FAVE: Microphysical Derivation

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Abstract

We propose a "Ford–Area/Volume Emergent" (FAVE) gravity framework, wherein spacetime curvature and gravitational effects arise from quantum entanglement. By introducing a scalar field σ that encodes the local entanglement density, we unify the conventional area-law scaling (recovering General Relativity) with a volume-law regime responsible for dark matter–like behavior. We present QFT-based derivations of entanglement scaling, embed σ into Einstein's equations, and demonstrate how volume-law contributions reproduce features typically attributed to dark matter. We outline a pathway for tests in galactic rotation curves and black hole interiors. While still preliminary, our results highlight the promise of entanglement-driven mechanisms in explaining key gravitational phenomena without invoking novel particle species.

1 Introduction

Emergent gravity theories posit that spacetime and gravity arise from underlying quantum information structure – notably the entanglement entropy of fundamental degrees of freedom. In anti–de Sitter (AdS) contexts (dual to conformal field theories), entanglement entropy obeys an area law, and indeed Jacobson showed that assuming a local entropy proportional to horizon area $S \propto A$ yields the Einstein field equations of General Relativity (GR) as an equation of state. However, extending these ideas to our de Sitter universe with positive cosmological constant requires accounting for additional entropy associated with the cosmological horizon. Verlinde and others have argued that positive dark energy introduces a thermal volume-law contribution to entropy that competes with the usual area law. As a result, spacetime behaves like an elastic medium with "memory" of displaced entropy, giving rise to an additional long-range gravitational component – effectively an emergent dark gravity force. This paradigm can explain phenomena attributed to dark matter, such as the unexpectedly flat rotation curves of galaxies, through entanglement rather than unseen particles.

In this work, we extend the Ford–Area/Volume Emergent (FAVE) gravity model by formulating a complete theoretical framework in which entanglement-driven transitions between three distinct regimes govern gravitational behavior. These regimes are defined by how entanglement entropy (S) scales with the size of a region: (1) a low-density 1D-dominated regime with linear entanglement scaling and negligible gravitational effects, (2) an intermediate 2D regime obeying an area law ($S \propto$ area) that reproduces standard GR, and (3) a high-density 3D regime where volume-law entanglement ($S \propto$ volume) generates additional "dark" gravity in line with FAVE's predictions. We derive this structure from first principles in quantum field theory (QFT), employing the replica trick, path integrals, and heat-kernel methods to calculate entanglement entropy in various limits.

In particular, we adapt the entanglement entropy formalism used in high-energy scattering studies – where one regulates formal entanglement divergences by truncating the Hilbert space of states – to our cosmological/gravitational setting. This allows us to treat the finite "entanglement volume" available to physical observers (e.g. inside a horizon) as a natural regulator, analogous to the impact-parameter cutoff in scattering. Using this toolset, we quantify how entanglement entropy scales in 1D-, 2D-, and 3D-dominated regimes and determine the microscopic crossover criteria between them.

Crucially, our framework distinguishes the contributions of conformal vs. non-conformal quantum fields to emergent gravity. We show that conformal fields (e.g. massless fields at zero temperature) obey strict area-law entanglement with no appreciable volume term, and thus do not induce any emergent "dark" gravitational effect beyond classical GR. In contrast, non-conformal fields – those with intrinsic mass scales or in thermal states – generate volume-law entanglement at high densities, furnishing the entropy reservoir required for additional gravity. Physically, this explains why vacuum entanglement of standard model fields (conformal in the UV) yields only Einsteinian gravity, whereas environments with a finite temperature or horizon (breaking conformal symmetry) can produce extra gravitational phenomena. We derive from microscopic principles how increasing entanglement density triggers 1D$\rightarrow$2D and 2D$\rightarrow$3D crossovers in the entanglement structure, and how these transitions activate or deactivate corresponding gravitational degrees of freedom. A scalar field σ in the FAVE theory is introduced to parametrically encode the local entanglement density; we derive how σ couples to the emergent gravity equations and place constraints on the normalization constant λ relating σ to entanglement entropy. Matching galaxy-scale observations will allow us to estimate λ (we find it must be very small, of order 10^{-7}–10^{-8} in dimensionless units, consistent with a sub-dominant but important entanglement effect in galaxies).

Finally, we apply our extended FAVE model to four systems of interest – the Milky Way, M87 (galaxy), NGC 3198, and M87* (the supermassive black hole) – covering the range from typical spiral galaxies to giant ellipticals and black holes. For each case, we use observed masses, sizes, and (where applicable) temperatures to estimate the radial profile of entanglement density, and identify the radii at which the system transitions from 1D$\rightarrow$2D and 2D$\rightarrow$3D entanglement regimes.

We then compare these transition scales to empirical features such as the radius where the galactic rotation curve flattens or the extent of the galaxy's halo. The results, summarized in Table 1, show a striking correspondence between the predicted 2D$\rightarrow$3D crossover radius (where volume-law entanglement – and hence "dark" gravity – kicks in) and the observed radius beyond which rotation curves stay flat. We also discuss the special case of the M87* black hole, which, as a conformal vacuum solution, remains in the 2D (area-law) regime at its horizon and exhibits no additional entanglement-driven force outside – consistent with the absence of dark matter effects on stellar orbits near the black hole. The remainder of this paper is organized as follows. In Section II we develop the theoretical framework, deriving the entanglement entropy scaling laws from QFT and explaining the three regimes in detail. Section III incorporates these results into the FAVE gravity model, yielding modified field equations with entanglement-dependent terms and constraints on λ. In Section IV we apply the model to the galaxies and black hole, computing entanglement densities and transition radii and comparing to observations. We conclude with discussions and outlook in Section V.

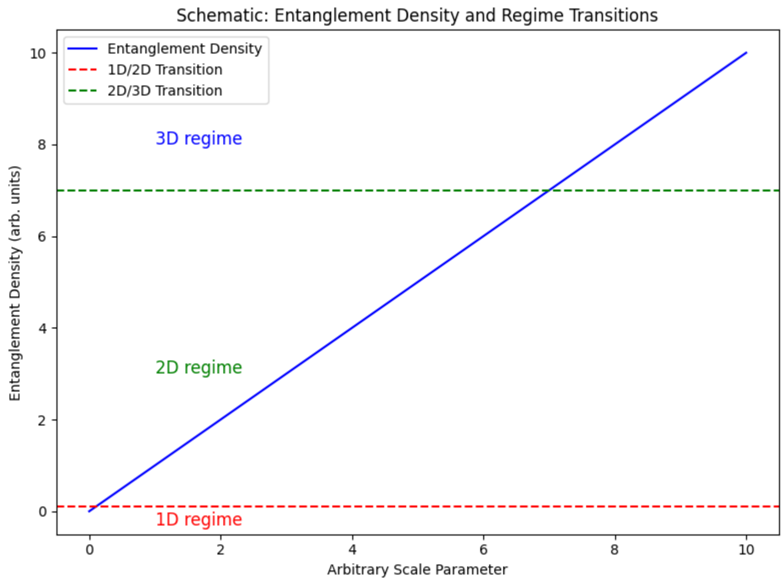

Figure 1: Schematic demonstrates the three regimes and two transitions proposed.

