Extending Dirac's Hamiltonian with Energy Terms for All Forces in QFT: A Renormalization Approach and Integration with Woit's Geometric Framework

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Abstract

Dirac's Hamiltonian, focused on electrodynamics, lacks energy terms for the weak and strong forces, limiting its applicability to the full Standard Model. Building on Nige Cook's critique and a renormalization approach (https://nigecook.substack.com/p/axiomatic-basis-for-quantum-field-fe6), where the bare core electromagnetic coupling ($\alpha = 1$) is reduced to the low-energy coupling ($\alpha_{\text{low}} \approx 1/137.036$), with the difference $\alpha - \alpha_{\text{low}}$ accounting for energy converted into mass and other forces, we propose a comprehensive Hamiltonian that includes all Standard Model forces, with total energy conserved in quantum field theory (QFT). This approach, supported by a geometric mean method for coupling constants and mixing angles (viXra:1111.0111v1), is integrated into Peter Woit's geometric framework (arXiv:hep-th/0206135), using Clifford algebras to represent energy sharing and flavor mixing geometrically. We explore predictions for mass, couplings, and mixing angles, offering a unified description of particle interactions.

1 Introduction

Dirac's Hamiltonian, derived from the Dirac equation, describes the relativistic dynamics of an electron in an electromagnetic field but is limited to electrodynamics (QED):

$$H = \vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta mc^2 + eA_0,$$

where $\vec{\alpha}$ and β are Dirac matrices, $\vec{p}$ is the momentum, $\vec{A}$ and A_0 are the electromagnetic potentials, e is the charge, and m is the mass. This formulation neglects the weak (SU(2)) and strong (SU(3)) forces, and its energy terms are incomplete for a quantum field theory (QFT) description.

Nige Cook's Substack post [1] critiques this simplicity, proposing that the bare core electromagnetic coupling ($\alpha = 1$, unshielded, high-energy) is reduced to the low-energy coupling ($\alpha_{\text{low}} \approx 1/137.036$, shielded, IR cutoff), with the difference $\alpha - \alpha_{\text{low}}$ converted into short-range effects like mass, weak, and strong forces. This renormalization approach, detailed in a geometric mean method for coupling constants and mixing angles [2], suggests a more comprehensive Hamiltonian that includes all forces, with total energy conserved in QFT. We extend this idea into Peter Woit's geometric framework [3], using Clifford algebras to represent energy sharing and flavor mixing, and explore predictions for Standard Model parameters.

2 Critique of Dirac's Hamiltonian

Dirac's equation in an electromagnetic field is:

$$(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\psi = 0,$$

where γ^μ are the Dirac matrices, $A_\mu = (A_0, \vec{A})$ is the electromagnetic four-potential, and ψ is the fermion field. The Hamiltonian is:

$$H = \vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta mc^2 + eA_0,$$

with $\vec{\alpha}_i = \gamma^0\gamma^i$, $\beta = \gamma^0$, and $\vec{p} = -i\hbar\nabla$. This includes:

- Kinetic energy: $\vec{\alpha} \cdot \vec{p}$,
- Rest mass energy: βmc^2,
- Electromagnetic interaction: $-e\vec{\alpha} \cdot \vec{A} + eA_0$.

Limitations:

- **Electrodynamics Only**: It only accounts for the electromagnetic force (U(1)), neglecting the weak (SU(2)) and strong (SU(3)) forces.
- **First Quantization**: It treats the wavefunction as a classical field, missing QFT effects like virtual particle exchange.
- **Energy Terms**: It lacks energy contributions from other forces and does not explicitly enforce energy conservation across all interactions.

3 Renormalization Approach to Energy Redistribution

Cook [1] proposes that the bare core electromagnetic coupling (high energy, unshielded) is $\alpha = 1$, while the low-energy (IR cutoff, shielded) coupling is $\alpha_{\text{low}} \approx 1/137.036$. The difference:

$$\alpha - \alpha_{\text{low}} = 1 - \frac{1}{137.036} \approx 0.9927,$$

represents the fraction of electromagnetic charge energy converted into short-range effects, including:

- **Mass**: Virtual particle interactions contribute to particle masses via self-energy corrections.
- **Weak and Strong Forces**: Energy is transferred to the weak (SU(2)) and strong (SU(3)) forces.
- **Mixing Angles**: The energy redistribution influences interaction amplitudes, such as the Weinberg angle and CKM matrix elements.

The viXra paper [2] formalizes this with a geometric mean method:

$$\alpha_w = \frac{\alpha_{\text{low}}}{\sin^2\theta_w}, \quad \alpha_s = \frac{\alpha_{\text{low}}}{\cos^2\theta_w},$$

where $\sin^2\theta_w \approx 0.231$, yielding $\alpha_w \approx 1/31.75$, $\alpha_s \approx 1/105.5$. The energy fraction $\alpha - \alpha_{\text{low}}$ is distributed as:

$$\text{Energy}_{\text{weak}} \propto \alpha_w, \quad \text{Energy}_{\text{strong}} \propto \alpha_s.$$

3.1 Mass Contribution

The mass contribution from $\alpha - \alpha_{\text{low}}$ must match observed masses. For the electron ($m_e \approx 0.511\,\text{MeV}$), the self-energy correction in QED is:

$$\delta m_e = \frac{3\alpha_{\text{low}}}{4\pi} m_e \ln\left(\frac{\Lambda^2}{m_e^2}\right),$$

where Λ is the UV cutoff. In Cook's framework, the total mass contribution from virtual particles is proportional to $\alpha - \alpha_{\text{low}}$:

$$m_e \approx m_{e,0} + (\alpha - \alpha_{\text{low}}) \times (\text{scaling factor}).$$

This requires detailed QFT calculations to determine the scaling factor, ensuring consistency with experimental masses.

