$$S^{(AB)}_{\text{interact}} = \sum_{j,k} \int dt \int dx \, \mathcal{I}_{jk}(\phi^{(A)}_j, \phi^{(B)}_k) \tag{22}$$

Total system amplitude:

$$\Psi_{\text{System}} = \int \mathcal{D}[\phi^{(A)}]\mathcal{D}[\phi^{(B)}]\, e^{\frac{i}{\hbar B}(S^{(A)}+S^{(B)}+S^{(AB)}_{\text{interact}})} \tag{23}$$

### 9.4 Example: Auditory–Speech Interaction

Speech by $A$ activates $T^{(A)}_{\text{Speech}}$, which propagates through space and enters $T^{(B)}_{\text{Audio}}$. The kernel $\mathcal{I}_{\text{Speech,Audio}}$ could involve convolution over acoustic transfer functions.

### 9.5 Example: Tactile Synchronization

Touch fields for $A$ and $B$ are entangled via direct skin-to-skin contact:

$$\mathcal{I}_{\text{Touch,Touch}} = \kappa(t) \cdot \phi^{(A)}_{\text{Touch}}(x,t) \cdot \phi^{(B)}_{\text{Touch}}(x,t) \tag{24}$$

This interaction may drive phase-locked neural synchrony in the somatosensory cortices of $A$ and $B$.

The path integral framework over multisensory tangent subspaces enables a first-principles model of interpersonal dynamics. Real-time shared experience arises from dynamic entanglement across the visual, auditory, speech, and tactile manifolds of interacting brains. Each observer's internal perceptual geometry constructs a synchronized multisensory wavefunction whose collapse manifests as shared perception and communication.

## 10 Multisensory Propagators for Interpersonal Path Integrals

We now construct the explicit propagators for each segment of the multisensory path integral in a two-person interactive system. Each propagator encodes the amplitude for the perceptual-motor field to evolve from an initial to a final configuration within a specific tangent subspace.

### 10.1 General Form

For a field $\phi^{(i)}_k(x,t)$ in subspace $T^{(i)}_k$ (individual $i = A, B$), the path integral propagator is:

$$K^{(i)}_k[\phi_2, t_2; \phi_1, t_1] = \int_{\phi(t_1)=\phi_1}^{\phi(t_2)=\phi_2} \mathcal{D}[\phi^{(i)}_k]\, e^{\frac{i}{\hbar B} S^{(i)}_k[\phi^{(i)}_k]} \tag{25}$$

with action:

$$S_k^{(i)} = \int_{t_1}^{t_2} dt \int d^n x \left[ \frac{1}{2}(\partial_t \phi)^2 - \frac{v_k^2}{2}(\nabla \phi)^2 - V_k(\phi) \right] \tag{26}$$

## 10.2 Visual Cortex (Vision)

Retinotopic percepts as delta functions:

$$K_{\text{Vision}}(x_2, t_2; x_1, t_1) = \left( \frac{1}{2\pi i \hbar^B (t_2 - t_1)} \right)^{n/2} e^{\frac{i}{\hbar^B} \frac{|x_2 - x_1|^2}{2(t_2 - t_1)}} \tag{27}$$

## 10.3 Auditory Cortex (Audio)

Modeled in cochlear-frequency space:

$$K_{\text{Audio}}(x_2, t_2; x_1, t_1) = e^{\frac{i}{\hbar^B} \left( \frac{(x_2 - x_1)^2}{2(t_2 - t_1)} - \omega_0^2 (t_2 - t_1) \right)} \tag{28}$$

## 10.4 Motor Cortex (Speech)

Speech planning as harmonic oscillator:

$$K_{\text{Speech}}(q_2, t_2; q_1, t_1) = \sqrt{\frac{m\omega}{2\pi i \hbar^B \sin(\omega T)}} \, e^{\frac{i m \omega}{2\hbar^B \sin(\omega T)} \left( (q_2^2 + q_1^2) \cos(\omega T) - 2 q_1 q_2 \right)} \tag{29}$$

## 10.5 Cerebellar Timing (Cereb)

Timing modulation as phase evolution:

$$K_{\text{Cereb}}(t_2, t_1) = e^{-i \Delta E_{\text{Cereb}} (t_2 - t_1)/\hbar^B} \tag{30}$$

## 10.6 Somatosensory Cortex (Touch)

Tactile input as skin-localized impulse:

$$K_{\text{Touch}}(x_2, t_2; x_1, t_1) = e^{\frac{i}{\hbar^B} \frac{|x_2 - x_1|^2}{2(t_2 - t_1)}} \tag{31}$$

If both individuals make contact:

$$K_{\text{Touch,AB}}(x, t) = \kappa(t) \, \delta(x^{(A)} - x^{(B)}) \tag{32}$$

### 10.7 System-Level Propagator

Total amplitude for joint perception and interaction:

$$K_{\text{System}} = \prod_k K_k^{(A)} \prod_k K_k^{(B)} \, e^{\frac{i}{\hbar B} S_{\text{interact}}^{(AB)}} \tag{33}$$

This propagator encodes coupled multisensory dynamics across individuals, forming the foundation for quantum field modeling of interpersonal experience.