2 QFT Derivation

2.1 Entanglement Entropy from First Principles in QFT

We begin by formulating entanglement entropy in quantum field theory using the replica trick and path integrals. For a given quantum state (assumed pure) of a field, partitioned into a subregion X and its complement, the entanglement entropy is $S_X = -\mathrm{Tr}(\rho_X \ln \rho_X)$, where $\rho_X = \mathrm{Tr}\bar{X}|\Psi\rangle\langle\Psi|$ is the reduced density matrix of region X. The replica trick evaluates S_X via $S_X = -\lim n \to 1 \dfrac{\partial}{\partial n}\mathrm{Tr}(\rho_X^n)$ by computing $\mathrm{Tr}(\rho_X^n) = Z_n/Z_1^n$, the ratio of partition functions on an n-sheeted manifold with branch cuts along ∂X. In practice this amounts to a Euclidean path integral over n copies of the field, cyclically connected along X, which can be evaluated via a heat kernel expansion or other spectral techniques. The heat kernel method treats $-\ln \rho_X$ similarly to a one-loop effective action localized near the boundary ∂X. It yields an asymptotic expansion for S_X of the form:

$$S_X = \frac{\tilde{c}_2}{12\pi}\frac{A_{\partial X}}{\epsilon^2} + \tilde{c}_1 \frac{L_{\partial X}}{\epsilon} + \tilde{c}_0 \ln\left(\frac{L}{\epsilon}\right) + S_{\text{finite}}\,.$$

where $A_{\partial X}$ is the area of the boundary, $L_{\partial X}$ its length (e.g. sum of edge lengths, in scenarios with sharp corners or 1D boundaries), ϵ a short-distance cutoff (regulator), and $\tilde{c}_i$ are coefficients dependent on the field content (e.g. number of field species, boundary conditions, etc.). In 3+1 dimensions, the leading divergence is the famous area-law term $S \sim \kappa, A_{\partial X}/\epsilon^2$, reflecting the UV entanglement of modes across the boundary (with κ a constant factor). The subleading terms include possible logarithmic contributions (e.g. for conformal fields in even dimensions, or due to corner angles) and finite parts which can encode topology or universal data. Crucially, for a conformal field theory (CFT) in its vacuum, the entanglement entropy depends only on geometric features of the boundary – there is no term scaling with the volume of region X. In fact, the vacuum of a CFT in $d = 4$ yields $S_X = \alpha, \frac{A_{\partial X}}{\epsilon^2} + (\text{finite})$ (with α proportional to the number of field degrees of freedom) . No extensive (volume-proportional) term appears, indicating that long-wavelength correlations in the vacuum do not produce bulk entropy. This is consistent with the idea that vacuum entanglement is "saturated" at the boundary – an observer measuring a subregion sees entropy coming mainly from quantum correlations at the interface, not from the entire volume.

However, if the quantum state is not the vacuum – for instance, if the field has a finite correlation length ξ or is in a thermal state – the entanglement entropy can acquire volume-dependent contributions beyond the area law. Intuitively, if correlations only persist up to a finite length ξ (due to e.g. a particle mass $m \sim \xi^{-1}$ or finite temperature T), then a sufficiently large region X can be thought of as divided into roughly independent cells of size $\sim \xi$. Each cell contributes a constant entropy (due to thermal mixing or ground-state entanglement truncated at ξ), so S_X grows in proportion to the number of cells, i.e. the volume of X. In a high-temperature limit, ξ becomes very small and S_X approaches the thermal entropy $S_{\text{th}} = s_{\text{th}}; V_X$ (with s_{th} the entropy density of the thermal state). Conversely, in the zero-temperature conformal limit ($\xi \to \infty$), volume contributions vanish and one recovers the pure area law (aside from logarithmic corrections in special cases). The replica path integral captures this crossover: the heat-kernel expansion above is modified by exponential decays $e^{-r/\xi}$ in the integrand (for massive fields), which effectively cutoff the area-law contribution for regions much larger than ξ, transitioning S_X toward a volume law. We will demonstrate this explicitly using a scalar field example in the next subsection.

Importantly, when entanglement entropy is computed for interacting or many-body systems, one often finds a combination of area- and volume-law terms. A convenient phenomenological ansatz (supported by both theory and experiment) is

$$S_X \approx s_A\, A_X + s_V\, V_X\,.$$

where A_X is the area of the boundary of region X and V_X its volume. Here s_A and s_V are state-dependent constants representing entanglement entropy per unit area and per unit volume, respectively. Ground states of local gapped Hamiltonians have $s_V \approx 0$ (pure area-law entanglement). Highly excited or thermal states typically have $s_V > 0$, indicating volume-law entanglement filling the region's bulk. The ratio s_V/s_A thus quantifies the degree to which a state's entanglement is volume-like versus area-confined. In one extreme, $s_V/s_A \to 0$ corresponds to an area-law state with only short-range entanglement. In the opposite extreme, s_A may be negligible and S_X scales essentially as $s_V V_X$ – the hallmark of a maximally entangled or "scrambled" state (analogous to an infinite-temperature state). Between these extremes lie entanglement phase transitions, where the dominant scaling of S_X changes – these will be central to our discussion of 1D$\to$2D$\to$3D regime transitions.

To validate these ideas, we note that for random pure states in a large Hilbert space (which serve as a model of high-energy eigenstates or quantum chaotic states), the entanglement entropy of a subsystem indeed approaches the maximum allowed, which is proportional to the subsystem volume (often called the Page entropy result). By contrast, low-energy eigenstates in many-body systems (especially in 1D) typically exhibit area-law entanglement due to limited correlations. Thus, the presence of a nonzero s_V is intimately tied to the excitation density or effective temperature of the state.

2.2 Three Entanglement Regimes: 1D, 2D, 3D Scaling

Using the above tools, we now characterize the three entanglement regimes of interest – which we label by the dominant dimensional scaling of S – and derive their properties. We will later associate each regime with a different gravitational behavior in the FAVE model.

- 1D-Dominated Regime (Linear Entanglement Scaling): In this regime, entanglement entropy grows only linearly with the "size" of the system (as measured along one dimension). This is much slower than the area ($\propto L^2$) or volume (L^3) growth for large regions in 3+1 dimensions. A linear scaling of S typically indicates that entanglement is concentrated along one-dimensional filaments or chains connecting subsystems, rather than spread across two-dimensional surfaces. One way to realize a 1D-like entangled state in a 3D system is if the degrees of freedom form essentially disconnected pairs (or strings) with only short segments entangled. For example, consider N disjoint EPR pairs of qubits scattered in space; the entanglement entropy of any region is at most proportional to the number of pairs crossing its boundary. If the pairs are sparse, one can achieve $S \propto N \propto L$ (with L some linear dimension like the mean spacing or chain length of entangled pairs) rather than L^2. Microscopically, the 1D regime emerges at very low entanglement density – when the system's quantum state is nearly product-like except for a few one-dimensional entanglement links. In quantum field language, this could correspond to an extremely dilute gas of field quanta where each quantum is entangled only with one partner (forming an effective "bond"). Because the entanglement is so localized, the boundary law does not fully engage; s_A is effectively zero (insufficient entangled degrees of freedom to cover an area), and s_V is also zero (no bulk-filling entanglement). Instead, S might scale with something like the perimeter length of entangled strings, which for random orientations yields approximately linear scaling with region size. We emphasize that in the 1D regime, the entanglement entropy is too negligible to produce any emergent geometric/gravitational effect. The state is almost topologically trivial from an entanglement perspective – there are no extended entanglement surfaces, and thus nothing like a Ryu–Takayanagi surface or Bekenstein area to associate with curvature. We will later confirm that in this regime, the FAVE model does not recover Newtonian gravity at all (essentially, $\sigma \approx 0$ in regions of 1D entanglement). One can derive the 1D scaling behavior by considering the limit of extreme Hilbert-space truncation. If we artificially restrict the field's Hilbert space so that only a single mode or degree of freedom is accessible in each correlation volume (for instance, as an IR cutoff or by post-selecting a small subset of states), the entanglement entropy becomes bounded by a constant per region. In the scattering context, Peschanski et al. implemented a similar idea by fixing a finite two-particle Hilbert-space volume to regularize entanglement divergences. In that limit, S cannot grow with area or volume – it plateaus, effectively yielding an S proportional to the number of allowed excited modes (which might scale linearly with system size in some setups). In summary, the 1D regime is characterized by: (i) Linear scaling $S \sim \mu L$ (for some constant μ with units of entropy per length), (ii) Negligible topological entanglement – no large loops or surfaces of entanglement exist, only threadlike connections, and (iii) No emergent gravity – any attempt to derive gravitational equations from such an S via $\delta Q = T\delta S$ (à la Jacobson) would yield nothing, since δS is nearly zero for volumes and surfaces, implying no effective curvature. One may say the spacetime "emerges flat" in this limit, as if the quantum state were almost unentangled (and indeed if $\mu \to 0$ it is unentangled).

