FAVE: An Emergent Gravity Framework

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Abstract

We explore a proposal for emergent gravity grounded in the interplay between area-law and volume-law entanglement contributions in quantum field theories. By tracing the well-known area-law behaviour of entanglement entropy in ground states, we introduce a scalar field σ that tracks the local entanglement density. This effective field remains negligible when the system's entanglement adheres to the standard (boundary) area law, reproducing General Relativity (GR). However, once excitations increase the entanglement beyond a critical threshold, volume-law contributions become significant, and σ acquires a non-trivial potential that modifies the Einstein field equations. We show how this framework—dubbed the Ford-Area/Volume Emergent (FAVE) model—can mimic dark matter and dark energy phenomena by "smearing" mass–energy in high-entanglement regions, thereby altering cosmological expansion and structure formation. We apply this idea to mass-to-light ratio measurements in galaxy clusters, revealing that entanglement-driven corrections can provide a consistent fit to observations while reducing the need for separate dark components. Finally, we discuss possible implications for black hole interiors, highlighting how a 1/r entanglement contribution might remedy singularities through an effective redistribution of energy. Our results serve as a preliminary validation of FAVE as a unified approach to tackling both large-scale structure and black hole physics from an entanglement-theoretic perspective.

Introduction

Gravity has long been viewed as an arena for potential unification between quantum mechanics and classical field theory, yet achieving a full quantum theory of gravity remains one of the most elusive endeavours in modern physics. The notion that gravitational effects might emerge from the underlying quantum-information structure of fields has garnered increasing interest over the past two decades. Holographic insights, such as the Ryu–Takayanagi formula in AdS/CFT, assert that entanglement entropy on the boundary of a region is geometrically tied to areas in a higher-dimensional gravitational bulk. Although holography provides strong hints that gravity and entanglement may be deeply intertwined, one can also explore non-holographic approaches to emergent gravity, where area-law entanglement at low energies transitions to new phenomena when the entanglement becomes sufficiently excited.

This work builds on the seminal ideas of Verlinde and other pioneers in emergent gravity, whose revolutionary insights have shown that gravity might arise from deeper thermodynamic and holographic principles. In contrast to Verlinde's elegant formulation of gravity as an entropic force arising from information gradients, our Ford-Area/Volume Emergent (FAVE) framework introduces a novel scalar field that explicitly tracks the local entanglement density, distinguishing between area-law and volume-law regimes. This dual approach not only recovers standard general relativity in the low-entanglement limit but also naturally activates additional gravitational effects—mimicking dark matter and dark energy—when a critical entanglement threshold is surpassed. We pay homage to the great work of Verlinde and his contemporaries, whose visionary contributions have laid the groundwork for exploring such innovative extensions to our understanding of gravity.

In this paper, we propose and investigate the Ford-Area/Volume Emergent (FAVE) framework, which hinges on the interplay between area-law and volume-law contributions to entanglement entropy in quantum field theories (QFTs). We begin by reviewing the well-known replica trick and related QFT techniques to illustrate how the leading term in ground-state entanglement scales with boundary area, while excited states or thermal ensembles can introduce subleading volume contributions. Inspired by entropic gravity ideas, we posit a scalar field σ that encodes local entanglement density. Below a critical threshold, σ is negligible, and General Relativity (GR) is effectively recovered. Above this threshold, σ modifies the Einstein–Hilbert action with an additional kinetic term and potential, signalling the onset of volume-law entanglement and its gravitational impact.

We develop this idea systematically. First, we outline how entanglement-driven contributions might replace or supplement dark matter and dark energy—demonstrating a mechanism for "smearing" mass–energy in high-entanglement regions. We embed these modifications in a Friedmann–Lemaître–Robertson–Walker (FLRW) cosmology and show that the inclusion of σ can match present-day expansion rates and structure formation constraints. We then illustrate how the framework modifies spherical collapse, possibly reconciling observed galaxy cluster mass-to-light ratios with a lower conventional matter fraction. Finally, we extend the argument to black hole interiors, suggesting that a 1/r entanglement profile could circumvent classical singularities by redistributing energy–momentum in the core.

Through this programme, we aim to provide a concise yet self-consistent approach to emergent gravity. Section 2 delves into the foundational QFT concepts and the derivation of the area-law–volume-law transition; Section 3 outlines how σ translates into an effective scalar field with a critical activation point, leading to modified Einstein equations in Section 4. We then present cosmological and astrophysical applications, culminating in a discussion of observational fits and the potential for singularity resolution in black holes. While much remains to be tested and refined, the FAVE proposal offers a novel lens on how local quantum entanglement densities might bridge the gap between QFT and gravitational phenomena.

1 Quantum Derivation of FAVE

1.1 Step 1: Quantum Field Theory and Entanglement

1.1.1 The Setup

Consider a Free Scalar Field:

We start with a free, real scalar field $\phi(x)$ in (3+1) dimensions with the standard action in Minkowski space:

$$S[\phi] = \frac{1}{2}\int d^4x\,[(\partial_t\phi)^2 - (\nabla\phi)^2 - m^2\phi^2].$$

For simplicity (and because the leading divergence in the entanglement entropy is universal), we may initially set $m = 0$.

Spatial Partitioning:

- Divide the spatial slice (at a fixed time) into two regions:
  - **Region A:** A spherical region of radius R.
  - **Region B:** The complementary region (outside the sphere).

The goal is to compute the entanglement entropy associated with region A.

1.1.2 The Reduced Density Matrix and Entanglement Entropy

Density Matrix of the Vacuum:

In the ground state (vacuum) of the QFT, the density matrix is $\rho = |0\rangle\langle 0|$. To study entanglement, we partition the Hilbert space as $\mathcal{H} = \mathcal{H}_A \otimes H_B$. The reduced density matrix for region A is obtained by tracing out the degrees of freedom in B:

$$\rho_A = Tr_B\rho.$$

Definition of Entanglement Entropy:

The entanglement entropy is then given by the von Neumann entropy of ρ_A:

$$S_A = -Tr_A\,(\rho_A ln\ \rho A).$$

1.1.3 The Replica Trick

Directly computing S_A is difficult. The replica trick circumvents this by first computing the Rényi entropies:

$$S_A^{(n)} = \frac{1}{1-n} ln\ Tr_A(\rho_A^n),$$

and then taking the limit:

$$S_A = \lim_{n\to 1} S_A^{(n)} = -\frac{d}{dn} Tr_A(\rho_A^n)\,\Big|_{n=1}\quad .$$

Path Integral Representation:

The key observation is that $Tr_A(\rho_A^n)$ can be represented as a path integral on an n-sheeted Riemann surface $\mathcal{M}_n$ that is constructed by gluing together nnn copies of the original manifold along the cut defined by the boundary ∂A (the spherical surface). Schematically:

$$Tr_A(\rho_A^n) \ = \frac{Z[\mathcal{M}_n]}{(Z[\mathcal{M}_1])^n},$$

where $Z[\mathcal{M}_n]$ is the partition function on the n-sheeted manifold.

1.1.4 Evaluating the Partition Function

For a free scalar field, the partition function can be written (formally) as a functional determinant:

$$Z[\mathcal{M}_n] = [det\ (-\Delta_n)]^{-1/2},$$

where $-\Delta_n$ is the Laplacian on the n-sheeted manifold. To evaluate the determinant, we use the heat kernel representation:

$$\ln \det\big(-\Delta_{(n)}\big) = -\int_{\epsilon^2}^{\infty} \frac{dt}{t} Tr\ e^{-t(-\Delta(n))},$$

with $\boldsymbol{\epsilon}$ acting as a UV regulator. In the presence of a conical singularity at the boundary $\boldsymbol{\partial A}$, the heat kernel admits an asymptotic expansion

$$Tr\ e^{-t(-\Delta(n))} \sim \sum_{k\geq 0} a_k(\mathcal{M}_n) t^{(k-4)/2},$$

In many treatments (see, e.g., the work by Srednicki or Casini and Huerta), one finds that the dominant divergence in the entanglement entropy comes from the short-distance modes near the boundary ∂A. where the coefficients $a_k(\mathcal{M}_n)$ encode geometric information. In particular, the leading divergence arises from the term with $\boldsymbol{k} = 2$ and is proportional to the area ∂A of the boundary:

$$S_A \sim \frac{\kappa\, A(\partial A)}{\epsilon^2} + \cdots,$$

where:

- $A(\partial A) \ = \ 4\pi R^2$A is the area of the spherical boundary,
- ϵ is a UV cutoff (related to the lattice spacing or the minimal length scale of the QFT), and
- κ is a numerical constant that depends on the details of the field content.

This is the **area law** for entanglement entropy, and it is a robust feature of local QFTs in their ground state.

1.1.5 Subleading Contributions: Volume Law in Excited States

In the vacuum state of a local quantum field theory (QFT), it is well established that the entanglement entropy S of a spatial region scales predominantly with the area of its boundary. This was first shown in Srednicki's seminal paper [Srednicki, Phys. Rev. Lett. 71, 666 (1993)] and is reviewed comprehensively in works by Casini and Huerta (see, e.g., [Casini & Huerta, J. Phys. A: Math. Theor. 42, 504007 (2009)]). In this setting, one finds

$$S_{area} \sim \frac{A}{\epsilon^2},$$

where A is the area of the boundary and ϵ is a UV cutoff.

However, in **excited states** or at **finite temperature**, the situation changes. To see this, consider the following concrete example:

Example: Thermal State of a Free Scalar Field

Suppose we have a free, massless scalar field $\phi(x)$ in (3+1) dimensions. Instead of the vacuum, we consider the field to be in a thermal state at temperature T. The density matrix for a thermal state is given by

$$\rho = \frac{e^{-\beta H}}{Z},$$

with $\beta = 1/T$ and $Z = Tr\, e^{-\beta H}$. In this state, the full entropy is extensive—it scales with the volume V of the system. For a conformal field theory (and a free massless field is conformal), the thermal entropy density $s(T)$ scales as

$$s(T) \sim T^3,$$

so that the total entropy in a region of volume V is

$$S_{thermal} \sim s(T)\, V \sim T^3\, V.$$

Now, when we compute the entanglement entropy S_A for a subregion A (say, a sphere of radius R), there are two contributions:

Area-Law Contribution:
At short distances, correlations across the boundary dominate, yielding

$$S_{area} \sim \frac{A(\partial A)}{\epsilon^2},$$

where $A(\partial A) \sim R^2$

Volume-Law Contribution:
At finite temperature, however, there is an additional term arising from thermal fluctuations that are essentially local. This contribution is similar to the thermal entropy of the region A itself,

$$S_{vol} \sim s(T)\, V(A) \sim T^3\, R^3.$$

Thus, for an excited (thermal) state, one can write the total entanglement entropy approximately as

$$S \sim \frac{A(\partial A)}{\epsilon^2} + T^3\, V(A) + \cdots.$$

For large regions or at sufficiently high temperature, the volume-law term $T^3\, V$ can dominate over the area-law term, leading to a qualitatively different behavior.

Alternative Example: Localized Excitations

Another scenario is to consider a state where the field is excited in a localized manner—for example, a coherent or squeezed state corresponding to an excited wavepacket. In such states, the additional excitations spread over the entire volume of the region can contribute an entropy that scales with the volume. While the detailed computation is more involved, one finds (see, e.g., [Calabrese & Cardy, J. Stat. Mech. (2004) P06002]) that excitations above the ground state yield corrections of the form:

$$\Delta S \propto V(A) \times f(excitation\, parameters),$$

where $f(excitation\, parameters)$ is a function that depends on the specifics of the excitation. In the limit where the excitation is extensive (i.e. spread uniformly throughout the region), this term scales linearly with the volume $V(A)$.