## 11 Entanglement Between Perceptual Subspaces

In the multisensory path integral framework, entanglement arises when perceptual or motor subspaces cannot be treated independently. This results in non-separable quantum states across different tangent spaces, either within a single brain or between individuals.

### 11.1 Non-Separable States

Let $\Psi_{\text{Total}}$ be the global perceptual wavefunction. If subspaces are entangled:

$$\Psi_{\text{Total}} \neq \Psi_{\text{Vision}} \otimes \Psi_{\text{Audio}} \otimes \Psi_{\text{Speech}} \otimes \cdots \tag{34}$$

Instead, we express the total state as a non-factorizable sum:

$$\Psi_{\text{Total}} = \sum_{i,j} C_{ij} \, \psi_i^{(A)}(x) \otimes \psi_j^{(B)}(x) \tag{35}$$

with entangled coefficients $C_{ij}$.

### 11.2 Entangled Action and Path Integral

Consider subspaces $T_j$ and $T_k$ with fields $\phi_j$ and $\phi_k$. The entangled action includes a coupling term:

$$S_{\text{ent}} = \int dt \, dx \, \mathcal{I}_{jk}(\phi_j, \phi_k) \tag{36}$$

The full path integral becomes:

$$\Psi = \int \mathcal{D}[\phi_j]\mathcal{D}[\phi_k] \, e^{\frac{i}{\hbar B}(S_j + S_k + S_{\text{ent}})} \tag{37}$$

## 11.3 Example: Vision–Audio Entanglement

In audiovisual integration (e.g., lip reading), vision and hearing are coupled:

$$\mathcal{I}_{\text{Vision,Audio}} = \lambda(t)\,\phi_{\text{vis}}(x,t)\,\phi_{\text{aud}}(x,t) \tag{38}$$

This term causes cross-modal enhancement or interference.

## 11.4 Example: Speech–Touch Entanglement

During physical interaction (e.g., handshake with speech):

$$\mathcal{I}_{\text{Speech,Touch}} = \beta(t)\,\partial_t\phi_{\text{Speech}}(x,t)\,\phi_{\text{Touch}}(x,t) \tag{39}$$

This links temporal articulation to tactile feedback.

## 11.5 Joint Propagator

The entangled propagator between two subspaces is:

$$K_{jk}[\phi_j,\phi_k] = \int \mathcal{D}[\phi_j,\phi_k]\,e^{\frac{i}{\hbar^B}(S_j+S_k+S_{jk}^{\text{ent}})} \tag{40}$$

## 11.6 Total Multisensory Entangled System

The full entangled amplitude across $n$ subspaces:

$$\Psi = \int \mathcal{D}[\{\phi_k\}]\,\exp\left[\frac{i}{\hbar^B}\left(\sum_k S_k[\phi_k] + \sum_{j<k} S_{jk}^{\text{ent}}\right)\right] \tag{41}$$

This formulation accommodates cognitive phenomena such as perceptual fusion, sensorimotor synchronization, and inter-brain coupling.

Entanglement between perceptual subspaces adds a rich quantum structure to multisensory modeling. These interactions are critical for real-time integration of diverse sensory inputs and motor outputs, enabling the emergence of coherent conscious experience.

# 12 Dynamic Entanglement Over Time and Between Groups

We now extend the entanglement model to include dynamic temporal entanglement and group-level interactions. This enables modeling of sustained multi-individual shared states, memory entrainment, and emergent group consciousness.

## 12.1 Time-Evolving Entangled States

Let $\phi_j(t)$ and $\phi_k(t)$ be fields in different subspaces or individuals. We allow their interaction term to evolve:

$$S_{jk}^{\text{ent}} = \int_{t_1}^{t_2} dt \int dx \, \lambda_{jk}(t) \, \phi_j(x,t) \, \phi_k(x,t) \tag{42}$$

Here $\lambda_{jk}(t)$ is a coupling strength that may vary with attention, learning, or context.

This allows entanglement to be:

- **Initiated** when sensory/motor alignment occurs.
- **Sustained** via feedback or synchrony.
- **Decayed** when attention shifts or contact is lost.

## 12.2 Time-Ordered Entanglement Propagation

We introduce a time-ordered exponential for the entangled propagator:

$$K_{jk}(t_2, t_1) = \mathcal{T} \exp\left[\frac{i}{\hbar^B} \int_{t_1}^{t_2} dt \, H_{jk}^{\text{ent}}(t)\right] \tag{43}$$

Where:

$$H_{jk}^{\text{ent}}(t) = \int dx \, \lambda_{jk}(t) \, \phi_j(x,t) \, \phi_k(x,t) \tag{44}$$

## 12.3 Group-Level Entangled States

Let $G = \{A, B, C, \dots\}$ be a group of individuals. Define the group perceptual field:

$$\Phi_G(t) = \bigotimes_{i \in G} \phi^{(i)}(t) \tag{45}$$

with group-wide entangled amplitude:

$$\Psi_G(t) = \sum_{\vec{n}} C_{\vec{n}}(t) \bigotimes_i \psi_{n_i}^{(i)}(t) \tag{46}$$

where $\vec{n} = (n_1, n_2, ..., n_N)$ encodes joint cognitive states.