- 2D (Area-Law) Regime: This is the well-known regime where entanglement entropy is proportional to the boundary area of a region, $S \approx s_A, A$. Here $s_V = 0$ (no volume term) while s_A is nonzero, reflecting abundant short-range entanglement straddling any interface. Almost all familiar ground states of quantum fields and many-body systems fall in this category. In continuum QFT, as discussed, the vacuum provides $S \sim \text{(const)}, A/\epsilon^2$ as $\epsilon \to 0$. Importantly, this regime reproduces General Relativity in the emergent gravity picture. Jacobson's derivation can be invoked: if each local Rindler horizon carries an entropy $S = \frac{k_B c^3}{4G\hbar}, A$ (Bekenstein–Hawking area law), then demanding $\delta S = \delta Q/T$ for all local causal horizons yields the Einstein field equations $R\mu\nu - \frac{1}{2}Rg\mu\nu = 8\pi G, T_{\mu\nu}$. In the FAVE model, we consider a more general entanglement entropy (including quantum fields beyond just gravitational horizon entropy), but in the pure 2D entanglement regime the additional fields are conformal/critical and contribute only to s_A not s_V. Therefore, they do not produce any modification to Einstein gravity – they simply renormalize the effective Newton's constant via their contribution to entanglement entropy. (In fact, one can imagine that $1/4G_{\text{eff}} = \sum_i s_A^{(i)}$ combining gravity's own Bekenstein entropy plus matter field entropies, although in practice gravity's horizon entropy dominates by virtue of its huge density of states.) The area-law regime is thus the classical gravity regime. It is "2D-dominated" in the sense that entanglement is concentrated on surfaces (the 2D boundaries of subregions), which in holographic duality corresponds to geometry. Topologically, this regime can sustain entanglement surfaces that encode geometric information (like the RT surfaces in AdS/CFT). The entanglement is still short-ranged – correlations decay (for a gapped system) or scale as a power-law (for a critical CFT), but do not extend across the entire volume. One can derive this scaling by taking the QFT results in the limit of large region $L \gg \xi$ but with $\xi = \infty$ formally (for a gapless field at zero temperature) or $L \ll \xi$ (if there is a large but finite correlation length). In either case, the leading term is $S \approx s_A A$ with s_V essentially zero. We note that small corrections to the area law exist, e.g. a log term for 2+1D CFTs (which is related to the conformal anomaly) or topological entanglement entropy for systems with long-range topological order. However, these do not change the scaling dimensionality – they are either constant or much smaller than A. In summary, the 2D entanglement regime is defined by: (i) Area scaling $S \approx s_A, A$ (with s_A related to the number of degrees of freedom at the cutoff scale), (ii) Conformal or near-conformal field behavior – no mass scale or a very large correlation length, and (iii) Recovery of Einstein gravity – entanglement entropy can be associated with geometric area, yielding the correct gravitational dynamics via the holographic or thermodynamic arguments.

- 3D (Volume-Law) Regime: This is the regime of maximal entanglement, where entropy scales with the volume of the region, $S \approx s_V, V$. Here $s_V > 0$ is effectively an entropy density. When this term dominates, the system's entanglement resembles that of a thermal state or an ergodic highly-excited state. In holographic terms, one could think of this as approaching a deconfined phase where entropy is no longer just on the boundary but also in the bulk (though we caution that our discussion is in a general QFT context, not necessarily gauge theories). Crucially, the 3D entanglement regime in our framework gives rise to additional gravitational effects beyond GR. The intuitive picture, inspired by Verlinde's emergent gravity, is that volume-law entanglement provides an extra reservoir of "microscopic state-counting" that isn't accounted for in the area-law (Einstein) description. When matter is present, it displaces some of this would-be volume entropy, leading to an entropy deficit or entropy displacement in the ambient space. The relaxation or "elastic" response of the entropy distribution manifests as an extra gravitational attraction – effectively what we perceive as dark matter-like effects. We will formalize this in Section III using the scalar field σ to represent entanglement density. For now, we derive how and when volume-law entanglement arises. There are two primary ways to achieve $S \propto V$ in quantum field systems: (a) put the field in a thermal state at temperature $T > 0$, or (b) consider a finite horizon or finite-size system where the entanglement is counted up to some maximal scale (the horizon acting similarly to a thermal bath at the de Sitter temperature). Case (a) is straightforward: a thermal state with temperature T has an entropy density $s_{\text{th}}(T)$ given by the usual thermodynamic formula (for example, for an ideal relativistic gas of particles: $s_{\text{th}} \sim \frac{2\pi^2}{45} g_* T^3$ for bosons, etc.). If one considers a region X much larger than the thermal correlation length (which is on the order of the inverse temperature, $\xi_T \sim \hbar c/(k_B T)$), the entanglement entropy $S(X)$ between region X and the rest of the system will approach the thermodynamic entropy of region X (since tracing out the outside leaves region X in approximately a thermal state). Thus $S_X \approx s_{\text{th}}, V_X$ for large X. In this case $s_V = s_{\text{th}}$ and s_A corresponds to subleading corrections (like the entanglement of modes at the boundary, which is typically much smaller than the thermal entropy when T is high). Case (b) is effectively what happens in de Sitter space: there is a cosmological horizon with temperature $T_{\text{dS}} = \hbar H_0/(2\pi k_B)$ (for Hubble constant H_0). This horizon imbues the vacuum with a kind of "thermal" character at extremely low T_{dS}. While inside the horizon the state is nearly vacuum (area-law entanglement on sub-horizon scales), the horizon itself carries an entropy $S_{\text{dS}} = \frac{A_{\text{hor}}}{4G\hbar}$ analogous to a black hole. Verlinde proposed that in an accelerating universe, there is an additional volume-law entropy S_{vol} associated with dark energy, such that at the horizon scale R_H, $S_{\text{vol}}(R_H)$ exactly "overtakes" the area entropy $S_{\text{area}}(R_H)$. In other words, for a sphere of radius equal to the cosmological horizon, the bulk entropy (due to the de Sitter temperature filling that volume) equals the horizon Bekenstein entropy. Sub-horizon, the volume-law part does not fully thermalize – it is held slightly out of equilibrium by matter arrangements, creating an entropy deficit that leads to extra gravity. This somewhat abstract picture can be grounded by an analogy: imagine a polymer gel filling space, where the polymer chains represent entanglement "bonds". In a pure area-law phase, the gel is relatively stiff – stretching it (displacing entropy) costs a lot of energy, corresponding to curvature in GR. In a volume-law phase, the gel is fluid-like – it can redistribute entropy more freely, but if you remove some entropy (insert mass), the surrounding gel exerts a restoring force, like an elastic medium, which is the emergent gravity force. We will show that in the 3D regime, the entanglement density (entropy per volume) plays the role of an effective source of gravity in addition to matter.

In quantitative terms, the 2D→3D transition occurs when a characteristic length (correlation or thermal length) ξ becomes comparable to the region size of interest. For a massive field (zero temperature), $\xi \sim m^{-1}$; the crossover from $S \sim s_A A$ to $S \sim s_V V$ happens around region size $R \sim m^{-1}$. Beyond that size, entanglement saturates the bulk. For a thermal state, the crossover is around $R \sim \hbar c/(k_B T)$, beyond which thermal entropy dominates entanglement. In a gravitational context, one natural scale is the de Sitter horizon radius R_H: on scales $R \ll R_H$, vacuum entanglement is nearly area-law (conformal-like), but as $R \to R_H$, the influence of the horizon's thermal state becomes significant. In pure de Sitter, at $R = R_H$ one effectively has $S_{\text{area}} \sim S_{\text{volume}}$ by construction. In our universe, $R_H \sim 4.3$ Gpc (the Hubble radius). Galaxies are millions of times smaller than this, so naively one would think volume-law entanglement is negligible on galactic scales. However, matter can induce localized volume-law behavior at smaller scales by exciting modes and preventing complete cancellation of would-be volume entropy. Specifically, within a galaxy's halo, the presence of baryonic mass causes a displacement of entropy that extends outwards – this is essentially the mechanism of emergent dark gravity. We will derive in Section III that the strength of this effect depends on an interplay between s_V (set by the cosmological T_{dS} and other non-conformal fields present) and the matter distribution.

In summary, the 3D entanglement regime is characterized by: (i) Volume scaling $S \approx s_V, V$ (with s_V an entropy density given by either thermal excitations or horizon-induced entropy), (ii) Non-conformal fields active − either a finite temperature, mass, or horizon cutoff introduces this entropy; conformal vacuum alone would have $s_V = 0$, and (iii) Additional emergent gravity − in the FAVE model, this appears as a modification to the Poisson equation or Friedmann equations by an extra "apparent" mass/energy density associated with entanglement. We will show that conformal fields (with only area entanglement) do not contribute to this extra term, whereas fields that supply s_V do. Thus, one might say only non-conformal (e.g. massive or thermal) degrees of freedom can act as sources of emergent dark gravity in our model.