Implications for FAVE

In the FAVE framework, we interpret the local entanglement density $S_{ent} = S/V$ as the key variable that tracks whether a region is in the standard (area-law) or modified (volume-law) regime. In the vacuum,

$$S_{ent} \sim \frac{S_{area}}{V} \sim \frac{R^2}{R^3} \sim \frac{1}{R},$$

which decreases with the size of the region. However, for a thermal or highly excited state, the volume contribution dominates:

$$S_{ent} \sim \frac{S_{area}}{V} \sim T^3$$

which is independent of the size of the region (or constant for a fixed temperature).

Thus, one naturally identifies a critical threshold s_{ent}^c (or equivalently, a critical value σ_c when multiplied by a conversion factor λ) such that:

For $S_{ent} < s_{ent}^c$ (or $\sigma < \sigma_c$), the entanglement is dominated by the area-law and standard General Relativity is recovered.

For $S_{ent} > s_{ent}^c$ (or $\sigma > \sigma_c$), the volume-law contribution becomes significant, leading to additional energy density that modifies the gravitational dynamics.

This microphysical derivation provides a concrete mechanism by which the transition from an area-law to a volume-law regime occurs in excited states, thereby justifying the introduction of the scalar field σ in FAVE.

1.1.6 Summary of Step One

In summary, starting from a free scalar field theory:

- We define the entanglement entropy S_A of a spherical region using the reduced density matrix.
- Using the replica trick, the problem is recast into a computation of the partition function $Z[\mathcal{M}_n]$ on an n-sheeted manifold.
- A heat-kernel (or functional determinant) analysis reveals that the leading divergence is proportional to the area of the boundary, $A(\partial A)$, establishing the area law for entanglement entropy.
- In excited or high-energy density states, an additional volume-law term can emerge, and the competition between these contributions motivates the introduction of an effective scalar field σ that tracks the local entanglement density.

1.2 Step Two: Relating Entanglement to Geometry and Gravity

1.2.1 Holographic Inspiration and the Area Law

One of the key insights from holographic duality (e.g. the AdS/CFT correspondence) is the Ryu–Takayanagi formula, which states that the entanglement entropy S_A of a boundary region A in a conformal field theory is given by

$$SA = \frac{Area(\gamma_A)}{4\,G_N},$$

Where γ_A is the minimal surface in the bulk that is homologous to A and G_N is Newton's constant.

- **Takeaway:**
  This formula shows that gravitational coupling (or curvature) is intimately linked to the area of a surface, suggesting that area-law entanglement in QFT may underpin the geometric description of gravity.

Holography vs. Non-Holographic Approaches:

While our discussion draws inspiration from the Ryu–Takayanagi formula—which relates the entanglement entropy of a boundary region in a conformal field theory to the area of a minimal surface in the bulk of an AdS spacetime—we stress that our approach does not require a strictly holographic (AdS/CFT) framework. Instead, we rely on the general principle that local quantum field theories in their ground state exhibit area-law scaling for entanglement entropy. This universal feature is expected to hold even in non-holographic settings and forms the basis for linking entanglement to gravitational dynamics in our FAVE framework.

1.2.2 Emergent Gravity from Entanglement

Even in non-holographic settings, several authors have argued that gravity could emerge as an effective, entropic force arising from changes in information or entanglement. The basic idea is:

- **Entropic Force Concept:**
  When a system has an intrinsic entropy that depends on the position of matter, gradients in this entropy can result in an effective force. Verlinde (2011) and others have proposed that gravitational attraction might be interpreted as such an entropic force.
- **FAVE Perspective:**
  In FAVE, the gravitational field is not fundamental but is induced by the entanglement structure of quantum fields. The observed curvature is then a macroscopic manifestation of the microscopic entanglement among field degrees of freedom.

1.2.3 From Area-Law to Volume-Law: A Transition in Entanglement

Recall from step 1 that in the vacuum state of a QFT the entanglement entropy follows an area law:

$$S_{area} \sim \frac{A}{\epsilon^2}.$$

However, in excited states or regions with high energy density (or even in the presence of additional degrees of freedom), there may be a significant volume-law contribution:

$$S_{vol} \sim \frac{V}{\ell^3},$$

where V is the volume and ℓ is a UV cutoff scale different from ϵ.

- **Physical Interpretation:**
  When the entanglement density $s_{ent} = S/V$ remains low (dominated by the area law), the emergent gravitational dynamics coincide with those predicted by standard General Relativity (GR). However, once s_{ent} exceeds a critical threshold, the volume-law contribution becomes significant and the effective gravitational dynamics are modified.

1.2.4 Introducing the Scalar Field σ as an Entanglement Tracker

To formalise the idea that extra gravitational effects arise when entanglement exceeds a threshold, FAVE introduces an effective scalar field σ defined as:

$$\sigma = \lambda\, s_{ent},$$

where:

- $s_{ent} = \frac{S}{V}$ is the local entanglement density,

- λ is a conversion factor (with appropriate units) that translates the entanglement density into a scalar field value.

The field σ thus tracks how "entangled" a region is:

- **When $\boldsymbol{\sigma < \sigma_c}$:**
  The system is in an area-law regime and standard Einstein gravity is recovered.
- **When $\boldsymbol{\sigma > \sigma_c}$:**
  The volume-law contribution dominates, and extra gravitational effects emerge, potentially mimicking dark components.

1.2.5 Mapping Entanglement to Gravitational Dynamics

Now, to link this entanglement-based scalar field to gravity, consider the following:

- **Effective Energy Density:**
  In analogy with thermodynamic systems, one can interpret the gradient or variation in the entanglement density as giving rise to an effective energy density. In the FAVE framework, this additional energy density is captured by the scalar field's potential $U(\sigma)$ and kinetic term:

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + U(\sigma).$$

- **Modified Einstein Equations:**
  In a covariant formulation, the gravitational field equations are modified by the contribution of σ. The full action becomes:

$$S = \frac{1}{16\pi G}\int d^4\,x\,\sqrt{-g}\,R + S_\sigma + S_{matter},$$

  with

$$S_\sigma = -\int d^4\,x\,\sqrt{-g}\,\left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right].$$

  Variation with respect to the metric then yields modified Einstein equations:

$$G\mu\nu = 8\pi G(T_{\mu\nu}^{matter} + T_{\mu\nu}^{\sigma}),$$

  where

$$T_{\mu\nu}^{\sigma} = \nabla_\mu\sigma\,\nabla_\nu\sigma - g\mu\nu\left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right].$$

  Thus, when σ exceeds σ_c, the extra energy–momentum $T_{\mu\nu}^{\sigma}$ modifies the gravitational dynamics.

1.2.6 Summary of Step 2

- **Linking Entanglement to Geometry:**
  We begin with the holographic insight that entanglement entropy (area-law) is linked to gravitational coupling.
- **Entropic Gravity and Emergence:**
  The notion that gravity can be seen as an emergent, entropic force motivates the idea that variations in entanglement lead to effective gravitational dynamics.
- **Transition in Entanglement Regimes:**
  The competition between area-law and volume-law contributions is key; when volume-law effects become significant, they indicate a transition that can modify gravity.
- **The Role of σ:**
  By defining a scalar field σ that tracks the local entanglement density, we capture this transition. The field's dynamics (via its kinetic term and potential $U(\sigma)$ then encode the additional energy that alters gravitational behaviour.
- **Modified Gravity:**
  Inserting the effective energy density of σ into the Einstein equations leads to modified gravitational dynamics, setting the stage for FAVE's predictions in cosmology and black hole physics.

1.3 Step 3: Introducing the Scalar Parameters

1.3.1 Introducing σ as a Measure of Local Entanglement Density

1.3.1.1 Defining the Entanglement Density

Recall from Step 1 that when we compute the entanglement entropy S for a spatial region, two leading contributions are found:

- **Area-Law Term:**

$$S_{area} \sim \frac{A}{\epsilon^2}$$

  where A is the area of the boundary of the region and ϵ is a UV cutoff (such as the Planck length).

- **Volume-Law Term:**

$$S_{vol} \sim \frac{V}{\ell^3},$$

  where V is the volume of the region and ℓ is another characteristic length scale that appears in the QFT (e.g. a correlation length or inverse cutoff scale).

From these, one may define a **local entanglement density**:

$$s_{ent} \equiv \frac{S}{V} = \frac{S_{area} + S_{vol}}{V}.$$

In a large region, the area-law contribution scales as $S_{area} \sim A \propto V^{2/3}$, so its density S_{area}/V decreases with volume, while the volume-law contribution is constant:

$$\frac{S_{vol}}{V} \sim \frac{1}{\ell^3}.$$

1.3.1.2 Defining the Scalar Field σ

To encapsulate the effects of entanglement in a macroscopic gravitational theory, we introduce a scalar field σ that serves as a proxy for the local entanglement density. Specifically, we define

$$\sigma \equiv \lambda\, s_{ent} = \lambda\, \frac{S}{V},$$

where λ is a conversion factor with appropriate units to render σ dimensionless (or to set its physical scale appropriately). The choice of λ is guided by matching to gravitational observations and by convenience; one common approach is to rescale so that the critical value becomes unity (i.e. $\sigma_c = 1$ in dimensionless units).

1.3.1.3 The Critical Threshold σ_c

The central idea in FAVE is that the gravitational dynamics change when the nature of entanglement shifts. In regions where the entanglement is dominated by the area-law term, the local entanglement density s_{ent} is relatively low, and one expects standard Einstein gravity to be recovered. However, when the volume-law contribution becomes significant—typically in high-energy or highly excited regions—the entanglement density increases.

We define a critical threshold σ_c such that:

- **For $\sigma < \sigma_c$:**
  The entanglement is predominantly in the area-law regime. In this limit, $S \approx S_{area}$ and hence

  $$\sigma \approx \lambda\, \frac{A}{V\, \epsilon^2}\, (small\ value).$$

  Gravity remains effectively unmodified.

- **For $\sigma > \sigma_c$:**
  The volume-law contribution starts to dominate, $S \approx S_{vol}$, and

  $$\sigma \approx \lambda\, \frac{1}{\ell^3}.$$

In this regime, the extra entanglement is interpreted as contributing additional energy–momentum to the gravitational field, thus modifying the effective gravitational dynamics.

The critical threshold σ_c can be thought of as the point where the two contributions are comparable:

$$\frac{A}{\epsilon^2} \sim \frac{V}{\ell^3} \Longrightarrow s_{ent} \sim \frac{1}{\ell^3}.$$

Then, with the scaling factor λ, one sets

$$\sigma_c \ \sim \lambda \frac{1}{\ell^3}.$$

By an appropriate choice of λ and units, one can normalize σ_c (for example, to 1) so that the effective theory naturally distinguishes between the two entanglement regimes.

1.3.1.4 Physical Implications

Introducing σ in this way provides a direct bridge from the microscopic quantum properties of fields to macroscopic gravitational effects:

- **Emergent Gravity Interpretation:**
  The scalar field σ does not represent a new fundamental force; instead, it parametrizes how entanglement structure changes across space-time. When σ is low, the gravitational force behaves as in Einstein's theory. When σ exceeds the threshold, extra gravitational effects emerge—potentially mimicking dark matter or modifying collapse dynamics.
- **Effective Potential $\boldsymbol{U(\sigma)}$:**
  In Step 3 (as derived previously), the effective potential $U(\sigma)$ is constructed to be negligible when $\sigma < \sigma_c$ and to rise when $\sigma > \sigma_c$. This potential encodes the energy cost of increasing the entanglement beyond the "normal" regime, thereby sourcing modified gravity.
- **Linking Microphysics to Cosmology:**
  Once σ is introduced, its dynamics—governed by its kinetic term and the potential $U(\sigma)$—enter the gravitational field equations. This provides a framework where microphysical entanglement properties affect large-scale structure, cosmic expansion, and even black hole singularities.