### 12.4 Multi-Group Interaction

Let $G_1$ and $G_2$ be two groups. Inter-group entanglement is defined via:

$$S^{\text{ent}}_{G_1,G_2} = \int dt \sum_{i \in G_1, j \in G_2} \int dx\, \mu_{ij}(t)\, \phi^{(i)}(x,t)\, \phi^{(j)}(x,t) \tag{47}$$

This framework supports modeling of crowd synchrony, team cognition, and cultural transmission.

### 12.5 Memory and Residual Entanglement

After disengagement, residual coupling may persist:

$$C_{ij}(t) = C_{ij}(t_0)\, e^{-\gamma(t-t_0)} \tag{48}$$

This decay governs memory traces and emotional resonance between individuals.

Dynamic and group-level entanglement enrich the multisensory path integral framework, enabling a temporal and collective view of consciousness. Interpersonal and transpersonal quantum cognition become formalizable via time-evolving, field-theoretic amplitudes across tangent perceptual spaces.

## 13 Justification for Using the Brain's Planck-Like Constant $\hbar^B$

In our formulation, we use a brain-specific Planck-like constant $\hbar^B$ in place of the universal Planck constant $\hbar$. This choice is motivated by the mesoscopic nature of neural processes and the necessity of scale-appropriate quantum dynamics in cognitive systems.

### 13.1 Mesoscopic Scaling of Action

While $\hbar = 6.626 \times 10^{-34}\,\text{J} \cdot \text{s}$ governs atomic and subatomic processes, the energy and temporal scales of the brain are several orders of magnitude larger. Neuronal spike energies ($\sim 10^{-10}\,\text{J}$) and gamma-band frequencies ($\sim 40\,\text{Hz}$) yield:

$$\hbar^B = \frac{E}{f} \approx \frac{10^{-10}}{40} = 2.5 \times 10^{-12}\,\text{J} \cdot \text{s} \tag{49}$$

This brings $S/\hbar^B$ into a computable regime ($\sim 1$) for meaningful path integral interference.

### 13.2 Analogy with Effective Constants

Effective constants are standard in condensed matter physics, e.g., effective mass $m^*$, quasi-particle charge $e_{\text{eff}}$, and phonon-induced potentials. Likewise, $\hbar^B$ models the emergent quantum behavior of neuronal assemblies, not elementary particles.

### 13.3 Coherence and the Quantum Zeno Effect

The Neuro-Biological Quantum Zeno Effect (NBQZE) depends critically on the relation:

$$\Delta E \cdot \Delta t \sim \hbar^B \tag{50}$$

Using $\hbar$ would yield unphysically short coherence windows. $\hbar^B$ supports time scales ($\sim$ 10–100 ms) consistent with perceptual and EEG rhythms.

### 13.4 Empirical Derivation from Brain Metabolism

From average brain power $P = 25$ W and an estimated firing rate $\bar{f} = 10^{12}$ Hz across $10^{11}$ neurons, we get:

$$\hbar^B = \frac{P}{\bar{f}} = 2.5 \times 10^{-11}\,\mathrm{J \cdot s} \tag{51}$$

This supports cognitive field quantization in a way that is physiologically grounded.

Replacing $\hbar$ with $\hbar^B$ in our cognitive path integral framework is essential for accurate scaling, interference preservation, and physiological realism. It allows neural quantum amplitudes to evolve meaningfully across perceptual, motor, and inter-brain domains.

## 14 Quantum Statistics and the Brain's Boltzmann-Like Constant $k^B$

To complement the Planck-like constant $\hbar^B$, we introduce the brain's Boltzmann-like constant $k^B$, tailored for quantum statistical descriptions of neural populations. This allows us to extend our path integral model to include Bose-Einstein and Fermi-Dirac statistics appropriate to distinct classes of neurons.

### 14.1 Definition of $k^B$

Classically, the Boltzmann constant $k_B$ relates energy to temperature:

$$k_B = 1.38 \times 10^{-23} \text{ J/K} \tag{52}$$

For the brain, with average neural excitation energy $\langle E \rangle \sim 10^{-10}$ J and operating temperature $T \sim 310$ K, we define:

$$k^B = \frac{\langle E \rangle}{T} \approx \frac{10^{-10}}{310} = 3.2 \times 10^{-13} \text{ J/K} \tag{53}$$

This mesoscopic $k^B$ enables thermal-like probabilistic occupation of neural states in the perceptual manifold.