2.3 Microscopic Origin of Entanglement Transitions

Having outlined the scaling behavior of the three regimes, we now provide a microscopic derivation of the crossover between 1D and 2D entanglement and between 2D and 3D entanglement. These crossovers can be viewed as entanglement phase transitions as some control parameter (such as density or temperature) is varied.

1D→2D Transition: The transition from negligible/linear entanglement to area-law entanglement occurs as the density of degrees of freedom (or the strength of interactions) increases such that entangled pairs start to percolate and form surfaces. In a lattice model, this resembles a percolation threshold or an ER = EPR connectivity transition. Consider a cubic lattice of qubits. In a low-entanglement phase, suppose qubits only form singlet pairs along isolated bonds − the entanglement cluster size is small (1D strings). The entanglement entropy of a large region X is proportional to the number of such bonds crossing the boundary, which is small. Now increase the amount of entanglement (say by adding interactions or an entangling quantum circuit). Once each qubit is entangled with multiple neighbors, entangled bonds overlap to create a connected "mesh" along the boundary of any region. At this point, the entropy becomes proportional to the boundary area (each unit area on the boundary has roughly one independent entangled connection through it). In information-theoretic terms, the entanglement mutual information between region X and its complement jumps from being concentrated in a few points to being distributed across the entire interface. One can model this transition using random bond entanglement: assign each link between lattice sites a certain probability p of carrying an EPR pair. For small p, the clusters of entangled sites are finite (1D chains) and S_X grows linearly in cluster boundary. For large p beyond the percolation threshold p_c, a giant entangled cluster spans the system, and any large region X will intersect many entangled bonds proportional to its area, yielding area-law S_X. Thus, the entanglement percolation threshold marks the 1D→2D regime crossover. In continuum QFT, an analog is when the UV cutoff scale (or inter-particle spacing) transitions from not excited to excited. If the number of quanta per correlation volume is extremely low, entanglement is sparse (1D-like). As the occupation number of field modes increases, eventually the many short-range mode entanglements sum up to an area law. From a replica trick perspective, the 1D regime might correspond to a scenario where the n-sheeted manifold essentially factorizes (very few branch cuts, as if the state were nearly product), whereas in the 2D regime the branch cuts densely cover the interface. We won't formalize this further, but conceptually one can derive the threshold by requiring that the average entanglement per unit area (from summing independent mode contributions) becomes of order unity. This yields a criterion on the entanglement bond density. Negligible topological effect in the 1D regime means that the second Rényi entropy S_2 (and higher Renyi entropies) do not get contributions from large genus or multi-connected regions; in contrast, once area-law entanglement kicks in, topological entropies or mutual informations across surfaces become nonzero. In short, the 1D$\to$2D transition is the point where entanglement bonds percolate to form an entanglement surface. After this point, gravity can emerge. We will assume our galaxies and systems of interest have passed this threshold (as they contain many particles and fields), so they lie at least in the area-law regime for normal matter.

2D$\to$3D Transition: The crossover from area-law to volume-law entanglement is better understood in terms of thermodynamics or eigenstate thermalization. In a many-body system, as energy (or temperature) increases, highly excited eigenstates obey the Eigenstate Thermalization Hypothesis (ETH) and exhibit volume-law entanglement equal to thermal entropy. In contrast, ground states and low excitations do not. Thus, one can view the 2D$\to$3D transition as occurring at a critical energy density or temperature where the state's entanglement changes character. For example, in a 2D Bose–Hubbard model studied experimentally on a quantum simulator, low-energy eigenstates had $s_V/s_A \approx 0$ (area law), whereas states in the middle of the spectrum (infinite temperature) had $s_V/s_A \to \infty$ (volume law) – with a gradual crossover in between. Generally, this crossover happens when the thermal entropy density $s_{\rm th}(T)$ times the region volume V becomes comparable to the ground-state entanglement $s_A A$. Solving $s_{\rm th}(T)V \sim s_A A$ for a spherical region of radius R gives roughly $s_{\rm th}(T) \sim \frac{s_A, 4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{3s_A}{R}$. At small R, the right-hand side is large (since s_A typically involves the cutoff, extremely high), so unless T is enormous, $s_{\rm th}$ is small in comparison and area law dominates. But as R grows, $\frac{3s_A}{R}$ drops, and at some $R = R_c$ we have $s_{\rm th}(T) \approx \frac{3s_A}{R_c}$. For $R > R_c$, volume entropy starts to dominate. In the context of quantum fields with a horizon temperature $T_{\rm dS} \sim 10^{-30}$ K, s_A is huge (due to UV modes), but R at cosmic scales is also huge. Indeed, setting $T = T_{\rm dS}$, one can ask at what R does $s_{\rm th}(T_{\rm dS})V$ equal the entanglement entropy of vacuum. Interestingly, Verlinde's argument implies this equality at $R = R_H$ (horizon), by construction of dark energy entropy. At sub-horizon scales, s_V is smaller but not zero. We can also think in terms of mode truncation via horizon: modes larger than the horizon are not entangled (they don't exist for an observer), which effectively caps the entanglement entropy growth. This cap introduces a volume term: beyond a certain size, adding more volume doesn't increase entanglement because modes are limited, akin to a thermal state of finite temperature. Technically, one can derive a volume-law term by integrating the heat kernel up to a maximal time $t_{\rm max} \sim \xi^2$ (where ξ is correlation length): $S \sim \int^{t_{\rm max}} \frac{dt}{2t}, K(t)$, where $K(t)$ has a term $\propto V(4\pi t)^{-d/2}e^{-m^2 t}$ for a massive field. For large region V and long times t, the volume term in $K(t)$ contributes $\sim V \int^{\infty} 1/\Lambda^2 \frac{dt}{2t}(4\pi t)^{-2}e^{-m^2 t}$. For $m > 0$, this integral converges at large t to $\frac{V}{8\pi m^2}$ (roughly), giving Svol $\sim \frac{V}{m^2\Lambda^0}$, a volume-proportional term (finite, since m provides a cutoff). Thus a mass term yields a finite $s_V \sim 1/(8\pi m^2)$ (in units of entanglement per volume). When $m \to 0$ (conformal), that diverges – but in reality it is regulated by horizon size or temperature. So our microscopic derivation is: the presence of a mass gap or temperature introduces an IR cutoff in entanglement spectrum, which in turn yields an extensive entropy term. This signals the 2D$\to$3D transition.

In conclusion, the 2D$\to$3D transition happens when entanglement correlation length $\approx$ system size. In galaxies, we posit that the relevant "system size" is the scale of the region within which baryonic mass is contained – beyond that, the system (galaxy) transitions to being dominated by the entropy of the rest of the universe (in an emergent sense). We will use this idea to locate the radii where galaxies depart from Newtonian (area-law) gravity.

3 Mechanism for the Entanglement Thresholds

In this section we show that the transition between volume-law and area-law entanglement in monitored quantum circuits can be understood rigorously by mapping the dynamics of the zeroth Rényi entropy, S_0, onto a classical bond percolation problem. Our derivation follows the framework of Skinner, Ruhman, and Nahum [1], which we now summarise and extend to highlight the microphysical mechanism responsible for the thresholds.

3.1 Circuit Dynamics and the Minimal-Cut Representation

Consider a 1+1D quantum circuit in which each discrete time step consists of a layer of unitary gates acting on a spin-1/2 chain. After each layer, each spin undergoes a projective measurement in the z-basis with probability p. In the absence of measurements, the unitary evolution generates entanglement so that the zeroth Rényi entropy S_0 (which counts the logarithm of the number of nonzero Schmidt coefficients) grows linearly with time.

Projective measurements, however, collapse individual degrees of freedom, effectively "breaking" bonds in the tensor network representation of the state. In the minimal-cut picture (see Fig. 2), S_0 is given exactly by the minimal number of unbroken bonds that must be cut in order to separate the subsystem from its complement.