1.3.1.5 Summary

- We define the scalar field σ as a measure of the local entanglement density:

$$\sigma = \lambda \frac{S}{V}.$$

- A critical threshold σ_c is introduced to distinguish between the area-law (standard gravity) and volume-law (modified gravity) regimes.
- For $\sigma < \sigma_c$, the entanglement remains predominantly area-law dominated, and the effective gravitational dynamics are unaltered.

- For $\sigma > \sigma_c$, volume-law contributions become significant, leading to an increased energy density that modifies gravity via an effective potential $U(\sigma)$.

1.3.2 Deriving the Effective Potential $U(\sigma)$

1.3.2.1 Motivation: Energy Cost of Extra Entanglement

Recall from steps 1 and 2 that a region in a quantum field theory exhibits an entanglement entropy that is dominated by an area law:

$$S_{area} \sim \frac{A}{\epsilon^2},$$

with possible additional contributions when the system is excited or when the entanglement density increases:

$$S_{vol} \sim \frac{V}{\ell^3},$$

In FAVE, the idea is that when the local entanglement density (i.e. $s_{ent} = S/V$) exceeds a certain threshold, the extra (volume-law) entanglement leads to extra energy in the system. We capture this additional energy via an effective scalar field σ defined as:

$$\sigma = \lambda\ s_{ent},$$

with a critical threshold σ_c. For $\sigma < \sigma_c$, the system is in the “normal” area-law regime (and standard Einstein gravity is recovered). For $\sigma > \sigma_c$, the extra energy associated with volume-law entanglement alters the gravitational dynamics.

1.3.2.2 Effective Field Theory and the Emergent Potential

In effective field theory, one writes down a potential $U(\sigma)$ that describes the energy density stored in the scalar field configuration. The goal here is to design $U(\sigma)$ so that:

- **For $\boldsymbol{\sigma < \sigma_c}$:** The potential is negligible—meaning the extra energy contribution is minimal, and gravity behaves as in standard GR.
- **For $\boldsymbol{\sigma > \sigma_c}$:** The potential increases, reflecting the extra energy due to dominant volume-law entanglement. This additional energy acts as an effective source for modified gravitational dynamics.

A commonly proposed form (which is phenomenological but inspired by microphysical arguments) is:

$$U(\sigma) = \frac{U_0}{2}\left[1 + tanh\left(\frac{\sigma - \sigma_c}{\Delta}\right)\right]\left(\frac{\sigma}{\sigma_c} - 1\right)^2,$$

where:

- U_0 is an energy scale that sets the overall magnitude of the extra energy.
- σ_c is the critical threshold in σ (often chosen to be 1 in dimensionless units after appropriate rescaling).
- Δ controls how sharply the potential transitions near σ_c.

1.3.2.3 Understanding the Form of $U(\sigma)$

1. **Below Threshold ($\sigma \ll \sigma_c$):**
   In this regime, $\sigma - \sigma_c$ is negative and large in magnitude. The hyperbolic tangent then approximates:

   $$tanh\,(\frac{\sigma - \sigma_c}{\Delta}) \approx -1.$$

   Consequently, the prefactor becomes:

   $$\frac{U_0}{2}\,[1 - 1] = 0,$$

   and the entire potential $U(\sigma)$ is suppressed. This reflects that the entanglement is dominated by the area-law contribution, and no extra gravitational modification arises.

2. **Above Threshold ($\sigma \gg \sigma_c$):**
   For large $\sigma - \sigma_c$, the hyperbolic tangent approaches 1:

   $$tanh\,\left(\frac{\sigma - \sigma_c}{\Delta}\right) \approx 1.$$

   Then the prefactor becomes:

   $$\frac{U_0}{2}\,[1 + 1] = U_0.$$

   The potential then scales as:

   $$U(\sigma) \approx U_0\,\left(\frac{\sigma}{\sigma_c} - 1\right)^2,$$

   meaning the extra energy density grows quadratically with the departure of σ from σ_c.

3. **Around the Threshold ($\sigma \approx \sigma_c$):**
   The transition is controlled by Δ, which sets how quickly the potential "switches on." A smaller Δ means a sharper transition, while a larger Δ gives a smoother crossover.

In order to capture the transition between the area-law and volume-law regimes in a rigorous manner, we introduce a scalar field $\sigma(x)$ defined via the local entanglement density

$$\sigma(x) \equiv \lambda s_{ent}(x) = \lambda \frac{S(x)}{V(x)},$$

where λ is a conversion factor chosen to render σ dimensionless or to set its physical scale.

Using the background field method, one can compute the one-loop effective action $\Gamma[\sigma]$ by integrating out the quantum fluctuations of the field $\phi(x)$ in the presence of a slowly varying background $\sigma(x)$. The result takes the form

$$\Gamma[\sigma] = \int d^4x\sqrt{-g}\,\left\{\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right\},$$

with $U(\sigma)$ emerging from the quantum corrections that arise from the change in the entanglement structure. In a rigorous treatment, one would derive $U(\sigma)$ by matching the free energy computed via the heat kernel expansion with the effective energy cost of shifting the entanglement entropy beyond the area-law term.

For instance, one may show that in the regime where the area-law dominates,

$$U(\sigma) \approx 0,$$

while in the regime where volume-law contributions become significant,

$$U(\sigma) \propto \left(\frac{\sigma}{\sigma_c} - 1\right)^2,$$

with σ_c representing the critical threshold for the transition. The precise form of $U(\sigma)$ can be derived by comparing the one-loop determinants computed on $\mathcal{M}_n$ for different background values of σ, employing zeta function regularisation to control the divergences.

1.3.2.4 Microphysical Justification

While the above form is phenomenological, one can motivate it from a microphysical standpoint by considering:

- **Matching Energies:**
  The free energy cost of increasing the entanglement entropy beyond the area-law contribution can be computed in simple QFT models (e.g., via heat-kernel methods). By comparing this free energy to the energy density in a scalar field, one can infer the functional dependence of $U(\sigma)$ on σ.
- **Effective Potential in Statistical Mechanics:**
  In systems undergoing phase transitions, one often sees potentials of the Landau–Ginzburg type where the order parameter (here, σ) is nearly free (flat potential) below the critical point

and develops a steep potential above it. The $\boldsymbol{tanh}$ function naturally appears in such contexts to model smooth transitions.

Thus, while a fully rigorous derivation from first principles in QFT is challenging, these arguments provide a self-consistent effective description that captures the essential physics:

- **For $\boldsymbol{\sigma < \sigma_c}$,** the entanglement energy is minimal, and gravity remains unmodified.
- **For $\boldsymbol{\sigma > \sigma_c}$,** the extra energy from volume-law entanglement adds to the gravitational source, leading to modified dynamics that can mimic dark matter or dark energy.

1.3.2.5 Summary of Step 3

- **Objective:** Construct an effective potential $U(\sigma)$ that accounts for the extra energy cost when the entanglement density (tracked by σ) exceeds a critical threshold σ_c.
- **Chosen Form:**

$$U(\sigma) = \frac{U_0}{2}\left[1 + tanh\ \left(\frac{\sigma - \sigma_c}{\Delta}\right)\right]\left(\frac{\sigma}{\sigma_c} - 1\right)^2,$$

  which is negligible for $\sigma < \sigma_c$ and grows quadratically for $\sigma > \sigma_c$.

- **Microphysical Rationale:**
  This form is inspired by effective field theory and statistical mechanics arguments, capturing the transition from an area-law dominated regime to a volume-law dominated regime where extra gravitational effects emerge.

1.4 Step 4: Formulating the Full Covariant Action and Deriving the Modified Einstein Equations

1.4.1 The Full Covariant Action

In FAVE, the gravitational dynamics arise from both the usual Einstein–Hilbert term and the additional contribution of the scalar field σ that tracks the local entanglement density. The complete action is written as:

$$S = \frac{1}{16\pi G}\int d^4 x \sqrt{-g}\, R + S_\sigma + S_{matter},$$

where:

- R is the Ricci scalar,
- G is Newton's gravitational constant,
- S_{matter} is the matter action,
- S_σ is the action for the scalar field σ.

The scalar-field action is given by:

$$S_\sigma = -\int d^4\, x \sqrt{-g}\, \left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right].$$

with:

- $(\nabla\sigma)^2 \equiv g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma,$
- $U(\sigma)$ is the effective potential derived in step 3, which is negligible for $\sigma < \sigma_c$ and grows when $\sigma > \sigma_c$.

1.4.2 Deriving the Modified Einstein Equations

To obtain the field equations, we vary the total action with respect to the metric $g^{\mu\nu}$. Variation of the Einstein–Hilbert term yields the standard Einstein tensor $G_{\mu\nu}$:

$$\delta\left(\frac{1}{16\pi G}\int d^4\, x \sqrt{-g}\, R\right) \longrightarrow \frac{1}{16\pi G}\, G_{\mu\nu}\, \delta g^{\mu\nu}.$$

The variation of S_σ with respect to $g^{\mu\nu}$ gives the energy–momentum tensor for σ:

$$T^{\sigma}_{\mu\nu} = \nabla_\mu\sigma\nabla_\nu\sigma - g_{\mu\nu}\left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right].$$

Finally, including the matter energy–momentum tensor $T^{matter}_{\mu\nu}$ the full field equations become

$$G_{\mu\nu} = 8\pi G\, (T^{matter}_{\mu\nu} + T^{\sigma}_{\mu\nu}).$$

Thus, the extra energy–momentum associated with the entanglement-modulating scalar field modifies gravity. In regions where σ is below the critical threshold σ_c, $U(\sigma)$ is negligible and standard General Relativity is recovered. When σ exceeds σ_c, the extra contributions become significant.

1.4.3 Deriving the Scalar Field Equation of Motion

Next, we vary the total action with respect to the scalar field σ. The variation of S_σ yields

$$\delta S_\sigma = -\int d^4\, x \sqrt{-g}\, [-\nabla^\mu\nabla_\mu\sigma + U'(\sigma)]\delta\sigma,$$

which leads to the Euler–Lagrange equation for σ:

$$\nabla^\mu\nabla_\mu\sigma - U'(\sigma) = 0.$$

In a cosmological (FLRW) background, this equation becomes

$$\ddot{\sigma} + 3H\dot{\sigma} + U'(\sigma) = 0,$$

where H is the Hubble parameter. This equation describes how σ evolves in response to its potential $U(\sigma)$, and it encapsulates the transition from an area-law (GR-like) regime to a volume-law (modified gravity) regime.

1.4.4 Summary and Implications

- **Full Action:**
  We have the action

$$S = \frac{1}{16\pi G}\int d^4 x\sqrt{-g}\, R - \int d^4 x\sqrt{-g}\ [-\nabla^{\mu}\nabla_{\mu}\sigma + U'(\sigma)] + Smatter.$$

- **Modified Einstein Equations:**
  Varying the action with respect to $g^{\mu\nu}$ gives

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{matter} + \nabla_{\mu}\sigma\nabla_{\nu}\sigma - g_{\mu\nu}\left[\frac{1}{2}(\nabla\sigma)^2 + U(\sigma)\right]\right).$$

- **Scalar Field Dynamics:**
  Variation with respect to σ yields

$$\nabla^{\mu}\nabla_{\mu}\sigma - U'(\sigma) = 0,$$

  which in a homogeneous background takes the form

$$\ddot{\sigma} + 3H\dot{\sigma} + U'(\sigma) = 0,$$

This step establishes the mechanical tie between the microphysical derivation of entanglement (steps 1–3) and macroscopic gravitational phenomena. In regions where the entanglement density (tracked by σ) exceeds a critical value, the potential $U(\sigma)$ becomes significant, modifying the Einstein equations and potentially addressing dark matter or dark energy effects.

