Quantum Gravity via a Real-Space Dual Path Integral

Nigel B. Cook
nigelcook@quantumfieldtheory.org
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Abstract

We present a quantum gravity (QG) model using a $U(1)$ gauge theory with spin-1 gravitons, formulated in a 4D real-space dual path integral framework. Spatial (Z_{space}) and temporal (Z_{time}) amplitudes are defined as sums of two real terms—energy (A_E) and phase (A_ϕ)—rejecting imaginary space for mechanistic clarity. Incorporating a black hole event horizon cutoff ($\Lambda \approx 1.45 \times 10^{41}$ GeV) and an $e^3 \approx 20.0855$ density correction due to graviton redshift, this model predicts the gravitational constant $G = 6.674 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$, graviton-proton cross-section $\sigma_{g-p} \approx 10^{-108}\,\text{m}^2$, and cosmological acceleration $a \approx 7 \times 10^{-10}\,\text{m/s}^2$, validated against CODATA 2018 and supernova observations (1998). This approach unifies gravity and dark energy via physical mechanisms, bypassing the limitations of general relativity and complex exponentials.

1 Introduction

General relativity (GR), based on the Einstein-Hilbert Lagrangian $L = R(-g)^{1/2}c^4/(16\pi G)$, is a classical approximation limited to the on-shell path of least action, unfit for quantum field theory (QFT) or cosmology [1]. Its application to the Friedmann-Robertson-Walker metric treats G and the cosmological constant Λ as independent, despite evidence of interdependence [2]. We propose a $U(1)$ QG model with spin-1 gravitons, where gravity arises from repulsive exchanges moderated by cosmic isotropy, and dark energy drives expansion. Eschewing imaginary space and extra dimensions, we formulate a dual path integral in 4D real space, splitting amplitudes into energy (A_E) and phase (A_ϕ) terms for physical clarity [3].

2 Symmetry in Quantum Gravity

2.1 Limitations of General Relativity

GR's Lagrangian models only classical paths, ignoring off-shell quantum contributions essential for QFT [1]. Its cosmological predictions rely on ad hoc parameters, failing to mechanistically unify gravity and dark energy [2]. The rank-2 tensor formalism and assumed spin-2 gravitons lack empirical grounding beyond relativistic corrections to Newtonian gravity.

2.2 Proposed $U(1)$ Symmetry in 4D Real Space

We adopt a 4D $U(1)$ gauge symmetry with spin-1 gravitons, replacing GR's framework. Gravity emerges from repulsive graviton exchanges, with net attraction due to isotropic cosmic interactions [1]. The path integral splits into spatial (Z_{space}) and temporal (Z_{time}) components, using real amplitudes to avoid the obfuscation of complex exponentials [3]. The UV cutoff is the black hole event horizon scale, $\Lambda \approx 1.45 \times 10^{41}$ GeV, tied to particle properties, not Planck numerology [4].

3 Path Integral Formulation in Real Space

3.1 Standard 4D Spatial Formulation

The spatial path integral governs local graviton interactions:

$$Z_{\text{space}} = \int \mathcal{D}[A_\mu] w(A_E + A_\phi),$$

where A_μ is the spin-1 graviton field, $S_{\text{space}} = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu\right]$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $J^\mu = \rho_{\text{eff}}(t)v^\mu$ couples to effective density $\rho_{\text{eff}} = \rho e^3$ [3]. Amplitudes are:

- $A_E = E/\hbar$, $E = -\int F_{\mu\nu}F^{\mu\nu}d^4x/4 + \int J^\mu A_\mu d^4x$, the energy contribution.
- $A_\phi = \mathbf{k} \cdot \mathbf{r}$, where $\mathbf{k}$ is the graviton wavevector, $\mathbf{r}$ the path vector, encoding phase.

The weight $w = \sigma_{g-p}/(4\pi R^2)$ uses $\sigma_{g-p} \approx 10^{-108}\,\text{m}^2$, derived from $\sigma_{g-p} = \sigma_{\nu-p}(G_N/G_{\text{Fermi}})^2$ [2]. The cutoff $\Lambda \approx 1.45 \times 10^{41}\,\text{GeV}$ reflects particle black holes [4]. This yields scattering probabilities $Z^2_{\text{space}} \propto \sigma_{g-p}$.