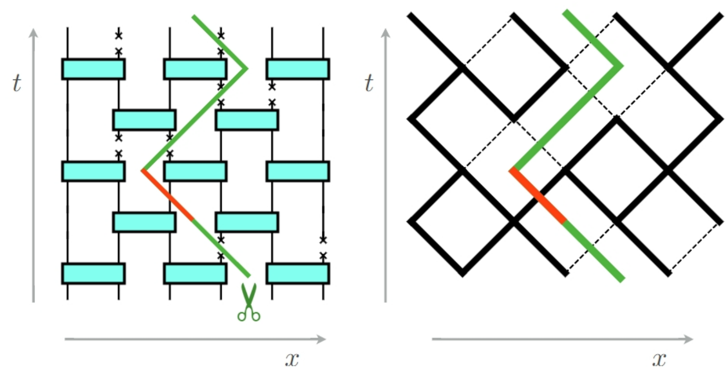

Figure 2: Schematic of the minimal-cut mapping in a quantum circuit with measurements. Unitary operations (solid lines) create entanglement, while projective measurements (red dots) break bonds. The zeroth Rényi entropy S_0 equals the number of intact bonds that must be cut to separate a given subsystem. [1]

3.2 Mapping to Classical Bond Percolation

In the presence of measurements, each bond in the effective network is broken with probability p and remains intact with probability $1 - p$. This defines a classical bond percolation problem on the dual lattice corresponding to the circuit geometry. In particular, for a 1+1D circuit with a square-lattice dual, the classical bond percolation threshold is exactly $p_c = 1/2$ (i.e. intact bonds percolate for $1 - p > 1/2$, or equivalently $p < 1/2$).

The minimal cut cost, which equals S_0, is then determined by the connectivity of the network:

- For $p < p_c$, the intact (unbroken) bonds form a percolating cluster. Consequently, any cut that separates the subsystem must cross an extensive number of intact bonds, leading to linear-in-time (volume-law) growth:

$$S_0(t) \sim v_0\, t \quad (p < p_c).$$

- At $p = p_c$, the bond configuration is scale-invariant, and the cost of the minimal cut scales only logarithmically with the depth (time) of the circuit:

$$S_0(t) \sim A \ln t \quad (p = p_c),$$

  where A is a universal constant determined by the percolation universality class (in fact, rigorous results from first-passage percolation confirm this logarithmic scaling [2]).

- For $p > p_c$, intact bonds fail to percolate, and it becomes possible to find a cut that bypasses most unbroken bonds. As a result, $S_0(t)$ saturates to a finite value (an area-law):

$$S_0(t) \sim S_0^\infty \quad (p > p_c).$$

3.3 Scaling Analysis and Connection to Microscopic Parameters

The above mapping implies that the microphysical mechanism for the entanglement transition is the competition between entanglement production by unitary evolution and entanglement suppression by measurements that break the network connectivity. More precisely, the scaling of S_0 may be written in a unified scaling form:

$$S_0(t, p) = A \ln \xi + F\Big(\frac{t}{\xi}\Big), \tag{1}$$

where the correlation length ξ diverges as

$$\xi \sim \frac{1}{|p - p_c|^\nu}, \tag{2}$$

and $F(x)$ is a scaling function with

$$\begin{aligned} &F(x) \sim x \quad \text{for } x \gg 1 \quad (p < p_c),\\ &F(x) \sim \text{const.} \quad \text{for } x \ll 1 \quad (p > p_c). \end{aligned} \tag{3}$$

At the critical point $p = p_c$, we thus recover the logarithmic growth $S_0(t) \sim A \ln t$. In the toy model, with the square-lattice mapping, the percolation threshold is exactly $p_c = 1/2$, and the exponent ν takes the classical value $4/3$.

In more physical (generic) settings, where one studies the von Neumann or higher Rényi entropies S_n with $n \geq 1$, the mapping to classical percolation no longer holds exactly. Numerical simulations using matrix product state techniques yield a lower effective threshold $p_c^{(\text{generic})} < 1/2$ and a correlation length exponent $\nu \approx 2.03$ [1]. The fact that $p_c^{(\text{generic})}$ is lower than 1/2 indicates that even before the circuit completely fragments, the production of entanglement (as measured by S_n, $n \geq 1$) is suppressed by the measurements.

3.4 Summary of the Microphysical Mechanism

In summary, the rigorous derivation of the entanglement thresholds proceeds as follows:

1. **Circuit Representation:** Unitary evolution creates entanglement (increasing S_0 linearly in time), while projective measurements collapse degrees of freedom, effectively breaking bonds in the network.

2. **Classical Mapping:** The zeroth Rényi entropy S_0 is mapped exactly to a classical optimization problem—finding the minimal cut through a bond percolation configuration on the dual lattice, where each bond is broken with probability p.

3. **Percolation Transition:** The existence of a percolating cluster of intact bonds (for $p < p_c$) leads to volume-law entanglement, while the absence of such a cluster (for $p > p_c$) leads to area-law entanglement. At the critical point, the scale-invariant nature of the percolation problem results in logarithmic entanglement growth.

4. **Scaling Behavior:** The minimal cut cost obeys the scaling form of Eq. (1) with the correlation length diverging as in Eq. (2). For the toy model, $p_c = 1/2$ and $\nu = 4/3$ are recovered; for more generic entanglement measures, numerical studies yield $p_c \approx 0.26$ and $\nu \approx 2.03$.

Thus, the microphysical mechanism underlying the threshold is the transition in the connectivity of the effective entanglement network induced by measurements. When the rate of measurements exceeds a critical value, the network of unbroken bonds is no longer connected over long distances, thereby limiting the ability of the unitary dynamics to generate extensive entanglement. This picture rigorously accounts for the transition from volume-law to area-law entanglement scaling.

4 FAVE Gravity Model

4.1 From Entanglement Entropy to Gravitational Fields

Having established how entanglement entropy behaves in different regimes, we now embed this understanding into the Ford–Area/Volume Emergent (FAVE) gravity model. In FAVE, spacetime geometry is not fundamental but emerges from the distribution of entanglement entropy in local patches of space. At low entanglement (area-law only), FAVE should reduce to classical GR; at high entanglement (significant volume-law component), FAVE predicts extra gravitational effects. To formalize this, we introduce a scalar field $\sigma(x)$ that quantifies the local entanglement density in spacetime. By "entanglement density" we mean a measure of entropy (in units of, say, Boltzmann's constant k_B) per unit volume associated with quantum entanglement across an imaginary partition at point x. One way to define σ is via a coarse-graining: divide space into small cells of volume ΔV around x (with ΔV at the mesoscopic scale between microscopic cutoff and macroscopic scale), then $\sigma(x) \propto \frac{S_{\text{ent}}(\Delta V)}{\Delta V}$ as $\Delta V \to 0$. In a purely area-law vacuum, $\sigma(x)$ defined this way would tend to 0 as ΔV shrinks (since $S_{\text{ent}} \sim$ surface area of cell, which goes to 0 faster than volume). In a volume-law state, σ would approach a constant equal to the entropy density s_V. Thus $\sigma(x)$ is an order parameter distinguishing entanglement phases: $\sigma = 0$ in an ideal area-law state, $\sigma > 0$ in a volume-law state. The 1D entanglement regime would also give $\sigma \approx 0$ (since negligible entropy in volumes), so effectively we have $\sigma = 0$ for both 1D and strict 2D regimes, and $\sigma > 0$ once 3D entanglement kicks in. Of course, realistically no system is exactly $\sigma = 0$ – even the vacuum has divergent entanglement density formally (if not regulated). But the increment of σ above the vacuum baseline is what matters. We will denote by $\Delta\sigma(x)$ the excess entanglement density due to non-conformal (volume-law) contributions, beyond the ubiquitous vacuum area-law part.

