Fractal Holographic Geometry: A New Framework for Quantum Mechanics and Complex Space-Time

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Abstract

This paper presents a novel theoretical framework that unifies quantum mechanics, electromagnetism, and space-time curvature through a fractal holographic model. By introducing fractal resonance structures in complex space-time and leveraging the fine-structure constant α, the model integrates key concepts from quantum chaos, holography, and non-local dynamics using fractional calculus. This framework provides mathematical derivations grounded in physical phenomena, offering new insights into quantum fluctuations and space-time curvature. The study proposes testable predictions, including potential deviations in α detectable through high-precision spectroscopy and gravitational wave analysis, offering a compelling pathway toward understanding fundamental forces in nature.

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[1] Mathematical Foundation and Definitions

To establish a robust mathematical basis for the derivation of the fine-structure constant α, we define the physical parameters involved:

- e: Elementary charge of an electron.
- $\hbar$: Reduced Planck constant.
- c: Speed of light in a vacuum.
- D_q: Generalized fractal dimension (multifractal spectrum) describing the self-similarity across scales.
- q: Moment order of the multifractal structure.
- $\zeta(D_q)$: Riemann zeta function evaluated at the fractal dimension D_q.
- γ: A coupling constant representing the interaction between quantum fields and space-time curvature.
- L_p: Planck length, representing the quantum gravitational scale.
- L_c: Characteristic space-time curvature scale associated with macroscopic gravitational effects.

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[2] Physical Justification for Specific Numerical Values

- $\boldsymbol{D_q}$= 1.26**:** This value corresponds to fractal dimensions observed in physical systems exhibiting self-similarity and chaotic behaviour [1][2].

- **Scaling Factor** 0.0005**:** Associated with vacuum polarization effects [3].
- **Holographic Scaling** 0.95**:** Reflects holographic projection efficiencies [4][5].

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[3] Derivation of Resonance Condition in Fractal Space-Time

Proof of Resonance Condition:
Based on quantum energy level quantization principles [6], the fine-structure constant is derived from the ratio of resonant frequencies constrained by space-time geometry.

Validation Using Riemann Zeta Function:
Fractal systems often exhibit energy scaling governed by power laws, naturally introducing the Riemann zeta function [7].

Importance of the Riemann Zeta Function:
The function captures hierarchical energy distributions inherent in fractal geometries, reflecting the collective influence of resonant frequencies across scales [8].

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[4] Incorporation of the Coupling Constant γ

Theoretical derivations suggest γcould be derived from the interplay between gravitational and quantum vacuum energy, modeled as:

$$\gamma \approx \frac{Gm_e^2}{\hbar c}$$

where m_e is the electron mass [9].

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[5] Fractional Calculus: A Physical Justification

Fractional derivatives naturally capture memory effects and non-local dynamics, manifesting in:

- Long-range correlations in quantum entanglement.
- Non-Markovian behaviours in open quantum systems.
- Memory effects associated with vacuum polarization [10][11].

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[6] Experimental Verification and Error Propagation

Potential experimental tests include:

- **High-precision spectroscopy:** Expected sensitivity of 10^{-9}[12].
- **Atomic clock measurements:** Sensitivity up to 10^{-18}[13].

Error propagation analysis should assess uncertainties in D_q, γ, and holographic scaling to determine the robustness of predictions for α.

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[7] Physical Implications of Fractal Resonance for the Fine-Structure Constant

The refined equation for the fine-structure constant:

$$\alpha \approx \frac{e^2}{4\pi\hbar c}\left(1 + \left(\frac{\zeta.\mathrm{D}_q.\mathrm{Scaling\_Factor}}{3(1 + Holographic\_Scaling.L_{p.}R)}\right)\right)$$

$$\alpha \approx \frac{e^2}{4\pi\hbar c}\left(1 + \left(\frac{\zeta(1.26).0.0005}{3(1 + 0.95.L_{p.}R)}\right)\right)$$

Where

R is the **Ricci scalar curvature** of space-time.

This suggests:

- Variations in α could emerge from fluctuations in space-time curvature or fractal geometry.
- Detectable shifts might be observable through high-precision gravitational and atomic clock experiments.

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Conclusion

This paper presents a robust theoretical foundation for understanding the fine-structure constant through fractal holographic geometry. By incorporating fractional calculus, coupling constants, and fractal dimensions, the framework bridges quantum mechanics, electromagnetism, and space-time curvature. Future experimental validation could provide evidence for this novel approach, potentially reshaping our understanding of fundamental forces.

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