### 14.2 Statistical Distributions

The average occupation number of a perceptual state with energy $\epsilon = h^B f$ becomes:

- **Bose-Einstein distribution (for excitatory populations)**

$$\langle n \rangle = \frac{1}{\exp\left(\frac{\epsilon-\mu}{k^B T}\right) - 1} \tag{54}$$

- **Fermi-Dirac distribution (for inhibitory populations)**

$$\langle n \rangle = \frac{1}{\exp\left(\frac{\epsilon-\mu}{k^B T}\right) + 1} \tag{55}$$

### 14.3 Neuron Classification

- **Bosonic Neurons:** Excitatory neurons (e.g., pyramidal cells) that exhibit synchronous firing and allow overlapping activation states.
- **Fermionic Neurons:** Inhibitory neurons (e.g., GABAergic interneurons) that show mutual exclusion and regulate sparse coding.

### 14.4 Cognitive Phenomena

- **Gamma Band Synchrony:** Modeled as Bose-Einstein-like condensation of excitatory neurons in visual cortex during attention.
- **Inhibitory Control:** Enforced via Fermi-Dirac suppression of multiple occupations in competing neural assemblies.

### 14.5 Summary

The inclusion of $k^B$ and the classification of neurons into quantum statistical categories enables deeper thermodynamic modeling of perception. This provides a temperature-dependent modulation of state amplitudes, bridging cognitive dynamics with mesoscopic field statistics.

This quantum statistical layer augments our Lagrangian and path integral structure, providing mechanisms for emergent coherence, inhibition, and thermal entropy in brain-based quantum fields.

# 1. Classification of Neurons: Bosonic vs Fermionic

| **Neuron Type** | **Classification** | **Justification** |
|---|---|---|
| Excitatory Neurons (Pyramidal Cells) | Bosonic | Synchronous firing, superposition allowed |
| Inhibitory Neurons (Interneurons) | Fermionic | Mutually exclusive activation, competitive inhib |
| Motor Neurons | Fermionic | Control discrete motor actions; binary spiking |
| Glial-Linked Ensembles | Bosonic | Slow, coherent field-like modulation |
| Hippocampal Place Cells | Bosonic | Encode continuous spatial fields; memory traces |

Table 1: Neuron classification based on quantum statistics

# 2. Summary Table: Classical vs Brain Constants and Statistics

| Concept | Classical | Brain Analog |
|---|---|---|
| Planck constant | $h$ | $h^B \sim 10^{-11}\,\mathrm{J\cdot s}$ |
| Boltzmann constant | $k_B$ | $k^B \sim 10^{-13}\,\mathrm{J/K}$ |
| Quantum statistics | Bosons / Fermions | Excitatory / Inhibitory Neurons |
| Quantum coherence | Bose-Einstein condensation | Gamma synchrony in V1 |
| Pauli exclusion | Fermionic blocking | Inhibitory neuron gating |

Table 2: Summary of quantum constants and analogies in brain modeling

# 15 Path Integrals and the Neurophysics of Dreaming

Dreams provide a natural context for interpreting brain activity as quantum superpositions over cognitive trajectories. Unlike the waking state, where perceptual collapse occurs continuously through attention and sensory input, dreams permit sustained evolution of field amplitudes without external measurement. This section formalizes the path integral structure underlying the dream state.

## 15.1 Dreams as Sum Over Cognitive Trajectories

In dreams, the brain samples internal perceptual field configurations $\phi(t)$ without collapse. The quantum amplitude is given by:

$$\Psi_{\text{dream}} = \int \mathcal{D}[\phi(t)]\, e^{\frac{i}{\hbar^B} S[\phi]} \tag{56}$$

where $S[\phi]$ is an internal cognitive action accumulated over time.

## 15.2 Replay and Time-Slicing

During REM sleep, hippocampal neurons replay time-sliced trajectories from waking experience. This is encoded as:

$$\Psi_{\text{dream}} \sim \sum_{\text{past paths}} e^{\frac{i}{\hbar^B} S[\text{trajectory}]} \tag{57}$$

Superposed memories interfere, forming dream narratives.

### 15.3 Altered Lagrangian Landscape

The internal Lagrangian in dreams is:

$$\mathcal{L}_{\text{dream}} = \frac{1}{2}(\partial_t \phi)^2 - \frac{v^2}{2}(\nabla\phi)^2 - V(\phi) \tag{58}$$

External sensory terms in $\nabla\phi$ are minimized, while $V(\phi)$ becomes a floating potential from memory, emotion, and archetype.

### 15.4 No Collapse Dynamics

Waking attention enforces projection:

$$\phi(t) \xrightarrow{\text{Attention}} \phi^*(t) \tag{59}$$

In dreams, no projection occurs:

$$\Psi_{\text{dream}} \text{ evolves freely} \tag{60}$$

This enables contradictory imagery, nonlinear narratives, and unfiltered transitions.

### 15.5 NBQZE is Inactive in Dreaming

The Neuro-Biological Quantum Zeno Effect (NBQZE) requires perceptual monitoring. During REM sleep:

$$P(t) \sim \left| \int \mathcal{D}[\phi] e^{\frac{i}{\hbar^B} S[\phi]} \right|^2 \tag{61}$$

The absence of observation allows wide trajectory exploration.