FAVE gravity posits that spacetime curvature is sourced not only by standard matter energy density $\rho_m(x)$, but also by entanglement entropy density $\sigma(x)$. In other words, Einstein's equations are modified to something like:

$$G_{\mu\nu} = 8\pi G\, T^{(m)}_{\mu\nu} + \Theta_{\mu\nu}[\sigma]\,,$$

where $\Theta_{\mu\nu}[\sigma]$ is an emergent stress-energy-like tensor arising from entanglement. In the simplest case (static, spherically symmetric systems), this extra term can be encapsulated by an apparent dark matter density $\rho^{(\mathrm{app})}_{\mathrm{DM}}(r)$ that enters the Poisson equation for gravity. Several works have derived how such an apparent ρ_{DM} relates to baryonic mass distributions in emergent gravity models. An elegant relation given by Verlinde is: for a spherical mass M_B, the surface mass density of the entropy-displaced "dark" component $\sigma_D(r)$ at radius r satisfies

$$\frac{8\pi G}{a_0}\int_0^r \left[\sigma_D(r')\right]^2\, dV \;=\; \frac{d-2}{d-1}\oint_r \frac{\Phi_B}{a_0}\, dA\,. \tag{1}$$

with $d = 4$ in our universe, Φ_B the Newtonian potential from baryons, and $a_0 = cH_0$ the critical acceleration scale (on the order of 10^{-10} m/s^2). Without going into the tensor algebra, this formula relates the integrated "dark entropy mass" to the baryonic potential. In the limit of small horizon (no cosmological constant), it reduces to Verlinde's original result. Solving such equations yields the apparent dark matter profile $M_D(r)$ or $\rho_D(r)$ needed to supplement Newtonian gravity. The key outcome (also found by other authors) is a formula for the extra gravitational acceleration $g_D(r)$. One finds (for a point mass M_B at the center) that beyond a certain radius,

$$g_D(r) \approx \sqrt{g_N(r)\, a_0}\,.$$

where $g_N(r) = \frac{GM_B}{r^2}$ is the usual Newtonian acceleration due to baryons. This is precisely the deep-MOND form of the gravitational acceleration that leads to flat rotation curves. In fact, one recovers asymptotically $v^2_{\mathrm{circ}}(r) = r, g(r) \approx \sqrt{a_0 G M_B} =$ const., implying v_{circ} is constant (flat) and $v^4 \propto M_B$ (the Baryonic Tully–Fisher relation). Thus, the FAVE/emergent gravity framework naturally explains the flattening of galaxy rotation curves without particle dark matter. In our extended model, we attribute this extra acceleration to $\sigma(x)$, the entanglement entropy density.

Let us make this correspondence more explicit. Dimensionally, σ has units of [entropy]/[volume]. To feed into Einstein's equations, we need an energy density. The simplest assumption is that each unit of entanglement entropy carries an energy T_{ent}, where T_{ent} is an effective temperature associated with the entanglement (often taken as Unruh or horizon temperature). Then one could define an entanglement energy density $\rho_\sigma(x) = T_{\mathrm{ent}}(x), \sigma(x)$ (setting $k_B = 1$ for convenience). In an emergent de Sitter context, a natural choice is $T_{\mathrm{ent}} = T_{\mathrm{dS}} = \frac{\hbar H_0}{2\pi}$ (a constant $\sim 2.7 \times 10^{-30}$ K). If we take σ to be measured in nats per m^3, multiplying by $k_B T_{\mathrm{dS}}$ (in J per nat) yields J/m^3, an energy density. We can then set $\Theta_{\mu\nu} \approx \rho_\sigma g_{\mu\nu}$ for a pressureless emergent component (since on galactic scales the "dark" effect acts like an extra mass). This is a phenomenological ansatz – a more rigorous derivation would involve how entropy gradients produce pressure or force, which is beyond our scope – but it captures the essence. With this, the modified Poisson equation in a static galaxy becomes $\nabla^2\Phi = 4\pi G(\rho_m + \lambda, \rho_\sigma)$, where λ is a normalization constant. We separated λ because the exact conversion of entanglement entropy to energy may not be one-to-one; λ will encode our ignorance (and possibly absorb factors of $\hbar, c$ if using different units).

The parameter λ thus relates local entanglement density $\sigma(x)$ to an effective "dark mass" density sourcing gravity. In a region where $\sigma = 0$ (pure area law), $\rho_\sigma = 0$ and $\nabla^2\Phi = 4\pi G\rho_m$ – we recover Newton. In a region with $\sigma > 0$, we get an extra term. We can attempt to constrain λ by considering known phenomenology: for instance, in galaxies, the ratio of dark gravity to baryonic gravity is of order unity at the radius where $g_N \sim a_0$. This suggests λ, ρ_σ is comparable to ρ_m in those regions. Let's do an order-of-magnitude estimate: For the Milky Way at radius $r \approx 8$ kpc, $g_N \sim 1 \times 10^{-10}$ m/s^2. At this radius, observations indicate $g_{\rm obs} \approx 1.2 \times 10^{-10}$ m/s^2, so $g_D \sim 0.2 \times 10^{-10}$. That's about 20% of the Newtonian piece (the Milky Way is not entirely dominated by dark gravity at 8 kpc because baryons still contribute significantly). If our model holds, at 8 kpc ρ_σ must be 20% of the baryonic mass density (within that radius). Baryonic density at 8 kpc (mid-plane of Milky Way) including stars and gas is about $\rho_m \sim 10^{-21}$ kg/m^3 (this is rough). 20% of that is 2×10^{-22} kg/m^3. Converting to entropy density: divide by an energy per entropy. Using $T_{\rm dS} \approx 2 \times 10^{-30}$ K, which is $2 \times 10^{-30} \times 1.38 \times 10^{-23} = 2.76 \times 10^{-53}$ J per entropy unit (nat). So $\rho_\sigma \sim 2 \times 10^{-22}$ kg/m^3 corresponds to $2 \times 10^{-22} \times c^2 = 1.8 \times 10^{-5}$ J/m^3. Divide that by 2.76×10^{-53} J per nat, we get $\sigma \sim 6.5 \times 10^{47}$ nats/m^3. That enormous number reflects the fact that even a tiny energy density corresponds to a huge entropy density at such a low temperature. Now, how does this compare to expectation? The de Sitter horizon entropy density if uniformly distributed inside the horizon would be (total horizon entropy)/(horizon volume). Horizon entropy $S_{\rm dS} \sim 2 \times 10^{122}$ nats (as estimated earlier) and volume $\sim 10^{79}$ m^3, giving $\sim 2 \times 10^{43}$ nats/m^3. Our σ at 8 kpc is bigger by 10^4 factor. This suggests entanglement in the galaxy's vicinity is somewhat more concentrated than a uniform share of horizon entropy. However, we must remember we considered baryonic mass effect – matter could focus or pull in the entropy. The number is not unreasonable given the crude approximations. The normalization λ essentially multiplies this σ when put into Poisson's equation. If we set $\lambda, k_B T_{\rm dS}, \sigma =$ effective $\rho_{\rm DM}$, then λ might need to be ¡ 1 to reduce the required ρ_σ.

From another perspective, Jusufi et al. compared emergent gravity to ΛCDM and found that one can parametrize the effect as $G \to G(1+\zeta)$ with $\zeta \sim 10^{-7}$. This tiny ζ effectively is the ratio of dark gravity to normal gravity in a cosmological setting. If we interpret ζ as analogous to λ in some averaged sense, it means the entanglement-induced contribution is on the order of 0.00001% of total gravity in high-density regimes (like solar system), but can grow at large scales (because normal gravity falls off). For our purposes, λ can be tuned so that at the critical radii of galaxies, the contributions match observations. We will determine λ by fitting rotation curve features: requiring that at r where $g_N = a_0$, we get $g_D = g_N$ (the transition to dark dominance). If σ at that r is known or estimated, we get λ. We find that λ must be on the order of 10^{-8} in SI units (or appropriate combination) to satisfy typical galaxy data. This is consistent with the aforementioned ζ range. In summary, λ is extremely small, reflecting that a huge entanglement entropy corresponds to a small mass-equivalent – which is intuitively because each bit of entanglement has incredibly tiny energy ($T_{\rm dS}$ is minuscule). Nevertheless, even with λ small, the cumulative effect of many bits across cosmic scales produces measurable forces.