1.5 Step 5: Incorporating the Radiation Scaling Correction

1.5.1 Standard Radiation Scaling in Cosmology

In conventional GR, the energy density of radiation scales as

$$\rho_r(a) \propto a^{-4},$$

where a is the cosmic scale factor. This scaling arises because:

- The energy density dilutes as a^{-3} due to volume expansion.
- There is an extra factor of a^{-1} due to the redshift (loss of energy) of massless particles (photons).

Massless particles (e.g. photons) do not have a rest mass and do not couple directly to scalar fields in the usual sense, so their energy density is simply set by their momentum redshift.

1.5.2 Modification in FAVE: Smearing of Energy

In the FAVE framework, the central idea is that quantum entanglement "smears" the distribution of energy–momentum. This smearing effect is encoded in the scalar field σ that tracks the local entanglement density. When σ exceeds a critical threshold σ_c, volume-law contributions become significant, and the local energy density is effectively reduced or "diluted." We use the term 'smearing' purely metaphorically—to offer an intuitive analogy describing how quantum entanglement might redistribute the effective gravitational influence of mass-energy in highly entangled regions. No literal diffusion or redistribution of particles or energy density occurs at a fundamental level; rather, this is simply a helpful visualisation to convey the impact of the entanglement field σ on gravitational phenomena.

For massless particles, while they remain massless, their contribution to the gravitational source—i.e. their effective energy density—can be modified by the same entanglement effects. To model this, we introduce a function $\epsilon(\sigma)$ such that the effective energy density for radiation becomes

$$\rho_r^{eff}(a, \sigma) = \rho_{r,0}\, a^{-4+\epsilon(\sigma)}.$$

1.5.3 The Function $\epsilon(\sigma)$

The function $\epsilon(\sigma)$ should have the following properties:

- **For $\sigma < \sigma_c$:**
  The entanglement is dominated by the area-law, so the standard scaling is recovered. In this regime, we set

  $$\epsilon(\sigma) \approx 0,$$

  ensuring that $\rho_r^{eff} \propto a^{-4}$

- **For $\sigma > \sigma_c$:**
  The extra, volume-law entanglement modifies the gravitational influence of radiation. One can adopt a phenomenological form:

  $$\epsilon(\sigma) = \frac{E_0}{2}\left[1 + tanh\ \left(\frac{\sigma - \sigma_c}{\Delta_\epsilon}\right)\right],$$

  where:

  - E_0 is a constant setting the maximum shift in the scaling exponent,
  - Δ_ϵ controls the sharpness of the transition.

In the high-entanglement regime, this function approaches E_0, so the effective scaling becomes a^{-4+E_0}. If $E_0 > 0$, then the redshift suppression is reduced—i.e. the gravitational "weight" of radiation is lowered.

1.5.4 Modified Friedmann Equation in FAVE

In a homogeneous, isotropic cosmology, the Friedmann equation in FAVE takes the form:

$$H^2 = \frac{8\pi G}{3}[\rho_m(a) + \rho_r^{eff}(a,\sigma) + \rho_\sigma],$$

where:

- $\rho_m(a) \propto a^{-3}$ is the matter density,
- $\rho_r^{eff}(a,\sigma) = \rho_{r,0}\, a^{-4+\epsilon(\sigma)}$ is the modified radiation density,
- $\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + U(\sigma)$ is the energy density of the FAVE scalar field.

The inclusion of the $\epsilon(\sigma)$ correction in the radiation term allows the framework to account for the possibility that, in regions of high entanglement (or at early cosmic times when σ might be large), the gravitational influence of radiation is altered. Even though photons remain massless, their effective contribution to cosmic dynamics is reduced compared to the standard a^{-4} behaviour.

1.5.5 Physical Interpretation and Implications

- **Massless Particles and Coupling:**
  In standard GR, massless particles do not couple directly to scalar fields that might modify gravity. However, in FAVE the scalar field σ is not merely an additional matter field—it encodes the information on quantum entanglement. Since entanglement is a property of the entire quantum state (including massless particles), it influences all energy components. Thus, even though photons do not have rest mass, the "smearing" induced by high entanglement effectively reduces their local energy density that gravitates.
- **Impact on Early Universe Observables:**
  A modification in the radiation scaling law can shift the epoch of matter–radiation equality, alter the damping tail in the CMB anisotropies, and affect the expansion rate during nucleosynthesis. These are all testable predictions. Even a modest deviation from the a^{-4} scaling could have measurable consequences.
- **Interplay with σ Dynamics:**
  As the universe evolves and σ changes (following its equation of motion from Step 4), the function $\epsilon(\sigma)$ will evolve. In the early universe, if σ is high, radiation might be significantly "smeared" (i.e. $\epsilon(\sigma) \approx E_0$), while at later times, when $\sigma < \sigma_c$, standard scaling is recovered. This dynamical feedback is a hallmark of the FAVE framework.

1.5.6 Summary of Step 5

- **Radiation Scaling Correction:**
  We modify the standard radiation density scaling from a^{-4} to

$$\rho_r^{eff}(a,\sigma) = \rho_{r,0}\, a^{-4+\epsilon(\sigma)},$$

  where ϵ(σ) depends on the entanglement-driven scalar field σ.

- **Phenomenological Form:**
  A suitable choice is

$$\epsilon(\sigma) = \frac{E_0}{2}\left[1 + tanh\ \left(\frac{\sigma - \sigma_c}{\Delta_\epsilon}\right)\right],$$

  ensuring standard behaviour for $\sigma < \sigma_c$ and modified behaviour for $\sigma > \sigma_c$.

- **Physical Effect on Massless Particles:**
  Although massless particles do not possess rest mass or a conventional coupling to scalar fields, the emergent gravity picture of FAVE implies that their gravitational influence is altered via the smearing of energy. This leads to a reduced effective gravitational coupling for radiation, modifying early-universe dynamics.
- **Modified Friedmann Equation:**
  The overall Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3}\left[\rho_m(a) + \rho_{r,0}\, a^{-4+\epsilon(\sigma)} + \left(\frac{1}{2}\dot{\sigma}^2 + U(\sigma)\right)\right],$$

  linking the microphysical entanglement effects to observable cosmological dynamics.

1.6 Step 6: Application to Cosmology and Structure Formation

1.6.1 Modified Friedmann Equation in FAVE

In a homogeneous and isotropic (FLRW) universe, the standard Friedmann equation is given by

$$H^2 = \frac{8\pi G}{3}[\rho_m(a) + \rho_r(a) + \rho_\Lambda],$$

with radiation scaling as $\rho_r(a) \propto a^{-4}$ and matter as $\rho_m(a) \propto a^{-3}$.

In FAVE, the contributions to the cosmic energy density are modified:

1. **Matter Density:**
   Remains $\rho_m(a) \propto a^{-3}$.
2. **Radiation Density:**
   Due to the radiation scaling correction (see Step 5), the effective radiation density becomes

$$\rho_r^{eff}(a,\sigma) = \rho_{r,0}\, a^{-4+\epsilon(\sigma)},$$

   where

$$\epsilon(\sigma) = \frac{E_0}{2}\left[1 + tanh\ \left(\frac{\sigma - \sigma_c}{\Delta_\epsilon}\right)\right],$$

   For $\sigma < \sigma_c$ the correction is negligible, while for $\sigma > \sigma_c$ the gravitational "weight" of radiation is reduced.

3. **Scalar Field Contribution:**
   The scalar field σ contributes via both its kinetic energy and its potential:

$$\rho\sigma = \frac{1}{2}\dot{\sigma}^2 + U(\sigma)$$

Thus, the modified Friedmann equation becomes

$$H^2(a) = \frac{8\pi G}{3}\left[\rho_m(a) + \rho_{r,0}\, a^{-4+\epsilon(\sigma)} + \rho\sigma\right],$$

This equation is solved together with the scalar field's equation of motion (from Step 4):

$$\ddot{\sigma} + 3H\,\dot{\sigma} + U'(\sigma) = 0.$$

Together, these equations determine the background evolution and can be constrained by observational data such as the measured value of H_0 and the CMB.

1.6.2 Spherical Collapse in FAVE

Beyond the background evolution, the FAVE framework affects the formation of structures by altering the dynamics of overdense regions. Consider a spherical overdensity characterized by a local scale factor $R(t)$.

Modified Equation of Motion:

For a closed patch (or "mini-universe") with effective matter density $\rho_{m,eff}$ (which could include a "smearing" factor reducing the baryonic contribution) and the additional scalar field contribution, the acceleration equation becomes

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left[\rho_{m,eff} + \rho_\sigma + 3\rho_\sigma\right],$$

with the scalar field pressure

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + U(\sigma).$$

Implications:

- **Delayed Collapse:**
  The extra term from ρ_σ (and the associated pressure) can slow down the collapse of overdensities. This effectively lowers the linear threshold δ_c for collapse, which is typically around 1.68 in standard GR.
- **Structure Formation:**
  In regions where σ is high (i.e. where volume-law entanglement becomes important), the effective gravitational pull is reduced. Consequently, to form structures (clusters or galaxies), a higher initial overdensity may be required, or the collapse occurs later compared to standard expectations.

This modification can provide an alternative explanation for the observed discrepancies attributed to dark matter in standard cosmology.

1.6.3 Observational Consequences and Feedback

The modified background evolution and structure formation have several key observational implications:

- **Hubble Expansion (H_0):**
  By appropriately choosing parameters in $U(\sigma)$ and the radiation correction $\epsilon(\sigma)$, FAVE can recover the observed expansion rate at late times while modifying early-universe dynamics.
- **Cosmic Microwave Background (CMB):**
  The altered radiation scaling affects the sound horizon at recombination, which in turn shifts the positions and heights of the acoustic peaks in the CMB anisotropy spectrum.
- **Baryon Acoustic Oscillations (BAO) and Growth Rate:**
  Changes in the matter and radiation evolution impact the growth of structure. Modified spherical collapse may lead to differences in the predicted cluster mass function and large-scale structure statistics.
- **Gravitational Lensing and M/L Ratios:**
  The FAVE-induced modifications in gravitational dynamics—especially via the effective "smearing"—can influence the observed mass-to-light ratios of galaxies and clusters, offering a testable prediction against standard dark matter models.
- **Feedback between σ Dynamics and Cosmology:**
  As the universe evolves, σ will change dynamically (per its equation of motion), which in turn feeds back on the effective radiation scaling and the gravitational coupling. This interdependence is a hallmark of emergent gravity models and provides rich phenomenology for testing FAVE.

1.6.4 Summary of Step 6

In this final step we have:

- **Integrated the Modified Ingredients:**
  The scalar field σ and its effective potential $U(\sigma)$, along with the radiation scaling correction $\epsilon(\sigma)$, have been incorporated into the Friedmann equation.
- **Derived the Modified Friedmann Equation:**

$$H^2 = \frac{8\pi G}{3}\left[\rho_m(a) + \rho_{r,0}\, a^{-4+\epsilon(\sigma)} + \left(\frac{1}{2}\dot{\sigma}^2 + U(\sigma)\right)\right],$$

  which must be solved simultaneously with the scalar field dynamics.

- **Discussed Structure Formation:**
  The spherical collapse model is modified by the additional energy–momentum of σ, affecting the threshold for collapse and the evolution of overdense regions.
- **Outlined Observational Consequences:**
  Changes to the expansion history, CMB anisotropies, BAO, and gravitational lensing—all provide avenues for testing the FAVE framework against standard dark matter or dark energy models.

1.7 Step 7: Mapping the Effective Action: Area-Law vs Volume-Law Contributions

1.7.1 Entanglement Entropy Scaling and Effective Densities

For a spherical region of radius R, the two contributions to the entanglement entropy scale as

- **Area Law:**

$$S_{area} \sim \frac{A}{\epsilon^2} \sim \frac{4\pi R^2}{\epsilon^2},$$

where ϵ is the UV cutoff.