### 15.6 Statistical Sampling in Dream Field

We may interpret dream evolution thermally:

$$P[\phi] \sim \frac{1}{Z} \exp\left( -\frac{E[\phi]}{k^B T} \right) \tag{62}$$

Dreams sample memory configurations weighted by $k^B$ and internal cognitive temperature.

| Feature | Waking State | Dream State |
|---|---|---|
| Collapse | Yes (via attention) | No (free evolution) |
| External input | Sensory-driven | Internally generated |
| NBQZE | Active | Inactive |
| Path integral | Collapses to dominant path | Full path sum persists |
| Memory access | Causal, logical | Nonlinear, associative |
| Potential $V(\phi)$ | Environmental | Archetypal / emotional |

Table 3: Comparison of brain dynamics in waking and dreaming states

### Summary: Waking vs Dreaming Dynamics

Dreaming represents the brain's intrinsic quantum path integrator, free from measurement-induced collapse. It explores the perceptual manifold using internal potentials shaped by memory, emotion, and identity. Path integrals provide a natural formalism to unify waking and dreaming cognition under a single dynamic field-theoretic model.

# 16 The Brain as a Network of Quantum Oscillators and Neuronal Path Integrals

We model the brain as a network of approximately $10^{11}$ quantum oscillators, each corresponding to a single neuron. These oscillators are distributed across spatial cortical coordinates and are coupled via synaptic and field-mediated interactions. The evolution of this system is governed by path integrals spanning across neuronal trajectories.

## 16.1 Neuronal Quantum Oscillator Definition

Each neuron $n$ is associated with a state function $\psi_n(t)$ governed by the frequency of its firing rate:

$$\nu_n(t) = \text{Instantaneous firing rate of neuron } n \tag{63}$$

The quantum energy level is given by:

$$E_n = h^B \nu_n(t) \tag{64}$$

## 16.2 Coupled Oscillator Field Structure

Define a cognitive field:

$$\Phi(x_n, t) = \text{Complex neural field amplitude at neuron's location } x_n \tag{65}$$

Interactions between neurons $n$ and $m$ are described via a coupling function $J_{nm}(t)$ representing synaptic strength or field connectivity.

### 16.3 Neuronal Path Integral Amplitude

The full cognitive amplitude is written as a network-level path integral:

$$\Psi[\{\psi_n\}] = \int \prod_n \mathcal{D}[\psi_n(t)]\, e^{\frac{i}{\hbar^B} S[\{\psi_n\}]} \tag{66}$$

The total action functional is:

$$S[\{\psi_n\}] = \int dt \left[ \sum_n \left( \frac{1}{2} |\dot{\psi}_n|^2 - V_n(|\psi_n|) \right) - \sum_{n \neq m} J_{nm}(t)\, \psi_n^* \psi_m \right] \tag{67}$$

### 16.4 Network Lagrangian and Quantum Coherence

Each node contributes to the Lagrangian:

$$\mathcal{L}_n = \frac{1}{2} |\dot{\psi}_n|^2 - V_n(|\psi_n|) + \sum_{m \neq n} J_{nm}(t)\, \mathrm{Re}(\psi_n^* \psi_m) \tag{68}$$

### 16.5 Interpretation

- $\psi_n(t)$ encodes the excitation amplitude of neuron $n$
- $J_{nm}$ encodes excitatory/inhibitory interactions
- $V_n$ may depend on neurotransmitter energy cost or saturation effects

### 16.6 Collective Wavefunction and Collapse

The total brain state evolves as:

$$\Psi(t) = \sum_{\{n\}} c_{\{n\}}(t)\, |n_1, n_2, ..., n_N\rangle \tag{69}$$

where $|n_1, ..., n_N\rangle$ is the excitation level configuration. Measurement or attention causes collapse onto a configuration:

$$\Psi \to |n_1^*, ..., n_N^*\rangle \tag{70}$$

Modeling the brain as a network of quantum oscillators provides a field-theoretic and computational framework to apply path integrals over cognitive state spaces. This approach allows memory, attention, and perception to be represented as quantum dynamical processes across a lattice of entangled neuronal oscillators.

# 17 Coupled Quantum Harmonic Oscillators as a Model of Cortical Neuron Dynamics

We now refine our brain model by treating neurons as a network of coupled quantum harmonic oscillators (QHOs). Each neuron's excitation state is modeled as a quantized oscillator, with coupling terms representing excitatory or inhibitory interactions. These connections are arranged both horizontally (within cortical layers) and vertically (across layers), forming a structured quantum neural mesh.

## 17.1 Single Neuron as a Quantum Harmonic Oscillator

Each neuron $n$ is modeled by the Hamiltonian:

$$H_n = \frac{p_n^2}{2m} + \frac{1}{2}m\omega_n^2 q_n^2 \tag{71}$$

where $q_n$ is a field-like amplitude representing membrane potential or neurotransmitter activity, $\omega_n = 2\pi\nu_n$ is the firing frequency, and $m$ is an effective cognitive mass.

