Quantum Tunneling as Logistic Regression: A Quantum-Inspired Classifier

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Abstract

We propose a novel quantum-inspired classifier that leverages the mathematical similarity between quantum tunneling and logistic regression. By modeling classification probabilities using a tunneling-derived activation function, we offer a fresh perspective on probabilistic classification. We demonstrate the feasibility of this approach on a toy dataset, laying the groundwork for future exploration into quantum-inspired machine learning architectures.

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1. Introduction

Logistic regression models the probability of binary class membership using a sigmoid function [1]. Interestingly, the quantum tunneling probability of a particle passing through a potential barrier exhibits a strikingly similar S-shaped curve [3]. This observation motivates the development of a machine learning model that mimics tunneling behavior.

This approach may provide new insights into the design of machine learning models and offer potential advantages in situations where decision boundaries are steep, energy-based, or resemble threshold phenomena.

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2. Mathematical Analogy

Logistic Regression Activation:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Quantum Tunneling Probability [3][6]:

$$T(E) \approx \frac{1}{1 + e^{2Ka}} \qquad K = \sqrt{\frac{2m(V - E)}{\hbar^2}}$$

We define a quantum-inspired classifier:

$$Q(x) = \frac{1}{1 + e^{\alpha(\beta - x)}}$$

Where:

- $\alpha \sim 2Ka$ (controls sharpness),
- $\beta \sim$ ~ barrier height (threshold).

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3. Table of Similarities

Concept	**Logistic Regression**	**Quantum Tunneling**
Function Form	Sigmoid	Tunneling probability
Curve Behavior	Smooth transition from 0 to 1	Smooth transition from 0 to 1
Threshold	Decision boundary	Barrier height VVV
Sharpness Control	Sigmoid slope	Effective mass/barrier width
Output Interpretation	Class probability	Transmission probability

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4. Toy Model and Implementation

We use the following function to classify data:

```
def quantum_tunneling(x, alpha=1, beta=0):
    return 1 / (1 + np.exp(alpha * (beta - x)))
```

To train the model, we:

- Use a synthetic binary classification dataset (make_classification).
- Optimize α and β by minimizing **binary cross-entropy loss**:

$$\mathcal{L} = -\frac{1}{N}\sum y_i \log\big(Q(x_i)\big) + (1 - y_i\ ) \log\big(1 - \ Q(x_i)\big)\big)$$

- Apply the **'BFGS' method** from scipy.optimize.minimize.

The model achieves performance comparable to standard logistic regression (accuracy ~92–93%).

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5. Physical Interpretation of Parameters

In quantum mechanics, the tunneling decay factor is [3][6]:

$$2Ka = 2a\sqrt{\frac{2m(V-E)}{\hbar^2}}$$

Mapping this to α, we interpret:

- Higher α: steeper decision boundary (like larger barrier or mass),
- β: decision threshold analogous to potential height V.

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6. Optional Visualization

Plot comparing sigmoid and quantum-inspired activation:

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(-10, 10, 200)
sigmoid = 1 / (1 + np.exp(-x))
quantum = 1 / (1 + np.exp(1.5 * (0 - x)))

plt.plot(x, sigmoid, label='Sigmoid')
plt.plot(x, quantum, label='Quantum-Inspired', linestyle='--')
plt.legend()
plt.title("Activation Function Comparison")
plt.grid(True)
plt.show()
```

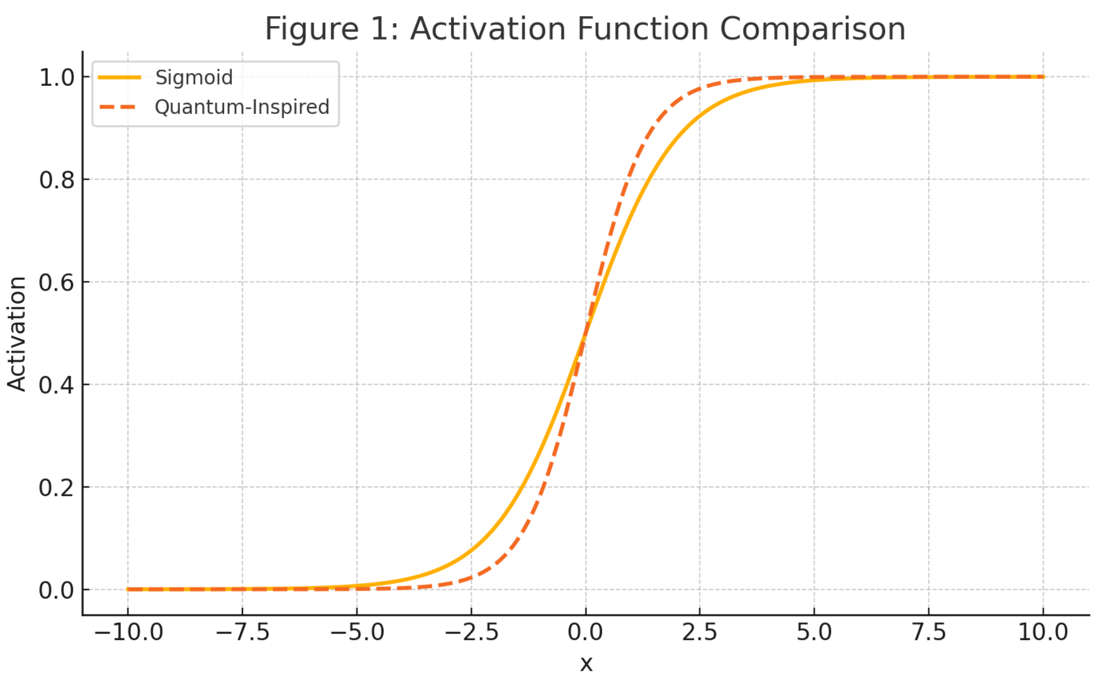

Figure 1, comparing the classic sigmoid function with the quantum-inspired activation based on tunneling probability. The quantum-inspired curve transitions more steeply, controlled by the α parameter — mimicking sharper potential barriers

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7. Conclusion & Future Work

We present a quantum-inspired approach to binary classification by using a tunneling-based activation function. The model is simple, interpretable, and performs comparably to logistic regression on synthetic data.

Future Directions:

- Explore multi-dimensional tunneling analogies.
- Compare **gradient flows** between sigmoid and quantum-inspired functions.
- Test on real-world datasets with non-linear boundaries.
- Extend to deep learning and quantum neural networks.
- Investigate new loss functions or tunneling-based regularization.

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References

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