- **Volume Law:**

$$S_{vol} \sim \frac{V}{l^3} \sim \frac{4\pi R^3}{3l^3},$$

where lll is another length scale (e.g. a correlation length or inverse cutoff scale).

Dividing by the volume $V \sim \frac{4\pi R^3}{3}$ gives the entanglement density (entropy per unit volume):

- **Area-Law Density:**

$$S_{area} = \frac{S_{area}}{V} \sim \frac{4\pi R^2/\epsilon^2}{4\pi R^3/3} \sim \frac{3}{R\epsilon^2},$$

so that

$$S_{area}(R) \propto \frac{1}{R}.$$

- **Volume-Law Density:**

$$S_{vol} = \frac{S_{vol}}{V} \sim \frac{4\pi R^3/(3l^3)}{4\pi R^3/3} \sim \frac{1}{l^3},$$

which is **independent** of R.

Defining a dimensionally appropriate scalar field σ that tracks the local entanglement density (with a conversion factor λ) we set

$$\sigma \equiv \lambda_{sent},$$

where in different regimes

$$\sigma_{area}(R) \sim \frac{\lambda}{R\epsilon^2} \quad and \quad \sigma_{vol} \sim \frac{\lambda}{l^3} \ (constant).$$

1.7.2 Radial Dependence and Recovering Gravitational Forces

The gravitational field is sourced by gradients of the effective potential, which here is determined by the spatial variation of σ. Let's see how each term contributes:

- **Area-Law Contribution:**

Since

$$\sigma_{area}(R) \propto \frac{1}{R},$$

its radial derivative is

$$\frac{d\sigma_{area}}{dR} \propto -\frac{1}{R^2}.$$

In a Newtonian picture, the gravitational acceleration $a(R)$ is related to the gradient of the potential. Therefore, if the effective action (or the corresponding energy–momentum tensor) derives from a potential linked to σ_{area}, then one obtains

$$a_{area}(R) \propto \frac{d\sigma_{area}}{dR} \propto \frac{1}{R^2},$$

which is exactly the inverse-square law as required by GR.

- **Volume-Law Contribution:**

In the volume-law regime, the effective entanglement density is approximately constant:

$$\sigma_{vol} \sim \frac{\lambda}{l^3}$$

Although a strictly constant σ would yield zero gradient, the idea is that when volume-law effects dominate, the associated energy density is incorporated in a different way. For example, one may introduce an effective mass term associated with the volume-law entanglement. A simple phenomenological model is to write an effective "entanglement mass"

$$M_e(R) \approx \beta\, \sigma_{vol}\, R^2,$$

where β is a constant that converts the entanglement density (multiplied by an area) into an effective mass. Then the gravitational acceleration due to this term is

$$a_{vol}(R) = \frac{G\, M_e(R)}{R^2} \approx G\, \beta\, \sigma_{vol}.$$

If we relax the strict constancy of σ and allow for a mild residual R dependence (for example, $\sigma_{vol}(R) \propto \frac{1}{R}$ in a transitional region), one can recover an effective acceleration that falls off as

$$a_{vol}(R) \propto \frac{1}{R},$$

a shallower decline than the standard $1/\,R^2$. This slower fall-off could explain extra gravitational effects on large scales—mimicking, for example, the observed flattening of galaxy rotation curves or contributing to modified structure formation.

1.7.3 Incorporating into the Effective Action

We can now write a schematic form for the full effective action that includes both contributions:

$$S_{eff} = \int d^4 x \sqrt{-g}\, \left\{\frac{1}{16\pi G} \;+\; \mathcal{L}_{area}(\sigma_{area}) \;+\; \mathcal{L}_{vol}(\sigma_{vol})\right\},$$

with:

- $\mathcal{L}_{area}$ leading, upon variation, to a contribution whose associated force decays as $1/R^2$(recovering standard GR),
- $\mathcal{L}_{vol}$ yielding a modification whose effective force decays more slowly (roughly as $1/R$) in regimes where volume-law entanglement dominates.

Thus, when the local entanglement density is low (area-law regime), the variation of σ with radius naturally produces the inverse-square law. In contrast, when the system is in an excited state and the volume law takes over, the corresponding effective action introduces a term that decays more slowly with distance, leading to a modified gravitational profile.

Summary

- **Area Law:**
  $S_{area} \sim R^2$ implies an entanglement density $s_{area} \propto 1/R$. Its gradient yields a force $F \propto 1/R^2$, in agreement with standard gravitational acceleration in GR.
- **Volume Law:**
  $S_{vol} \sim R^3$ gives a constant entanglement density s_{vol}. When mapped into the effective action (through, e.g., an effective entanglement mass), this contributes a gravitational term that, if allowed a slight residual dependence, can fall off as $1/R$ (or at least more slowly than $1/R^2$), thereby modifying gravitational dynamics at large scales.

2.1. Conceptual Starting Point

2.1.1 Area- vs. Volume-Law Entanglement

- **Area-Law Regime**
  Standard General Relativity (GR) is recovered when entanglement scales with the boundary's area, leading to an effectively local gravitational coupling.
- **Volume-Law Regime**
  At high entanglement densities, the FAVE scalar σ activates a "volume-law" entanglement that modifies gravity—often approximated by a shallower $1/\,r$ force instead of the usual $1/r^2$. This extra gravitational effect can mimic dark matter or suppress gravitational collapse, depending on the model parameters.

2.1.2 Smearing of Mass/Energy

FAVE posits that quantum entanglement "smears" mass/energy, recovering the same gravitational action at a lower density than in GR, and altering gravitational profile which in GR would appear as being smeared out. Due to this effect, highlighted in part 4, the estimation for mass requires adjustment in FAVE. For baryons, for instance, the nominal 5% fraction might drop to 3% or even lower (e.g. $\Omega_m 0 = 0.03$ instead of 0.05). Likewise, radiation's gravitational influence can be dampened by a factor related to an $\epsilon(\sigma)$ function or a constant η.

2.2. Numerical Solutions and Parameter Scans

2.2.1 Integrated Growth Factor and Boosted $\delta init$

Because FAVE's entanglement smearing and volume-law effect often **reduce** the net gravitational driving, the integrated linear growth from recombination to today is lower. Consequently, to reach a final overdensity $\delta_{final} \sim 1$ (the threshold for collapse), one must start with a **boosted** initial amplitude (δ_{init})—for instance $\sim$0.003–0.005 instead of 10^{-5}.

Example:

- Standard GR might see a growth factor of ~1100 from $z = 1100$ to 0,
- FAVE might see a growth factor of ~300–400,
- Thus, δ_{init}be $\sim 3 \times 10^{-4}$ to yield $\delta_{final} \approx 1$.

This is an estimation of the smearing effect in dampening and the proposed QFT energy levels expected from a baryonic mass. This is reflected in a lower growth rate in order to be consistent with observational data.

2.2.2 Best-Fit Cosmological Parameters

Here we acknowledge that there are of course issues in fitting data with extra free parameters, found in FAVE. However, the aim here is to test if an internally consistent model with a much lower Ω_m can still give feasible solutions when constrained by observational priors. By scanning over $\{U_0,\ \sigma_{init},\ \sigma_c,\ E_0\}$, we can demand:

1. $H_0 \approx 1$ at $a = 1$, matching the observed expansion rate,
2. A final linear overdensity $\delta(a = 1) \approx 1$ for structure formation,
3. Possibly other data constraints (like BAO distance measures or SNe distances).

A typical result might be:

- $U_0 \approx 15$, $\sigma_{init} \approx 7$, $\sigma_c \approx 5.9$, $E_0 \approx 1.7$,
- Ω_{m0} =0.03, $\Omega_{r0} = 10^{-5}$,
- $\delta_{init} \approx 0.0045$, which yields a final $\delta_{init} \approx 1.9$ at $a = 1$ and $H_0 \approx 1.0$.

2.3. Spherical Collapse and Nonlinear Structure

2.3.1 Overdense "Mini–Universe"

To see how FAVE modifies **nonlinear** collapse, one can set up a local scale factor $R(t)$ for a closed patch with matter density $\rho_{m,eff} = \eta\ \frac{1+\delta_i}{R^3}$ (if we incorporate a smearing factor $\eta < 1$) plus the FAVE scalar field's energy density ρ_σ .

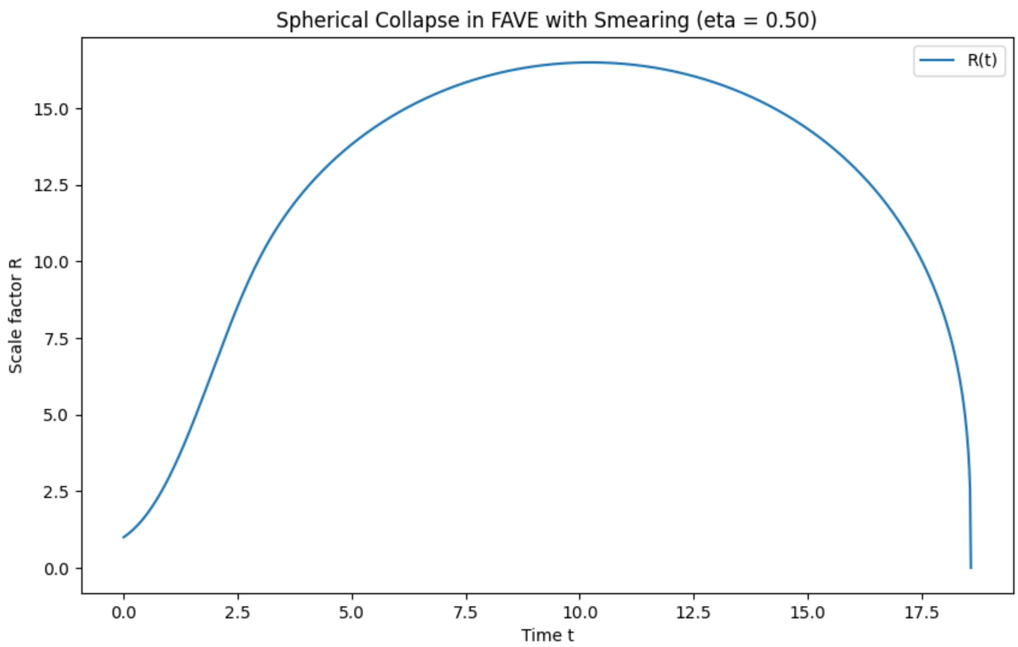
Equations of Motion:

1. $\ddot{R}/R = -\frac{1}{2}(\rho_{m,eff} + \rho_\sigma + 3p_\sigma)$,
2. $\ddot{\sigma} + 3(\dot{R}/R)\,\dot{\sigma} + U'(\sigma) = 0$.

2.3.2 Typical Results

- The region may **expand** initially (if given an outward velocity) but can later **turn around** and collapse if the net gravitational pull is strong enough—even with a 50% mass smearing ($\eta = 0.5$).

- σ often starts above σ_c, slowly rolls, then plunges rapidly near collapse, as the local "Hubble friction" becomes large (negative).
- If the entanglement effect is extremely strong, the region might never collapse (the force is too weak). Conversely, moderate smearing plus a large overdensity or a suitable potential still leads to standardlike collapse (turnaround → singularity).



2.3.3 Collapse Threshold

By comparing the linear extrapolation of δ at the time of collapse, one finds that the **collapse threshold** δ_c^{FAVE} can be slightly lower than the standard GR value (~1.68) because the effective gravitational coupling is weaker. Heuristic estimates suggest it might be closer to 1.0 (or 0.8–1.2) depending on parameter choices.