## 17.2 Coupled Hamiltonian for Networked Neurons

When neurons are coupled, the total Hamiltonian becomes:

$$H = \sum_n \left( \frac{p_n^2}{2m} + \frac{1}{2}m\omega_n^2 q_n^2 \right) + \sum_{n \neq m} \kappa_{nm} q_n q_m \tag{72}$$

Here, $\kappa_{nm}$ defines the nature of interaction:

- $\kappa_{nm} > 0$ for **excitatory** coupling
- $\kappa_{nm} < 0$ for **inhibitory** coupling

## 17.3 Cortical Geometry: Horizontal and Vertical Coupling

Let $q_n^{(l)}$ denote the oscillator at position $n$ in cortical layer $l$. Then:

$$H_{\text{total}} = H_0 + \sum_{n,m,l} \kappa_{nm}^{\text{horiz}} q_n^{(l)} q_m^{(l)} + \sum_{n,l \neq l'} \kappa_{ll'}^{\text{vert}} q_n^{(l)} q_n^{(l')} \tag{73}$$

### 17.4 Network Path Integral

The cognitive evolution is described by a path integral over all oscillator trajectories:

$$\Psi = \int \mathcal{D}[q_n(t)] \, \exp\left(\frac{i}{\hbar^B}\int dt\, \mathcal{L}[q_n, \dot{q}_n]\right) \tag{74}$$

where the network Lagrangian is:

$$\mathcal{L} = \sum_n \left(\frac{1}{2}m\dot{q}_n^2 - \frac{1}{2}m\omega_n^2 q_n^2\right) - \sum_{n\neq m} \kappa_{nm} q_n q_m \tag{75}$$

### 17.5 Interpretation

- $q_n(t)$: firing amplitude of neuron $n$
- $\kappa_{nm}$: synaptic coupling (positive for co-firing, negative for inhibition)
- Synchronization emerges from constructive coupling
- Desynchronization arises from inhibitory competition

### 17.6 Collapse and Perception

Measurement or attentional projection collapses the field configuration:

$$\Psi[q] \to \delta[q - q^*(t)] \tag{76}$$

Real-time perception selects dominant trajectories from the quantum ensemble.

This framework models the cortex as a field of coupled quantum harmonic oscillators, capturing the interplay of excitation, inhibition, and coherent oscillatory behavior. It provides the mathematical machinery for simulating wave propagation, resonance, and suppression in neuronal quantum fields.

## 18 Matrix Formulation of Neuronal Coupling in the Quantum Oscillator Brain Model

We now express the coupling dynamics of the quantum oscillator brain model in matrix form. This formulation captures the connectivity and propagation behavior of cortical neurons as a lattice of coupled quantum harmonic oscillators.

## 18.1 State Vector and Coupling Matrix

Let $\mathbf{q}(t) = [q_1(t), q_2(t), \ldots, q_N(t)]^T$ be the column vector of neural oscillator displacements, and let $\mathbf{K}$ be the $N \times N$ symmetric coupling matrix with entries:

$$K_{nm} = \begin{cases} m\omega_n^2 + \sum_{j\neq n} \kappa_{nj}, & \text{if } n = m \\ -\kappa_{nm}, & \text{if } n \neq m \end{cases} \tag{77}$$

## 18.2 Quadratic Hamiltonian in Matrix Form

The total Hamiltonian of the coupled system is:

$$H = \frac{1}{2}\mathbf{p}^T M^{-1}\mathbf{p} + \frac{1}{2}\mathbf{q}^T \mathbf{K}\mathbf{q} \tag{78}$$

Here:

- $\mathbf{p}(t)$ is the conjugate momentum vector.
- $M = m\mathbf{I}$ is the mass matrix (scalar multiple of the identity).
- $\mathbf{K}$ encodes all pairwise coupling information.

## 18.3 Path Integral over Matrix System

The transition amplitude between initial and final configurations is given by:

$$\Psi[\mathbf{q}_i, \mathbf{q}_f; T] = \int \mathcal{D}[\mathbf{q}(t)] \exp\left[\frac{i}{\hbar^B}\int_0^T dt\, \left(\frac{1}{2}\dot{\mathbf{q}}^T M\dot{\mathbf{q}} - \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}\right)\right] \tag{79}$$

## 18.4 Eigenmode Decomposition

Diagonalizing $\mathbf{K}$ yields normal modes:

$$\mathbf{K} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^T, \quad \text{with } \boldsymbol{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_N) \tag{80}$$

where $\lambda_i$ are the eigenvalues (mode frequencies squared), and $\mathbf{U}$ is the orthonormal mode basis. The transformed field $\boldsymbol{\xi} = \mathbf{U}^T\mathbf{q}$ evolves as independent harmonic oscillators:

$$\mathcal{L}_{\text{diag}} = \sum_i \left(\frac{1}{2}m\dot{\xi}_i^2 - \frac{1}{2}m\lambda_i\xi_i^2\right) \tag{81}$$

The matrix formulation allows compact expression of the cortical oscillator lattice. Eigenmode analysis reveals collective excitations, wave propagation, and resonance structures across brain regions, enabling spectral analysis of cognitive dynamics and synchronization patterns.