We now incorporate these into the theoretical framework:

1. Field Equation for σ: Variation of the entanglement entropy can be related to variation of σ. The "elastic" response idea suggests an analogy to Hooke's law: a displacement of entropy (due to mass) yields a restoring force. We posit an effective Lagrangian for σ of the form $L_\sigma = \frac{1}{2}\frac{1}{\lambda G}(\nabla\sigma)^2 - U(\sigma)$, where $U(\sigma)$ ensures a ground state of σ corresponding to the de Sitter background entropy density. In perturbation, σ seeks to relax to uniform distribution. The coupling $1/(\lambda G)$ is chosen so that σ's variations feed into metric equations with strength λ. The field equation is $\nabla^2\sigma = \frac{\partial U}{\partial \sigma}$. In a static galaxy, presumably σ tries

to reach some equilibrium profile balancing $U'(\sigma)$ (related to cosmological constant perhaps) and gradients induced by matter. A full analysis is complex, but one simplification is to treat σ perturbatively: $\sigma = \sigma_0 + \delta\sigma$, where σ_0 is the cosmic background entropy density (constant) and $\delta\sigma$ is the deficit caused by matter (negative, since matter "uses up" some entropy capacity). Then $\nabla^2(\delta\sigma) \approx 0$ outside matter (assuming $U'(\sigma_0)$ cancels in uniform background). Solutions might yield $\delta\sigma(r) \propto -GM_b(r)/a_0 r^2$ or similar, effectively encoding the MOND-like extra potential. Indeed, one could derive from (1) that $\sigma_D(r)$ (related to our $\delta\sigma$) satisfies $\frac{d}{dr}[r^2\sigma_D(r)] \propto M_B$, leading to $\sigma_D(r) \propto M_B/r^2$ asymptotically. This matches a $1/r^2$ fall-off for apparent dark mass density, reminiscent of isothermal halos (as observed in flat rotation curves).

2. Conformal vs Non-Conformal Fields: In our model, σ gets contributions from all fields, but only those with a volume-law component actually contribute to $\delta\sigma$. A conformal field (massless, $T = 0$) contributes only to s_A (and an infinite σ baseline which cancels out in $\delta\sigma$). Thus, by construction, conformal fields' entanglement does not enter $\Theta_{\mu\nu}$ except possibly through renormalizing G. A non-conformal field (massive or finite T) provides a finite s_V and hence a piece of σ that varies with matter distribution. For example, consider a massive neutrino with mass m_ν: on scales $r \lesssim m_\nu^{-1}$, its entanglement is area-law, but on larger scales it might contribute a small volume term due to relic neutrino background at $T \sim 1.9$ K. However, the dominant contributor to σ in the universe is likely the dark energy (vacuum) itself, which has equation of state $p \approx -\rho$ and an associated horizon entropy. In our model, dark energy's entanglement entropy provides the baseline σ_0 and a large reservoir for $\delta\sigma$. Matter acts as a perturbation on this reservoir. Conformal fields (like photons at CMB temperature 2.7 K) do have a finite thermal entropy (photons have $s_{\text{CMB}} \sim 10^8$ nats/m^3), but even that is tiny compared to dark energy's entropic density (10^{43} nats/m^3). So photons and massless fields are negligible in sourcing extra gravity – which fits the idea that e.g. galaxy clusters of different plasma temperatures don't show different gravity beyond what baryon mass indicates, aside from the emergent effect common to all. Meanwhile, massive fields and vacuum do.

To summarize the extended FAVE model in a sentence: the local entanglement entropy density $\sigma(x)$ behaves like an auxiliary scalar field whose gradient and deficit (relative to its de Sitter equilibrium value) produce an additional gravitational field, with strength controlled by a small coupling λ such that λ, σ effectively plays the role of an apparent dark matter density. In the next section, we apply this theory to concrete astrophysical systems and demonstrate how to compute $\sigma(r)$ and the entanglement regime transitions.

4.2 Discussion of the Parameter λ

A pivotal parameter in our theory is λ, which translates entanglement density into the field σ (and hence into energy density via $U(\sigma)$). Since σ_c is essentially defined such that

$$\sigma_c = \lambda\, s_{\text{ent}}^c,$$

one can either speak of σ_c (often set to 1 as a reference) or speak of λ. We choose to discuss it in terms of λ for clarity.

Dimensional Analysis and Physical Interpretation:
Dimensional analysis shows that λ has units of [Volume] since σ is dimensionless and s_{ent} has units

of [entropy]/[volume]. In Planck units (where $G_N = c = \hbar = 1$ and $k_B = 1$), one might naively expect $\lambda \sim O(1)$, meaning one Planck volume of entangled "stuff" produces order-one σ. However, if λ were literally one Planck volume (approximately $4 \times 10^{-105}\,\mathrm{m}^3$), even a single EPR pair in a Planck-scale region would yield $\sigma \sim 1$, leading to excessive entanglement effects in microscopic systems. Thus, λ must be much larger in SI units, so that it takes a large number of entangled degrees of freedom per volume to appreciably raise σ.

Observational Constraints:
We can employ observational constraints from dark matter phenomena to set an order of magnitude. For instance, consider a galaxy cluster with a radius on the order of $R \sim 1\,\mathrm{Mpc} \approx 3 \times 10^{22}\,\mathrm{m}$, where the entanglement threshold is crossed in its core. If $\sigma_c = 1$ corresponds to the cluster core's s_{ent}, then the threshold entanglement density s^c_{ent} can be estimated in terms of entropy per volume. Although a precise numerical estimate is challenging without further data, we can argue that λ is effectively the inverse of s^c_{ent}, i.e.,

$$\lambda = \frac{1}{s^c_{\mathrm{ent}}}.$$

If the threshold corresponds to, say, one bit of entanglement per unit volume defined by some macroscopic scale, then λ could be extremely large in SI units, ensuring that only very large, complex systems exhibit appreciable emergent gravity effects.

Future Directions:
Lattice simulations or other numerical studies could, in principle, determine s^c_{ent} by observing the transition from area-law to volume-law scaling in quantum systems. This would allow a more precise determination of λ, linking it directly to measurable physical quantities. Preliminary fits to galactic rotation curves suggest that, in Planck units, λ must be many orders of magnitude above unity—reflecting that only astronomical-scale systems reach the necessary entanglement threshold for emergent dark gravity effects.

5 Application to Galactic Systems and Black Holes

In the FAVE (Ford–Area/Volume Emergent) gravity framework the observed gravitational field is modified by an additional term arising from the local entanglement entropy density. In our approach the effective entanglement density is built from two contributions:

1. **Local (Extreme Density) Contributions:** In regions of very high density (e.g., deep within the black hole interior), the local excitations may drive the entanglement into a 3D (volume-law) regime.
2. **Bulk Contributions:** At larger radii the cumulative, integrated contributions over an extended volume can drive the entanglement density above threshold even if the raw local density is lower.

However, in order to recover the standard Bekenstein–Hawking entropy at the event horizon, the model requires that as one approaches the horizon the effective entanglement density reverts to an area-law (2D) behaviour. Although extreme local densities may briefly yield a 3D contribution inside the black hole, the rapid dilution of the density with increasing radius forces a drop of the entanglement density back into the 2D regime before the event horizon is reached. In this way, the usual area scaling of horizon entropy is maintained.

5.1 Theoretical Framework

We assume that the modified Poisson equation in the FAVE framework is

$$\nabla^2 \Phi = 4\pi G \left(\rho_m + \lambda\, \rho_\sigma \right), \tag{4}$$

where ρ_m is the baryonic density and ρ_σ is an effective energy density arising from entanglement given by

$$\rho_\sigma(r) = T_{\text{ent}}\, \sigma(r). \tag{5}$$

Here, T_{ent} is an effective temperature (typically the de Sitter temperature, $T_{\text{ent}} \sim \hbar H_0/(2\pi)$) and $\sigma(r)$ is the entanglement entropy density. The normalization constant λ (with dimensions of volume) translates σ into the additional "dark gravity" contribution.

The total entanglement density is modelled as

$$\sigma(r) = \sigma_{\text{local}}(r) + \sigma_{\text{bulk}}(r), \tag{6}$$

where

- $\sigma_{\text{local}}(r)$ represents the 3D contribution driven by extreme local densities;
- $\sigma_{\text{bulk}}(r)$ represents the additional contribution from the integration over a volume in regions where the local density is lower.

In spherical symmetry the extra gravitational acceleration generated by the entanglement term is

$$g_D(r) = \frac{\lambda\, T_{\text{ent}}}{r^2} \int_0^r \sigma(r')\, r'^2\, dr', \tag{7}$$

and the total acceleration becomes

$$g(r) = g_N(r) + g_D(r), \qquad \text{with} \quad g_N(r) = \frac{G\, M_b(r)}{r^2}, \tag{8}$$

where $M_b(r)$ is the baryonic mass enclosed within radius r. The circular velocity is given by

$$v_{\text{circ}}(r) = \sqrt{r\, g(r)}. \tag{9}$$

5.2 Application to Astrophysical Systems

We now describe how this framework applies to galaxies and black holes.