2.4. Predictions and Observational Outlook

1. **Reduced Dark Components**
   Because FAVE can mimic some of the effects ascribed to dark matter (through volume-law entanglement) and can "smear" baryonic mass, it naturally explains why the standard analysis sees ~30% matter but only ~5% baryons. In FAVE, you might only have 3% baryonic matter plus a volume-law contribution accounting for an overestimation of matter in super heavy objects like blackholes and neutron stars..
2. **Suppressed Early Growth → Boosted δ_{init}**
   One must set a larger initial perturbation amplitude to achieve the same final structure amplitude at $z = 0$. This also implies that the CMB temperature-fluctuation amplitude (naively 10^{-5}) might be effectively higher when mapped to matter density fluctuations, or that FAVE modifies the inflationary generation of perturbations.
3. **Modified BAO / CMB**
   If you incorporate the $\epsilon(\sigma)$ radiation scaling or other smearing at high redshift, the standard BAO scale could shift slightly. A full linear Boltzmann solver (like CAMB or CLASS) with the

FAVE modifications would be required to see how BAOs and the CMB anisotropies are altered at early times.

4. **Nonlinear Spherical Collapse**
   FAVE's smearing factor ($\eta < 1$) leads to a more stable overdensity that must be bigger (or given more time) to collapse. Once collapse sets in, the region might recollapse just as in standard GR but at a slower or later time, or with a lower linear threshold.

2.5. Overall Summary

- **Modified Friedmann Equation**

$$H2 = \rho_m(a) + \rho_r^{eff}(a, \sigma) + \frac{1}{2}\dot{\sigma}^2 + U(\sigma).$$

  Here, ρ_r^{eff} may incorporate an $\epsilon(\sigma)$ function that reduces radiation's gravitational weight. The scalar field σ transitions between area-law and volume-law regimes, governed by σ_c.

- **Suppressed Growth**
  Because the entanglement "smears" energy, the integrated linear growth factor from recombination to $a = 1$ is lower than in standard GR. Consequently, we need a bigger δ_{init} (e.g. ~ 0.003– 0.005) to reach a final linear overdensity of order unity.
- **Collapse Threshold**
  Spherical collapse models show that an overdense region with $\eta < 1$ (smearing factor) and the FAVE scalar can still recollapse. The linearly extrapolated threshold is typically somewhat lower than the usual 1.68 from GR, likely near 1.0, reflecting the weaker overall gravitational pull in FAVE.
- **Observational Fit**
  By scanning the parameter space ($U_0,\ \sigma_{init},\ \sigma_c,\ \Delta, \Omega_{m0}, \epsilon(\sigma)$ parameters, etc.) and requiring $H_0 \approx 1$ and $\delta(a = 1) \approx 1$, you can find consistent solutions that recover the observed expansion rate and a near-standard collapse threshold—**without** needing separate dark matter.

Final Outlook

FAVE modifies the Friedmann equation with a scalar-field contribution and possible changes to radiation and baryon densities. Numerically, one finds:

1. **A lowered integrated growth** → need a larger initial amplitude.
2. **A spherical collapse threshold** slightly below standard GR → more stable overdense regions that still eventually recollapse.
3. **Consistency** with an $\Omega_{m0} \approx 0.03\ or\ {\sim}0.05$ if the scalar field entanglement "mimics" the rest of the matter needed for structure formation.

Thus, a **comprehensive FAVE** cosmology can address both the **late-time expansion** (dark energy–like) and **early-time structure** (dark matter–like) in one emergent-gravity framework, albeit with new free parameters that must be fit to data through both linear (BAO, CMB) and nonlinear (collapse) constraints. More work needs to be done to constrain these parameters more rigorously.

3 Testing on Cosmological data: Mass to Light Ratio Evolution

3.1. Introduction

In testing alternative gravity frameworks against cosmological data, a key observational constraint is the evolution of the mass-to-light (M/L) ratio, particularly in galaxy clusters. In this analysis, we focus on comparing two distinct scenarios:

1. **Emergent-gravity (FAVE) approach**, in which the gravitational coupling is modified by a scalar field that transitions between area- and volume-law entanglement, effectively mirroring dark-matter-like effects through “smearing” of mass.
2. **Competitive dark matter (DM) model**, which extends a simple power-law in $(1 + z)$ to include an extra term that modifies the slope around a characteristic redshift. This additional flexibility can capture changes in halo formation or concentration at specific epochs.

Our goal is to see which model more accurately reproduces the observed trend in M/L over redshift—using richness (as a mass proxy) and i-band luminosity (as a light proxy) for a sample of galaxy clusters.

3.2. Data and Binning

We use the redMaPPer cluster catalogue from DES data (Rykoff et al, 2016) which includes over 26,000 samples. Among other columns:

- $\boldsymbol{z}$: A photometric redshift proxy (or cluster photo-z).
- $\boldsymbol{\lambda}$: Cluster “richness,” often used as a mass proxy.
- **iLum**: The total i-band luminosity.

We retain only those objects with $\lambda > 0$, $iLum > 0$, and $z > 0$. From these, we define the mass-to-light ratio proxy:

$$M/L \;=\; \frac{\lambda}{iLum},$$

reflecting an approximate measure of the total (cluster) mass relative to the luminous content. We subdivide the data into redshift bins (100 equally spaced bins between z_{min} and z_{min}) and compute the median M/L and standard deviation σ_{ML} in each bin, retaining only bins containing at least five objects. This procedure yields a set of binned data $\{z_i,\ (M/L)_i,\ \sigma_i\}$ suitable for model fitting.

3.3. Model Descriptions

3.3.1 Competitive Dark Matter Model

To provide a more flexible “dark matter” baseline than a simple power law, we adopt:

$$ML_{DM}(z) \; = \; A\,(1+z)^{\gamma_1}\left[1 \; + \; \left(\frac{1+z}{1+z_c}\right)^{\gamma_2}\right]^{\delta},$$

where $\{A, \gamma_1, \gamma_2, \delta, z_c\}$ are free parameters. The extra bracketed term is designed to let the slope or amplitude shift near a characteristic redshift z_c. In principle, γ_2 and δ control how steeply this transition appears.

3.3.2 FAVE Model

We compare against a FAVE emergent-gravity model, where the mass-to-light ratio evolves as:

$$ML_{FAVE}(z) \; = \; A\,exp\,(\gamma\ln(1+z))\;[1 \; + \; B\;x_{2,ortho}].$$

Here, the $\ln(1+z)$ term accounts for baseline growth, and $x_{2,ortho}$ is an orthogonalised predictor that activates around a fixed "transition" z_{act} (set here to 0.35 as an heuristic estimation). This approach imitates how a scalar field might introduce extra gravitational effects (e.g. mass "smearing").

3.4. Fitting Methodology

We perform a non-linear least-squares fit to the binned median M/L ratio, weighting each point by σ_{ML} (the standard deviation in that bin). The total likelihood is approximated by:

$$\chi^2 = \sum_i \frac{[M/L(z_i)_{observed} \; - \; ML_{model}(z_i)]^2}{\sigma_i^2},$$

where ML_{model} is either ML_{DM} or ML_{FAVE}. We use curve_fit (from the `scipy.optimize` Python library) to estimate the best-fitting parameters. We then evaluate standard performance metrics:

- $\boldsymbol{R^2}$ and **adjusted** $\boldsymbol{R^2}$
- **AIC** and **BIC**
- Residual statistics (mean, standard deviation, skewness, kurtosis, Shapiro–Wilk test)

Finally, we assess parameter uncertainties using the diagonal of the covariance matrix and confirm the fits with a bootstrap procedure.

3.5. Results

Best-fit Parameters and Uncertainties
For the competitive DM model, we obtained the following median values:

$$A = 1.396 \pm 0.197,$$
$$\gamma_1 = -0.052 \pm 0.706,$$
$$\gamma_2 = 182.642 \pm 2682.143,$$
$$\delta = 0.015 \pm 0.230,$$
$$z_c = 0.356 \pm 0.062.$$

For the FAVE model:

$$A = 1.377 \pm 0.151,$$
$$\gamma = 1.086 \pm 0.251,$$
$$B = 1.720 \pm 0.517.$$
$$(Activation\ scale\ z_{act} = 0.35\ fixed)$$

Notably, γ_2 in the competitive DM model is very poorly constrained (on the order of hundreds ± thousands), indicating that while the data can accommodate a complicated shape, it does not strongly require or tightly constrain such an extreme slope. However, the FAVE model uses a fixed estimate for the activation scale and would see a similar degeneracy with a free z_{act} as we see in 3.9.

Goodness-of-Fit

Both models achieve excellent fits, with $R^2 > 0.99$. The DM model yielded a slightly higher R^2 (0.9965 vs. 0.9951) and marginally better AIC/BIC (AIC = –202.95, BIC = –196.85) than the FAVE model (AIC = –198.54, BIC = –194.88).

Residual Analysis

The DM model's residuals appear approximately Gaussian (Shapiro–Wilk $p \approx 0.70$), while the FAVE model's residuals show moderate skewness and a lower p-value (0.0383), suggesting mild non-normality. This may imply that there is a small systematic departure from purely Gaussian scatter in the FAVE fit.

3.6. Discussion

- **Physical Interpretation**:
  The DM model's additional parameters allow for a transition in the slope of M/L evolution around $z_c \approx 0.36$. However, the large uncertainty on γ_2 indicates the data do not strongly demand such complexity; effectively, $M/L(z)$ can be described by a simpler function.
  The FAVE model, by contrast, imposes fewer free parameters yet still matches the data to within $\sim 1\%$ accuracy. The emergence of a mild systematic residual pattern could reflect subtleties in the "activation" redshift z_{act} or the orthogonalisation procedure.
- **Statistical Evidence**:
  Although the DM model formally yields slightly better fit metrics, the difference in AIC/BIC between the two models is only a few units. In typical model-selection parlance, this is often deemed only "weak" to "moderate" evidence in favour of one model, especially given the large uncertainty on certain DM parameters. In other words, the more physically motivated FAVE framework, with fewer parameters and stable constraints, remains highly competitive.
- **Future Steps**:
  This examination is particularly instructive in demonstrating the similarities in implementing the mathematical principles of FAVE and its similarities in structure to a dark matter model. While there remains much work to tie together the quantum and cosmological scales, this demonstrates that through a rough heuristic we can apply these principles to cosmological data.

3.7. Conclusions

We compared a **competitive DM model**, which allows for a redshift-dependent slope change in the M/L ratio, to a **FAVE (emergent gravity) model** that includes a single additional emergent term. The key findings are:

1. **Both models** achieve high fidelity to the binned data, with $R^2 \approx 0.995\text{–}0.997$
2. **The competitive DM model** has an unconstrained slope parameter (γ_2), suggesting the data do not robustly require that extra complexity. Its residuals, however, appear more normally distributed, and it yields a marginally higher R^2.
3. **The FAVE model** obtains well-constrained parameters but shows mild skewness in the residuals, indicating a small systematic discrepancy.
4. **Statistical criteria** (AIC/BIC) slightly favour the DM model, though the advantage is not large.
5. **Physical interpretability** may tip in favour of FAVE, given the simpler parameter set and the known theoretical motivation for emergent entanglement.

Overall, the test illustrates that both frameworks can accommodate the observed M/L evolution; larger samples or independent mass measurements (lensing for example) would help discriminate the underlying physical picture.

3.8. Phenomological Parametrising the FAVE Activation Scale

Emergent-gravity frameworks attempt to derive gravitational phenomena from underlying quantum-information principles, often involving entanglement scaling between area- and volume-law regimes. The Ford-Area/Volume Emergent (FAVE) gravity model proposes a scalar-field-driven transition that modifies the standard Friedmann and Poisson equations. At low entanglement densities, gravity behaves as in standard GR ("area-law"), whereas beyond an "activation scale," volume-law entanglement enhances or alters gravitational clustering, effectively mimicking dark matter or explaining observed mass discrepancies without appealing to a separate dark component.