# 19 Neural Assemblies as Quantum Oscillator Super-Nodes

We extend the quantum oscillator brain model by grouping neurons into assemblies or functional units, each of which acts as a collective quantum oscillator. This enables a coarse-grained view of large-scale brain dynamics across cortical and subcortical regions.

## 19.1 Assemblies as Effective Quantum Oscillators

Let $\mathcal{A}_k$ denote the $k^{\text{th}}$ neural assembly comprising $N_k$ neurons. Define:

- $Q_k(t)$: collective amplitude of the assembly
- $\Omega_k$: effective oscillation frequency
- $M_k = \sum_{i \in \mathcal{A}_k} m_i$: effective mass

Then each assembly evolves as a quantum harmonic oscillator:

$$H_k = \frac{P_k^2}{2M_k} + \frac{1}{2} M_k \Omega_k^2 Q_k^2 \tag{82}$$

## 19.2 Inter-Assembly Coupling

Assemblies are coupled via cross-population interactions:

$$H_{\text{int}} = \sum_{k \neq l} \Lambda_{kl} Q_k Q_l \tag{83}$$

with $\Lambda_{kl}$ representing the effective synaptic or field-level connectivity:

- $\Lambda_{kl} > 0$: excitatory resonance
- $\Lambda_{kl} < 0$: inhibitory suppression

## 19.3 Lagrangian for Oscillator Assemblages

The total Lagrangian becomes:

$$\mathcal{L}_{\text{assemblies}} = \sum_k \left( \frac{1}{2} M_k \dot{Q}_k^2 - \frac{1}{2} M_k \Omega_k^2 Q_k^2 \right) - \sum_{k \neq l} \Lambda_{kl} Q_k Q_l \tag{84}$$

### 19.4 Path Integral Across Assemblies

The full amplitude evolves as:

$$\Psi[Q] = \int \mathcal{D}[Q_k(t)] \exp\left(\frac{i}{\hbar^B}\int dt\, \mathcal{L}_{\text{assemblies}}\right) \tag{85}$$

### 19.5 Applications and Examples

- Cortical column: $Q_{\text{cortex}}$ (10–40 Hz)
- Hippocampal theta loop: $Q_\theta$ (6 Hz)
- Gamma-band ensemble: $Q_\gamma$ (30–80 Hz)
- Basal ganglia module: $Q_{\text{BG}}$ (motor filtering)

### 19.6 Collapse and Dominance

Attention or perceptual convergence collapses the state onto a single assembly trajectory:

$$\Psi[Q] \to Q_k^*(t) \tag{86}$$

where $Q_k^*$ is the maximally coherent or attended trajectory.

Neural assemblies modeled as quantum oscillator super-nodes provide a mesoscale framework for understanding competition, resonance, and selection among brain regions. This enables multi-scale modeling of cognition through collective excitations and path integrals over cognitive subspaces.

## 20 Quantum Oscillator Model of the Cerebellum: Purkinje Cell Integration

Purkinje cells in the cerebellum are among the most structurally and functionally complex neurons, receiving input from approximately $10^4$ parallel fiber connections arising from granule cells. We model the Purkinje cell as a central quantum oscillator receiving weighted excitatory inputs from a dense set of smaller quantum oscillators, corresponding to presynaptic granule cells.

## 20.1 System Architecture

Let $g_i(t)$ be the oscillator amplitude of the $i^{\text{th}}$ granule neuron, and $P(t)$ be the state of the central Purkinje oscillator. The total granule-to-Purkinje input is:

$$Q_{\text{input}}(t) = \sum_{i=1}^{N} \kappa_i \, g_i(t), \quad N \sim 10^4 \tag{87}$$

Each $g_i(t)$ obeys a harmonic oscillator dynamic:

$$H_i = \frac{p_i^2}{2m} + \frac{1}{2} m \omega_i^2 g_i^2 \tag{88}$$

The Purkinje oscillator is modeled as:

$$H_P = \frac{P_P^2}{2M_P} + \frac{1}{2} M_P \Omega_P^2 P^2 + \sum_i \lambda_i P g_i \tag{89}$$

## 20.2 Total Lagrangian and Interaction

The system Lagrangian is:

$$\mathcal{L} = \frac{1}{2} M_P \dot{P}^2 - \frac{1}{2} M_P \Omega_P^2 P^2 + \sum_{i=1}^{N} \left( \frac{1}{2} m \dot{g}_i^2 - \frac{1}{2} m \omega_i^2 g_i^2 \right) - \sum_i \lambda_i P g_i \tag{90}$$

## 20.3 Path Integral Formulation

The total system evolves according to:

$$\Psi_P = \int \mathcal{D}[P(t)] \prod_i \mathcal{D}[g_i(t)] \, \exp\left( \frac{i}{\hbar^B} \int dt \, \mathcal{L} \right) \tag{91}$$

This encodes summation, interference, and collapse of integrated input signals.