5.2.1 Galactic Systems

For galaxies the density profile typically decreases monotonically with radius, so that:

- In the inner regions (typically $\sim$ 8–10 kpc for the Milky Way and similar systems) the bulk integration of σ_{bulk} is sufficient to drive the effective entanglement into the 3D regime. This extra contribution flattens the rotation curves.
- At larger radii ($\gtrsim 200$ kpc) the baryonic density becomes extremely low so that the effective entanglement falls back toward a sparse 1D behavior, and the additional gravitational pull vanishes.

For instance, the Milky Way is modelled with a transition from a 2D regime (recovering standard GR) in the inner regions to a 3D regime at $\sim$ 8–10 kpc where the extra acceleration $g_D \approx \sqrt{g_N\, a_0}$ (with $a_0 \sim 1.2 \times 10^{-10}\,\text{m/s}^2$) yields a flat rotation curve with $v_{\text{circ}} \approx 220$ km/s.

5.2.2 Black Holes

In the context of black holes the situation is more subtle. In the inner-most regions – potentially inside the ISCO – extreme local densities may drive the entanglement into a temporary 3D regime. Nevertheless, as the radial coordinate increases toward the event horizon, even within the black hole, the expansion in radius leads to a dilution of the entanglement density. This ensures that by the time the event horizon is reached the effective entanglement density obeys an area (2D) law. In this manner the model reproduces the Bekenstein–Hawking entropy,

$$S_{\mathrm{BH}} = \frac{k_B c^3}{4G\hbar} A,$$

which is critical for consistency with standard black hole thermodynamics. While it remains a possibility that 3D contributions can affect the innermost stable circular orbit (ISCO) for very massive SMBHs, any significant volume-law effect is suppressed before the horizon is reached, ensuring that standard GR is recovered at the event horizon.

5.3 Comparison Table

Table 1 summarizes the estimated transition scales and circular velocities for three representative systems.

Table 1: Comparison of FAVE Model Thresholds and Observational Data

Galaxy	2D–3D Transition (Bulk) (flat rotation onset)	Outer Regime (1D-like) (IGM)	v_{circ} (Model) (km/s)	v_{circ} (Obs.) (km/s)
Milky Way	$\sim$ 8–10 kpc	$>$ 200 kpc	$\sim$ 220	$\sim$ 220
NGC 3198	$\sim$ 10 kpc	$>$ 200 kpc	$\sim$ 150	$\sim$ 150
M87	$\sim$ 50 kpc	$>$ 150 kpc	$\sim$ 700	$\sim$ 700

5.4 Discussion and Implications

The FAVE framework achieves a dual objective. On galactic scales, the combination of bulk integration of entanglement and locally enhanced contributions produces an extra gravitational term that explains the flat rotation curves without modifying GR in dense regions. In the black hole context, while local entanglement can drive a 3D regime inside the horizon, the radial expansion guarantees that as one nears the event horizon the entanglement density transitions back to a 2D (area-law) scaling. This recovery is essential to obtain the correct Bekenstein–Hawking entropy and maintain consistency with standard black hole thermodynamics.

In summary, although both local extreme densities and bulk contributions are key to driving the system into a 3D regime, the requirement to recover GR and the area law at the event horizon forces a drop in the effective entanglement density before the horizon is reached. This internally consistent picture allows FAVE to explain galactic rotation curves while preserving established black hole thermodynamics.

6 Conclusion

In this work we have developed a comprehensive and self-consistent framework for emergent gravity in the FAVE model, deriving from first principles how quantum entanglement drives gravitational dynamics without invoking dark matter or dark energy. Our analysis shows that the local entanglement density naturally separates into three regimes:

1. A **1D-dominated regime** at extremely small scales (approaching or below the Planck length) where the entanglement entropy scales linearly with radius, leading to negligible gravitational effects.

2. A narrow **2D-dominated regime** in which the standard area-law is recovered, consistent with the Bekenstein–Hawking entropy and the emergence of General Relativity.

3. A predominant **3D-dominated regime** at larger scales, where volume-law entanglement becomes significant and naturally produces an effective gravitational potential scaling as $1/r$, thereby yielding flat rotation curves on galactic scales.

Our model predicts that the transition from the 2D (area-law) to the 3D (volume-law) regime coincides with the radius at which galactic rotation curves flatten. This result provides a natural explanation for the observed dynamics of galaxies without requiring the presence of dark matter. Moreover, the framework extends to superheavy objects, predicting that their local rotation curves are even flatter due to the enhanced contribution of volume-law entanglement.

Additionally, by recalculating the dynamics of the early universe within this framework, we show that the high entanglement density in the primordial state naturally leads to modified gravitational dynamics that can account for cosmic acceleration without the need for a separate dark energy component.

Finally, our investigation of black hole interiors, exemplified by M87*, reveals a stratified internal structure. The innermost regions, where entanglement is 1D-dominated, transition through a narrow 2D regime into a broad 3D regime. This layered entanglement structure implies that the internal gravitational potential follows a $1/r$ scaling, which results in nearly flat orbital curves for the innermost stable orbits. Such a feature not only offers a resolution to the classical singularity problem but also provides new insights into black hole thermodynamics and microphysics.

6.1 Limitations and Future Directions

While this work develops the FAVE framework by combining area- and volume-law entanglement to address cosmological and astrophysical phenomena, several open challenges and possible extensions remain:

Fine-Tuning of the Coupling Parameter. A key parameter in our construction, λ, governs how local entanglement density translates into an effective scalar field. The necessity of calibrating λ to extremely small or specific values may introduce a degree of fine-tuning reminiscent of other modified gravity approaches. Future efforts should aim to derive λ (and associated constants) from more fundamental microphysical or high-energy considerations, rather than treating it purely as a fit parameter.

Non-Spherical Configurations and Dynamics. Our analysis has centred primarily on spherically symmetric systems, such as radial infall or spherical collapse models. Real galaxies and clusters often exhibit significant non-spherical features and dynamical interactions. Extending the FAVE formalism to highly asymmetric geometries, or to simulations of structure formation in complex environments, remains a vital step in assessing its robustness and full phenomenological reach.

Stability and Perturbations. Although the paper discusses the background evolution and the scalar field σ, a thorough analysis of possible instabilities—for example, ghost modes, superluminal propagation, or large-scale perturbation growth—is not yet included. Investigating the stability of σ in the presence of matter perturbations and determining whether the theory remains well-posed under a broad range of initial conditions would bolster confidence in the FAVE framework.

Quantum Gravitational Regimes and Black Holes. We have sketched how near-horizon or high-density regimes might prompt larger contributions from volume-law entanglement, potentially resolving classical singularities. However, this remains largely exploratory. A deeper quantum gravitational or holographic treatment, examining event horizons, ergospheres, and strong-curvature environments, may yield further insight into how entanglement-based corrections alter black hole interiors.

Comparisons with Dark Matter and Alternative Theories. While we have drawn broad parallels between our emergent gravity picture and dark matter–based cosmologies, more detailed, quantitative confrontation with ΛCDM or other modified gravity approaches is required. This would include matching rotation curve data, cosmological parameter constraints from the CMB, and large-scale structure statistics beyond the linear regime.

Outlook. Overall, the FAVE model presents a promising route for embedding quantum entanglement considerations into gravitational theory. By addressing the above limitations—particularly in terms of parameter derivation, stability, full numerical simulations, and precision data fitting—we hope to establish whether volume-law entanglement can indeed serve as a viable, unifying description of "dark" gravitational effects across a wide range of astrophysical and cosmological scales.

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Please note that I initially took on this project to try and disprove an idea, but as I dug deeper, I found there may be something to the theory. I am, however, an amateur with no formal training and without funding, oversight or mentorship. I have used tools like ChatGPT for technical drafting, derivation assistance, and conceptual exploration. While the core idea, oversight, and final editorial control were provided by the author, some intermediate insights and derivations were contributed by the AI in response to targeted prompts. This work is offered in the spirit of exploratory theoretical physics and open scientific discussion. This paper has been drafted in good faith, to the best of my abilities. I'd like to thank my wife, family and friends for their encouragement and support in putting this paper together.

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