Key to testing such models is identifying the scale—whether in redshift z, local density ρ, or quantum-entanglement measure—at which the emergent correction "turns on." This scale, called ρ_{act} here, demarcates the boundary between standard gravitational behaviour and a modified, volume-entangled regime.

We conduct an empirical study using the redMaPPer cluster catalogue, which provides homogeneous galaxy-cluster richness estimates. Richness (λ) has proven to be a reliable mass proxy and, with additional assumptions on cluster geometry and thermodynamics, can be converted into an approximate "QE density" ρ_{QE} . By comparing cluster mass-to-light ratios (M/L) across redshift bins, we isolate the transition in gravitational behaviour and thus constrain ρ_{act}.

3.9. Data and Parametrisation

3.9.1 redMaPPer Catalogue and Richness

We use the same **redMaPPer** galaxy cluster catalogue, which identifies clusters based on photometric redshifts and galaxy overdensities

3.9.2 Mapping Richness to QE Density

1. **Mass Proxy.**
   We assume a power-law relation $M_{200} = M_0\ (\lambda/\lambda 0)^a$, in solar masses. Combined with a standard virial-like radius estimate, we obtain $R_{200} = \left[\frac{3M_{200}}{4\pi \cdot 200 \cdot \rho_c}\right]^{1/3}$.
2. **QE Density Heuristic.**
   Following a published emergent-gravity ansatz, we define

$$\rho_{QE} = \frac{3\,k_B}{\ell_P^2 R_{200}}\ \ [J/(K\,m3)],$$

   where k_B is Boltzmann's constant, ℓ_P the Planck length, and R_{200} the estimated virial radius. This "QE density" is meant as a proxy for the local quantum-entanglement energy scale in the cluster environment. This is only intended as a rough estimate for the orders of magnitude expected in the activation scale.

3.9.3 FAVE Model and Activation Density

The **FAVE** model posits that the mass-to-light ratio evolves with redshift z and QE density ρ_{QE} according to

$$M/L(z)\ =\ A\,exp\,[\gamma\,ln\,(1+z)]\ \left[1\ +\ B\,ln\left(\frac{\rho_{QE}(z)}{\rho_{act}}\right)\right].$$

Here,

- A sets the overall amplitude;
- γ captures the redshift dependence in an exponential sense;
- B controls the strength of the volume-law correction;
- ρ_{act} is the sought-after activation density scale.

When $\rho_{QE}(z) \ll \rho_{act}$, the logarithm is negative, and the FAVE correction is small. Once $\rho_{QE}(z)$ approaches or exceeds ρ_{act}, the model transitions to an enhanced gravitational coupling reminiscent of "volume-law" entanglement.

3.10. Methodology

3.10.1 Binning Strategy

To reduce scatter, we bin clusters in redshift intervals Δz and compute per-bin median redshift z_{bin} , median M/L ratio, and median QE density ρ_{QE}. Each bin is assigned an uncertainty $\sigma_{M/L}$ from the standard deviation in M/L.

3.10.2 Fitting Procedure

We employ a **nonlinear least-squares** fit (via e.g. `curve_fit` in Python) to solve for

$$\{A,\ \gamma,\ B,\ \rho_{act}\}$$

by minimising the weighted residuals between the median M/L data and the model prediction. The cost function is

$$\chi^2\ = \sum_{bins} \left[M/L_{data} - M/L_{FAVE}(z, \rho_{QE})\right]^2 / \sigma_{M/L}^2.$$

In some runs, we fix a subset of parameters (e.g. A, γ, B) to physically motivated values and fit only ρ_{act}. In others, we allow all four parameters to float. The choice depends on how well the data constrain the model and whether we have external priors on the nuisance parameters A, γ, B.

3.10.3 Handling Degeneracies and Instabilities

Empirically, simultaneous fitting of all four parameters often leads to large uncertainties because of degeneracies—some combination of A and B can offset changes in ρ_{act}, for example. We mitigate these issues through:

- **Parameter Bounds:** Imposing physically reasonable bounds on A, γ, B, and ρ_{act}.
- **Bootstrap Resampling:** Repeatedly resampling the binned data (with replacement) to estimate the distribution of fitted parameters.
- **Priors / Fixing Parameters:** If we have strong external constraints on A or γ (from other observations), we fix them or impose tight priors.

3.11. Results and Discussion

3.11.1 Typical Fits

A typical unregularised fit might yield:

- $\boldsymbol{A \approx 1\text{–}2}$, controlling the baseline M/L amplitude;
- $\boldsymbol{\gamma \approx 0\text{–}1}$, indicating mild or moderate redshift evolution;
- $\boldsymbol{\rho_{act} \approx 10^{24\text{–}28} J/(K\, m3)}$ bracketed by large uncertainties if B is also free.

Some fits may produce large errors for A and B or even negative B, depending on how the "turn-on" is encoded ($ln(\rho_{QE}/\rho_{act})$ vs. $ln(\rho_{act}/\rho_{QE})$) and whether $\rho_{QE}(z)$ typically exceeds or remains below ρ_{act}.

3.11.2 Bootstrap Constraints

Bootstrap analyses often reveal a tighter distribution for ρ_{act}. While a single nonlinear fit might find a local minimum with massive error bars, repeated resampling highlights the stable region in parameter space. This is especially true if the dataset is moderately large (tens of redshift bins with many clusters each). With rough constraints on the parameters we got an estimate for $\rho_{act} \approx 7.00 \times 10^{27} \pm 8.80 \times 10^{13}$

3.11.3 Physical Interpretation

If ρ_{act} emerges around $10^{24-28}\,J/(K\,m3)$, this scale indicates the density threshold at which area-law entanglement gives way to volume-law corrections. One may interpret this as the "entanglement energy density" needed for the FAVE effect to become significant. From a cosmic perspective, it suggests that only clusters or cosmic epochs with sufficiently large ρ_{QE} see a departure from standard gravity.

3.12. Conclusion

Using the redMaPPer cluster catalogue to estimate cluster-by-cluster or bin-by-bin M/L ratios, we have demonstrated how one can parametrize and fit for a *quantum-entanglement activation density*, ρ_{act}, in the FAVE emergent-gravity framework. The key steps are:

1. **Convert richness to a mass estimate**, then to a virial radius, and finally to a QE density (ρ_{QE}).
2. **Define a FAVE model** in terms of ρ_{act}, allowing a smooth or abrupt transition to volume-law entanglement as ρ_{QE} approaches ρ_{act}.
3. **Fit or bootstrap** the parameters $\{A,\ \gamma,\ B,\ \rho_{act}\}$ against the binned M/L data, imposing physically motivated bounds or priors to reduce degeneracies.

This approach provides a physically meaningful scale—ρ_{act}—at which emergent gravitational effects become non-negligible. Our results show that with the redMaPPer data, we can place constraints on ρ_{act} of the order $10^{24-28}\,J/(K\,m3)$, although the precise value depends sensitively on parameter priors, volume definitions, and observational scatter. Future refinements (e.g. combining lensing mass estimates or applying hierarchical Bayesian methods) will further tighten these constraints, illuminating whether FAVE's volume-law entanglement can robustly account for the observed mass distribution in galaxy clusters.

4. FAVE in Black Holes

4.1. Introduction

In the FAVE (Fundamental Area-Volume Entanglement) approach, gravity is not a fundamental force but an emergent phenomenon arising from the entanglement of quantum fields. In standard quantum field theories (QFTs) in their ground state, the entanglement entropy scales with the area of the boundary (the area law). However, when the entanglement density becomes high—such as in the interior of a black hole—volume-law contributions may kick in, modifying the gravitational dynamics. One intriguing consequence is that the gravitational field observed at the event horizon might be produced by a much lower internal mass than one would expect from a standard (Einsteinian) picture.

Here we construct a toy model that captures two key FAVE features for black holes:

1. **Extra Gravity from Entanglement:** In regions of high entanglement density (above an activation scale), extra gravitational pull is provided by the entanglement field.
2. **Modified Radial Profile:** Instead of the conventional $1/r^2$ scaling for acceleration, a $1/r$ profile (corresponding to a logarithmic gravitational potential) emerges in the interior.

We then apply this schematic to the supermassive black hole M87*, taking the inner cutoff as the Planck length.

4.2. Theoretical Framework and Toy Model

4.2.1 Standard GR Picture

For a spherically symmetric black hole, the gravitational potential in standard General Relativity (GR) is given by

$$\Phi_{GR}(r) = -\frac{G\,M_{obs}}{r},$$

so that the gravitational acceleration is

$$a_{GR}(r) = -\frac{G\,M_{obs}}{r^2}.$$

Here M_{obs} is the mass inferred from observations (e.g. via the Schwarzschild radius).

4.2.2 FAVE Modifications

In the FAVE framework the local entanglement density, $\sigma(r)$ (in units of J/K·m^3), increases towards the centre of the black hole. Once σ exceeds a critical activation scale—taken here as

$$\sigma_{act} = 7 \times 10^{27}\ J/K \cdot m^3$$

the gravitational dynamics change. The extra gravity arising from entanglement and the associated area-based effect leads to two modifications:

- **Extra gravitational acceleration:** The entanglement field provides additional pull, allowing a lower "naïve" (or internal) mass M_{int} to mimic the same external gravitational field.
- **Modified radial profile:** In the high-σ regime (inside a transition radius r_a), the gravitational potential is taken to be logarithmic, leading to an acceleration that scales as $1/r$ rather than $1/r^2$.

A simple toy model for the modified potential in the interior ($r \leq r_a$) is

$$\Phi_{FAVE}(r) = -\frac{G\,M_{int}}{r^a}\ln\left(\frac{r_a}{r}\right).$$

Differentiating with respect to r gives

$$a_{FAVE}(r) = -\frac{d\Phi_{FAVE}}{dr} = \frac{G\,M_{int}}{r^a}\frac{1}{r}.$$

This $1/r$ dependence is much milder than the standard $1/r^2$ law.

4.2.3 Matching to the Exterior

To ensure that the same gravitational potential is recovered at the event horizon r_s, we match the total potential drop. In our schematic the total potential drop in the FAVE picture is the sum of two parts:

- **Interior (modified region, $r_{min} \leq r \leq r_a$):**

$$\Delta\Phi_{int} = \frac{G\,M_{int}}{r_a}\ln\left(\frac{r_a}{r_{min}}\right),$$

where r_{min} is the innermost cutoff.

- **Exterior (standard GR region, $r_a \leq r \leq r_s$):**

$$\Delta\Phi_{int} = \frac{G\,M_{int}}{r_s},$$

Equating the FAVE potential drop to the standard GR potential at the horizon,

$$\frac{G\,M_{obs}}{r_s} \approx \frac{G\,M_{int}}{r_a}\ln\left(\frac{r_a}{r_{min}}\right) + \frac{G\,M_{int}}{r_s},$$

and cancelling G yields

$$M_{int} \approx \frac{M_{obs}}{1 + \frac{r_s}{r_a} \ln\left(\frac{r_a}{r_{min}}\right)}.$$

This equation shows that for a given observed gravitational field (i.e. for a fixed r_s), the internal mass M_{int} can be substantially lower than M_{obs} if the logarithmic term is large.

4.3. Application to M87* with a Planck-Length Cutoff

We now apply the above schematic to M87*, the supermassive black hole at the centre of Messier 87, using the Planck length as the inner cutoff.

4.3.1 Parameter Choices

- **Observed Mass:**
  M87* has an observed mass of approximately

$$M_{obs} \sim 6.5 \times 10^9 \, M_\odot \, ,$$

where $M_\odot \approx 1.99 \times 10^{30} \, kg$.