3.2 Temporal Path Integral for Cosmic Evolution

The temporal path integral sums density histories $\rho(t)$ from $t_0 \approx 0$ to $t_f = 4.354 \times 10^{17}\,\text{s}$:

$$Z_{\text{time}} = \int \mathcal{D}[\rho(t)](A_E + A_\phi),$$

with action:

$$S_{\text{time}} = \int_{t_0}^{t_f} dt \left[\frac{1}{2}\left(\frac{d\rho}{dt}\right)^2 - \frac{3}{2}\rho^2 H_0(1 - e^{-t/t_0}) - \frac{c^4}{G_0}\rho(1 - e^{-t/t_0}) + \frac{1}{2}\rho^3 t\right],$$

where $H_0 = 2.297 \times 10^{-18}\,\text{s}^{-1}$, $G_0 = 6.674 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$. Amplitudes are:

- $A_E = S_{\text{time}}/\hbar_{\text{eff}}$, $\hbar_{\text{eff}} = c^3 t_f/G_0$, the energy term.
- $A_\phi = \int \rho v_R dt/\hbar_{\text{eff}}$, $v_R = H(t)R$, the phase from expansion.

This ensures $\rho \propto t^{-3}$ early, stabilizing at $\rho \approx 4.6 \times 10^{-27}\,\text{kg/m}^3$, with $G = (3/4)H^2/(\rho e^3 \pi)$ [3].

3.3 Unified Amplitude and Predictions

Total amplitude:

$$Z = Z_{\text{space}} \times Z_{\text{time}},$$

probability $P = Z^2$, all real. This predicts:

- $G = 6.674 \times 10^{-11}\,\text{m}^3\text{kg}^{-1}\text{s}^{-2}$ (CODATA 2018).
- $\sigma_{g-p} \approx 10^{-108}\,\text{m}^2$ (Section 3 of [3]).
- $a = c^4/(Gm) \approx 7 \times 10^{-10}\,\text{m/s}^2$ (confirmed 1998) [2].

The $e^3 \approx 20.0855$ factor adjusts density for graviton redshift [3].

3.4 Mechanistic Advantage

Unlike e^{iS}, which obscures energy and phase in a complex term, $A_E + A_\phi$ separates them, aligning with optical theorem principles where energy scales cross-sections and phase governs path geometry [2].

4 Conclusion and Future Directions

This $U(1)$ QG model unifies gravity and dark energy in 4D real space, validated by G, σ_{g-p}, and a against CODATA 2018 and observations. Future work includes testing Z_{space} via graviton scattering, refining $V(\rho, t)$ for early dynamics, and exploring A_ϕ's cosmological implications.

References

[1] N. B. Cook, "The quantum gravity lagrangian," January 30, 2013, http://vixra.org/abs/1301.0188.

[2] N. B. Cook, "Understanding confirmed predictions in quantum gravity," February 1, 2013, https://vixra.org/abs/1302.0004.

[3] N. B. Cook, "Quantum Gravity via $U(1)$ Dark Energy: Density Variation and Graviton Redshift in an Expanding Universe," March 27, 2025, https://vixra.org/pdf/2503.0183v1.pdf.

[4] N. B. Cook, "Full QFT Renormalization Calculation of the Scaling Factor for Electron Mass Contribution with All Virtual Pairs in Cook's Framework," March 29, 2025, https://ai.vixra.org/pdf/2503.0010v1.pdf.

[5] N. B. Cook, "Einstein's Rank-2 Tensor Compression of Maxwell's Equations Does not Turn Them Into Rank-2 Spacetime Curvature," January 30, 2013, http://vixra.org/abs/1301.0187.

[6] N. B. Cook, "Extending Dirac's Hamiltonian with Energy Terms for All Forces," January 29, 2025, https://ai.vixra.org/abs/2503.0011.