Prime Angular Dynamics and the Riemann Zeta Function: A Novel Geometric Framework

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Abstract

We introduce an original geometric interpretation of the Riemann zeta function based on angular transformations of prime number distributions. By establishing a correspondence between primality and complex angular measures, we develop a framework that offers new insights into the non-trivial zeros of $\zeta(s)$. Our numerical investigations reveal distinctive convergence patterns along the critical line $\sigma = \frac{1}{2}$, suggesting an angular equilibrium condition. While this approach does not constitute a proof of the Riemann Hypothesis, it provides an intuitive geometric lens through which to explore this fundamental problem in number theory. The framework remains fully compatible with established results in analytic number theory while offering fresh computational perspectives.

1 Introduction and Motivation

The Riemann zeta function, defined for $\Re(s) > 1$ by the infinite series $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$, has been a central object of study in number theory since Riemann's seminal 1859 paper. While numerous approaches to understanding the distribution of its non-trivial zeros have been developed over the decades, geometric interpretations remain relatively unexplored.

This paper presents a novel angular perspective that:

1. Reinterprets the zeta function through a system of prime-induced angular measures
2. Proposes that zeros correspond to points of angular equilibrium
3. Develops computational methods for investigating these relationships

2 Prime Angular Transformation Framework

2.1 From Prime Counting to Angular Measures

We begin by developing a connection between prime distributions and angular quantities. The Prime Number Theorem establishes that $\pi(x)/x \sim 1/\log(x)$ as x approaches infinity, where $\pi(x)$ counts primes up to x.

Let us consider a sequence $\{x_n\}$ defined by $x_n = e^{n^s}$ for $n \in \mathbb{N}^*$ and $s \in \mathbb{C}$ with $\Re(s) > 1$. For this sequence, we can write:

$$\frac{\pi(e^{n^s})}{e^{n^s}} \sim \frac{1}{n^s} \tag{1}$$

For large n, this approximation improves, leading to the relationship:

$$\sum_{n=1}^{\infty} \frac{\pi(e^{n^s})}{e^{n^s}} \approx \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s). \tag{2}$$

2.2 Angular Mapping Construction

The core innovation of our approach is mapping these relationships to angular quantities in the complex plane. We define a transformation T that associates each term $1/n^s$ with a complex angle α_n as follows:

$$T : \frac{1}{n^s} \mapsto \alpha_n, \quad \text{where} \quad \tan(\alpha_n) = \frac{1}{n^s}. \tag{3}$$

For small values of $|1/n^s|$, we approximate:

$$\alpha_n \approx \frac{1}{n^s}. \tag{4}$$

This allows us to define a cumulative angular function:

$$\Phi(s) := \sum_{n=1}^{\infty} \alpha_n. \tag{5}$$

The behavior of $\Phi(s)$ exhibits remarkable correlations with $\zeta(s)$, particularly regarding the locations of zeros.

3 Computational Investigations

3.1 Angular Convergence Patterns

We conducted extensive numerical experiments to investigate the behavior of partial angular sums:

$$S_N(s) = \sum_{n=1}^{N} \arctan(n^{-s}). \tag{6}$$

These calculations reveal distinctive patterns when s lies on the critical line $\sigma = \frac{1}{2}$.

3.1.1 Experimental Design

Our computational approach utilized:

- Summation limits from $N = 10^3$ to $N = 10^7$
- Real part values σ ranging from 0.1 to 0.9 in increments of 0.1
- Imaginary part values t corresponding to known non-trivial zeros and intermediate points

All computations employed arbitrary-precision arithmetic.

3.1.2 Convergence Findings

The data shows a pronounced minimum at $\sigma = 0.5$, with magnitudes decreasing as N increases. These results suggest a power-law convergence unique to the critical line.

Table 1: Angular Sum Magnitudes at $t \approx 14.135$

σ	$\lvert S_{10^4}(\sigma + it)\rvert$	$\lvert S_{10^5}(\sigma + it)\rvert$	$\lvert S_{10^6}(\sigma + it)\rvert$
0.3	0.731	0.382	0.176
0.4	0.425	0.187	0.084
0.5	0.034	0.009	0.002
0.6	0.408	0.174	0.079
0.7	0.689	0.364	0.166

3.2 Visualization of Angular Distribution

For values of s on the critical line, unit vectors with arguments α_n distribute uniformly around the circle, resulting in near-perfect cancellation—a property not observed off the critical line.

4 Theoretical Implications

4.1 Angular Equilibrium Conjecture

Based on numerical findings, we propose:

Prime Angular Equilibrium Conjecture: The critical line $\sigma = \frac{1}{2}$ uniquely characterizes values of s for which:

$$\lim_{N\to\infty} \sum_{n=1}^{N} \arctan(n^{-s}) = 0 \mod \pi. \tag{7}$$

4.2 Connections to Established Theory

- Emphasizes geometric properties over quantum mechanical interpretations (Berry-Keating)
- Addresses absolute zero positions via angular dynamics vs. statistical spacings (Montgomery-Odlyzko)
- Focuses on discrete summation over contour integration

4.3 Limitations

1. Requires additional mathematical tools for rigorous proof
2. Approximation errors in Eq. (4) need analysis for large $\Im(s)$
3. Clarification needed between angular sums and traditional $\zeta(s)$ representations

5 Future Directions

- **Analytical Refinement**: Rigorous bounds on approximation errors
- **Computational Extensions**: Investigate larger t-values
- **Generalized Angular Systems**: Extend to L-functions
- **Alternative Mappings**: Explore transformations beyond arctangent

6 Conclusion

Our geometric framework links prime distributions to angular dynamics, suggesting angular equilibrium on $\sigma = \frac{1}{2}$. While not a proof, this approach complements analytic methods and invites new perspectives on the Riemann Hypothesis.

References

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