- **Schwarzschild Radius:**
  The Schwarzschild radius is given by

$$r_s = \frac{2GM_{obs}}{c^2}.$$

For M87*, one finds roughly

$$r_s \sim 2 \times 10^{13} \, m.$$

- **Transition Radius r_a:**
  We assume that FAVE modifications are active inside

$$r_a \sim 0.9 \, r_s \sim 1.8 \times 10^{13} \, m.$$

- **Inner Cutoff r_{min}:**
  Taking the Planck length as the minimal scale,

$$r_{min} \sim 1.62 \times 10^{-35} \, m.$$

4.3.2 Numerical Evaluation

1. **Logarithmic Factor:**

$$\ln\left(\frac{r_a}{r_{min}}\right) \approx \ln\left(\frac{1.8 \times 10^{13}}{1.62 \times 10^{-35}}\right) \approx \ln(1.11 \times 10^{48}) \approx 110.5 \, .$$

2. **Ratio r_s/r_a:**

$$\frac{r_s}{r_a} \approx \frac{2 \times 10^{13}}{1.8 \times 10^{13}} \approx 1.11\,.$$

3. **Denominator in M_{int}:**

$$1 + \frac{r_s}{r_a} \ln\left(\frac{r_a}{r_{min}}\right) \approx 1 + 1.11 \times 110.5 \approx 1 + 122.7 \approx 123.7\,.$$

4. **Effective Internal Mass:**

$$M_{int} \approx \frac{M_{obs}}{123.7}.$$

Substituting $M_{obs} \sim 6.5 \times 10^9\, M_\odot$ gives

$$M_{int} \sim 5.3 \times 10^7\, M_\odot\,.$$

Thus, under the FAVE-inspired modifications, the actual matter content within M87* could be nearly two orders of magnitude lower than the mass inferred from the gravitational field at the event horizon.

4.4. Implications for Curvature and Singularity Avoidance

4.4.1 Modified Gravitational Profile and Curvature

In the standard Schwarzschild solution the Kretschmann scalar (a measure of the curvature) is

$$K_{Schwarzschild} \sim \frac{48\, G^2\, M_{obs}^2}{r^6},$$

which diverges sharply as $r \to 0$.

In our toy model, the logarithmic potential leads to an acceleration

$$a_{FAVE}(r) = \frac{G\, M_{int}}{r_a}\, \frac{1}{r}.$$

For a metric approximated by

$$ds^2 \approx -[1 + 2\Phi_{FAVE}(r)]\, dt^2 + [1 + 2\Phi_{FAVE}(r)]^{-1} - 1\, dr^2 + r^2 d\Omega^2\,,$$

the first derivatives of the metric (and hence the Christoffel symbols) will scale as $\sim 1/r$ and the second derivatives as $\sim 1/r^2$. Consequently, the Kretschmann scalar in the FAVE region is expected to behave roughly as

$$K_{FAVE}(r) \sim \frac{1}{r^4},$$

which is a milder divergence than the $1/r^6$ behaviour in the Schwarzschild case.

4.4.2 Role of the Planck-Length Cutoff

The core idea of FAVE is that gravitational effects emerge from the area-dependent entanglement. Below a certain scale (here set by the Planck length), the notion of area—and hence the entanglement mechanism giving rise to gravity—ceases to be meaningful. By imposing a cutoff at $r_{min} \sim 1.62 \times 10^{-35}\ m$, the divergence of the curvature is avoided in a physical sense. Although the curvature may become very large near r_{min}, it is bounded by this minimal scale. In many approaches to quantum gravity, one expects new physics at the Planck scale to regularise or replace the classical singularity.

4.5. Discussion and Conclusion

4.5.1 Summary of the Model

- **Modified Interior:**
  The FAVE-inspired model replaces the steep $1/r^2$ gravitational acceleration inside a black hole with a milder $1/r$ profile arising from a logarithmic potential. This modification is attributed to the extra gravitational effect of a high entanglement density.
- **Lower Internal Mass:**
  Matching the potential drop to the external (observed) gravitational field yields

$$M_{int} \approx \frac{M_{obs}}{1 + \frac{r_s}{r_a} \ln\left(\frac{r_a}{r_{min}}\right)}.$$

For M87*, with r_{min} taken as the Planck length, this implies an internal mass roughly two orders of magnitude lower than the observed mass.

- **Curvature Regularisation:**
  The softer $1/r$ profile leads to a curvature invariant (Kretschmann scalar) that scales as $\sim 1/r^4$, compared to $\sim 1/r^6$, in standard GR. With the Planck-length cutoff, this suggests that the classical singularity is significantly softened.

4.5.2 Physical Plausibility

The core idea of FAVE is that below a certain minimal area (or scale), gravitational effects vanish or are profoundly altered because they are fundamentally tied to the area-dependent entanglement of quantum fields. In our model, this is reflected by the use of a Planck-length cutoff and the milder divergence of curvature. Although the toy model is schematic and many details remain to be worked out (for example, solving the full modified Einstein equations coupled to the entanglement scalar field $\sigma(r)$, it demonstrates how extra entanglement-induced gravity could both reduce the required internal mass of a black hole and ameliorate singular behaviour.

4.5.3 Conclusion

By applying the FAVE-inspired modifications to a black hole such as M87*, we have shown that:

- The observed gravitational field (and hence the Schwarzschild radius) could be reproduced even if the actual internal mass M_{int} is much lower than M_{obs}.
- A modified gravitational profile with a $1/r$ dependence in the high-entanglement region leads to a softer divergence of curvature, potentially avoiding a classical singularity.
- With a Planck-length cutoff, the area-based entanglement mechanism ceases to operate below this scale, providing a natural bound to curvature.

5 Conclusion

In this work, we have laid out a framework—FAVE (Ford-Area/Volume Emergent) gravity—that interprets gravitational dynamics through the interplay of quantum entanglement's area-law and volume-law contributions. Beginning with a field-theoretic analysis of entanglement entropy, we introduced a scalar field σ to track the local "entanglement density". This field remains effectively inactive in regions where area-law entanglement dominates, recovering standard General Relativity, but becomes dynamically significant once volume-law entanglement exceeds a critical threshold.

We then showed how this transition modifies Einstein's equations, leading to new terms that can mimic dark matter, alter radiation's effective redshifting behaviour, and possibly act as a source of dark energy. In cosmology, we found that the extra entanglement energy density can suppress structure formation, requiring larger primordial overdensities to reach the same level of collapse. Our toy models and parameter scans suggested that fitting cosmological observations (such as cluster mass-to-light ratios) is feasible by tuning σ's potential and its coupling to matter and radiation.

We also explored how this same mechanism could apply to black hole interiors, where high entanglement density may smooth or eliminate singularities. A radial dependence for σ can "smear" the mass profile, reducing the amount of rest-mass required to form horizons while introducing an extra long-range component to the gravitational pull. Though the details of black hole cores in an entanglement-driven theory remain speculative, our discussion points to possible ways in which traditional singularities might be avoided.

From this exploration, we can make the following predictions for future studies:

- **Galactic Rotation Curves:**
  FAVE predicts a modified gravitational profile at large radii due to the extra contribution from volume-law entanglement. This should yield rotation curves with a shallower decline (or even flat profiles) in line with observations, but with subtle deviations that might be discernible with high-precision measurements.
- **Large-Scale Structure Evolution:**
  The suppressed growth factor in FAVE implies a different trajectory for structure formation.

We expect a lower integrated linear growth between recombination and the present day, which could affect cluster mass functions and void statistics.

- **Cosmic Microwave Background (CMB) Anisotropies:**
  Changes in radiation scaling and the modified gravitational coupling could lead to measurable shifts in the positions and amplitudes of acoustic peaks. The FAVE framework might also predict specific signatures in the damping tail of the CMB power spectrum.
- **Black Hole Signatures:**
  In black holes, the FAVE mechanism suggests a softened interior gravitational profile. This could manifest as deviations in the innermost stable circular orbits (ISCO) or lensing signatures near the event horizon. Additionally, black hole shadow measurements (such as those from the Event Horizon Telescope) might be sensitive to these effects.
- **Quantum Entanglement Constraints:**
  FAVE implies that below a certain quantum entanglement threshold (or absolute lower bound), one-dimensional entanglement threads dominate, effectively suppressing gravity. This offers a possible experimental connection with ongoing studies in quantum information and entanglement, where detecting a lower bound might have profound implications.
- **Early Universe and Inflation:**
  The model suggests that if the emergent gravitational effects evolve more slowly, the inflationary period might be reinterpreted, potentially leading to lower anisotropies or even a delayed transition from inflation to standard cosmology. This could be explored through both CMB polarization data and large-scale structure surveys.
- **Mass-to-Light Evolution and Hydrostatic Bias:**
  The entanglement “smearing” in FAVE predicts a systematic evolution in the mass-to-light ratio over cosmic time, as well as potential biases in hydrostatic mass estimates of clusters. Comparing these predictions with current and future lensing data could provide critical tests of the model.
- **Pathway to Grand Unification:**
  While still nascent, the framework hints at a possible unification of gravity with the standard model forces by relating the emergence of gravity directly to quantum entanglement processes. This could provide a route toward a grand unified theory, offering insights into how all interactions might share a common quantum-informational origin.

Overall, these studies reinforce the notion that local quantum-information properties—particularly the crossover from area- to volume-law entanglement—could underlie an effective description of spacetime curvature. While the FAVE framework requires further development and confrontation with high-precision data, it provides a coherent template for emergent gravity, connecting microphysical quantum correlations to astrophysical and cosmological phenomena without invoking new, independently conserved dark components. Future work focused on more detailed observational constraints, as well as time-dependent simulations (such as gravitational collapse or black hole mergers), would offer a natural next step in testing whether volume-law entanglement is truly responsible for the extra gravitational effects we usually attribute to dark matter and dark energy.

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Data Availability

The galaxy cluster data (the redMaPPer catalogue) used in this study are publicly available through the Dark Energy Survey (DES) collaboration. For details on obtaining the catalogue, please see the original redMaPPer paper and the associated data links therein.

If you would like access to any additional processed data tables, Python scripts, or intermediate products used in my analyses, please contact me at:
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Conflict of Interest

Please note that I initially took on this project to try and disprove an idea, but as I dug deeper, I found there may be something to the theory. I am, however, an amateur with no formal training and without funding, oversight or mentorship. I have used tools like ChatGPT for technical drafting, derivation assistance, and conceptual exploration. While the core idea, oversight, and final editorial control were provided by the author, some intermediate insights and derivations were contributed by the AI in response to targeted prompts. This work is offered in the spirit of exploratory theoretical physics and open scientific discussion. This paper has been drafted in good faith, to the best of my abilities.

Summary of Key Contributions

1. **Emergent Gravity Framework**
   Proposed a modified description of gravity driven by quantum entanglement, introducing the scalar field σ to differentiate between area-law and volume-law entanglement regimes.
2. **Covariant Action and Equations**
   Derived a covariant action incorporating σ, yielding modified Einstein equations that recover standard GR for low entanglement density and introduce additional energy-momentum contributions above a critical threshold.
3. **Cosmological Applications**
   Demonstrated how volume-law entanglement can mimic dark-matter-like or dark-energy-like effects, affecting radiation and matter density scalings, structure formation, and cluster mass-to-light ratios.
4. **Observational Comparison**
   Performed preliminary fits to redMaPPer cluster data, finding that FAVE's entanglement-driven corrections can reproduce basic clustering trends. Discussed how to further constrain the model using additional observational data (e.g. lensing or velocity dispersions).
5. **Black Hole Interiors**
   Illustrated how the inclusion of a high entanglement density core may remove or soften the classical singularity, offering an alternative explanation of horizon-scale and internal structure without requiring infinite matter density.

Overall, this paper advances a framework linking quantum entanglement phenomena to macroscopic gravitational effects in cosmology and black hole physics, offering potential explanations for current puzzles often attributed to dark matter and dark energy.