## 20.4 Timing Precision and NBQZE

The Neuro-Biological Quantum Zeno Effect (NBQZE) constrains the perceptual resolution:

$$\Delta E \cdot \Delta t \sim \hbar^B \Rightarrow \Delta t \sim \frac{\hbar^B}{\Delta E} \tag{92}$$

Given cerebellar temporal precision in the millisecond range, this implies internal energy changes per input must match the cognitive Planck constant scale.

### 20.5 Functional Implications

- **Climbing Fiber Input:** Acts as a nonlinear perturbation or projective measurement on $P(t)$.
- **Long-Term Depression (LTD):** Modeled as modulation of coupling coefficients $\lambda_i \to \lambda_i(t)$.
- **Temporal Coordination:** Emerges from coherent phase-locking of $g_i(t)$ oscillators.

This framework models the Purkinje cell as a mesoscopic quantum integrator of excitatory signals from thousands of input oscillators. The path integral formalism allows computation of collective amplitude distributions and collapse behavior that underlies cerebellar precision, motor prediction, and learning.

## 21 The Cognitive Planck Constant Scale $\hbar^B$

In quantum physics, the Planck constant $\hbar$ governs the fundamental limit to precision in measurements of action, linking energy and time, or momentum and position. In the brain, however, we propose a rescaled version of the Planck constant—denoted $\hbar^B$—to accommodate the mesoscopic nature of neuronal energy scales and cognitive timescales.

### 21.1 Motivation

The canonical uncertainty relation in quantum mechanics is:

$$\Delta E \cdot \Delta t \sim \hbar \tag{93}$$

However, in neuronal systems:

- Typical energy change per neural event: $\Delta E \sim 10^{-10}$ to $10^{-11}$ J
- Cognitive processing time: $\Delta t \sim 1$ to 100 ms
- Standard Planck constant: $\hbar \sim 10^{-34}$ J $\cdot$ s is too small to be physiologically relevant

Thus, we define a brain-specific constant:

$$\hbar^B = \frac{\Delta E}{\Delta f} \sim 10^{-11} \text{ J} \cdot \text{s} \tag{94}$$

where $\Delta f$ is a representative firing frequency ( 1–100 Hz).

## 21.2 Functional Role in Neural Path Integrals

The Feynman path integral formalism requires:

$$\Psi = \int \mathcal{D}[q(t)]\, e^{\frac{i}{\hbar} S[q]} \tag{95}$$

In brain dynamics, using $\hbar$ makes $S[q]/\hbar \gg 1$, suppressing all non-classical paths. But with $\hbar^B$:

$$S[q]/\hbar^B \sim \mathcal{O}(1) \tag{96}$$

allowing meaningful interference, Zeno effects, and probabilistic superposition.

## 21.3 NBQZE and Cerebellar Precision

The Neuro-Biological Quantum Zeno Effect (NBQZE) becomes biologically plausible under $\hbar^B$. For a cerebellar Purkinje cell with input energy $\Delta E \sim 10^{-11}$ J, the perceptual resolution time is:

$$\Delta t \sim \frac{\hbar^B}{\Delta E} \sim 1\,\text{second} \quad \text{(low energy)} \tag{97}$$

Or:

$$\Delta t \sim 1\text{--}10\,\text{ms} \quad \text{(for faster/synchronous input)} \tag{98}$$

This matches observed precision in cerebellar timing, motor control, and conditioning.

## 21.4 Interpretation

- $\hbar^B$ allows perceptual collapse and interference at cognitive scales
- Defines a new "quantum of action" for brain dynamics
- Supports scaling of the path integral formalism to mesoscopic neuroscience

The cognitive Planck constant $\hbar^B$ provides a physiologically meaningful scale at which quantum dynamics may be preserved in the brain. It enables application of path integrals, quantum collapse, and temporal resolution phenomena in a biologically consistent manner, bridging microscopic theory with cognitive neuroscience.

# 22 Conclusion

n this paper, we have constructed a quantum field-theoretic model of the brain by treating neurons and neuronal assemblies as mesoscopic quantum harmonic oscillators. Using Feynman's path integral formalism and a biologically-scaled Planck constant $\hbar^B$, we developed the concept of neuronal action as propagating amplitude over cortical space and time.

Our results include formalisms for single-neuron dynamics, coupled oscillator networks, and assembly-level quantum interactions. We applied this framework to Purkinje cells and the cerebellum, demonstrating how precise timing and inhibition can emerge naturally from oscillator coupling. The introduction of the cognitive Planck scale allows temporal and energetic precision consistent with observed neural behavior, and offers a mechanism for quantum collapse, phase-locking, and information propagation.

This model unifies diverse cognitive phenomena—from perception and timing to coordination and learning—within a single quantum dynamic framework. Future work will include simulation and validation of these quantum structures using neural data, and exploration of consciousness as a macrostate emergent from oscillator coherence.

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