3.2 Coupling Constants and Mixing Angles

The weak and strong couplings are derived from the low-energy electromagnetic coupling:

$$\alpha_w = \frac{\alpha_{\text{low}}}{\sin^2\theta_w}, \quad \alpha_s = \frac{\alpha_{\text{low}}}{\cos^2\theta_w},$$

with $\sin^2\theta_w \approx 0.231$. The CKM matrix elements (e.g., $|V_{us}| \approx 0.225$) are derived from the energy fractions, as shown in Fig. 35 of [2], where decay amplitudes are proportional to the energy allocated to each interaction channel.

4 A Comprehensive Hamiltonian in QFT

The Standard Model Lagrangian includes all forces:

$$\mathcal{L} = \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{Higgs}},$$

where:

- $\mathcal{L}_{\text{fermions}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$, with:

  $$D_\mu = \partial_\mu + ieA_\mu + ig_w\tau^a W_\mu^a + ig_s\lambda^a G_\mu^a,$$

  where A_μ, W_μ^a, and G_μ^a are the gauge fields for U(1), SU(2), and SU(3), and τ^a, λ^a are the generators.

- $\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu}$, with:

  $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

  $$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_w\epsilon^{abc}W_\mu^b W_\nu^c,$$
  $$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc}G_\mu^b G_\nu^c.$$

- $\mathcal{L}_{\text{int}}$: Interaction terms.

- $\mathcal{L}_{\text{Higgs}}$: Higgs sector.

The Hamiltonian density is:

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L},$$

yielding:

$$\mathcal{H} = \mathcal{H}_{\text{free}} + \mathcal{H}_{\text{int}},$$

with:

$$\mathcal{H}_{\text{free}} = \bar{\psi}(i\gamma^i\partial_i + m)\psi + \frac{1}{2}(\partial_i A_j)^2 + \frac{1}{2}(\partial_i W_j^a)^2 + \frac{1}{2}(\partial_i G_j^a)^2,$$

$$\mathcal{H}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu - g_w\bar{\psi}_L\gamma^\mu\tau^a\psi_L W_\mu^a - g_s\bar{\psi}\gamma^\mu\lambda^a\psi G_\mu^a + (\text{gauge self-interactions}).$$

The total energy is:

$$E = \int d^3x\,\mathcal{H},$$

and is conserved due to time translation invariance, ensuring energy sharing among forces through interactions.

5 Extending Woit's Geometric Framework

Woit's framework [3] uses $\text{Cl}(4,0)$, with the spin representation:

$$\Lambda^*(\mathbb{C}^2) = \Lambda^0(\mathbb{C}^2) \oplus \Lambda^1(\mathbb{C}^2) \oplus \Lambda^2(\mathbb{C}^2),$$

decomposing under $\text{U}(2) \subset \text{SO}(4)$:

Component	$\text{SU}(2) \times \text{U}(1)$ Charges	Particles
$\Lambda^0(\mathbb{C}^2)$	$(0,0)$	ν_R
$\Lambda^1(\mathbb{C}^2)$	$(1/2,-1)$	ν_L, e_L
$\Lambda^2(\mathbb{C}^2)$	$(0,-2)$	e_R

Table 1: Lepton generation in Woit's spin representation.

To include all forces:

- **Embed SU(3)**: Extend to $\text{U}(2) \times \text{SU}(3)$, possibly using $\text{Cl}(6,0)$, where $\text{Spin}(6) \cong \text{SU}(4)$, and project onto $\text{SU}(2) \times \text{U}(1) \times \text{SU}(3)$.
- **Geometric Energy Terms**: Represent gauge fields as Clifford algebra elements, with interaction terms as projections of spinors. The energy fractions α_{low}, α_w, and α_s are encoded in the projection coefficients, with $\alpha - \alpha_{\text{low}}$ determining the energy converted to short-range effects.
- **Mixing Angles**: Derive the Weinberg angle and CKM matrix elements from the energy redistribution, consistent with $\sin^2\theta_w \approx 0.231$.

6 Critical Examination

- **Consistency with Standard Model**: The predicted couplings ($\alpha_w \approx 1/31.75$, $\alpha_s \approx 1/105.5$) may represent bare couplings, consistent with renormalization group flow.
- **Mixing Angles**: The geometric derivation of $\sin^2\theta_w \approx 0.231$ and CKM elements (e.g., $|V_{us}| \approx 0.225$) requires experimental validation.
- **Mass Prediction**: The mass contribution from $\alpha - \alpha_{\text{low}} \approx 0.9927$ implies that 99.27% of the bare core electromagnetic energy is converted into mass and other forces, which must match observed masses (e.g., $m_e \approx 0.511\,\text{MeV}$, $m_W \approx 80.4\,\text{GeV}$).

7 Conclusion

Using Cook's corrected renormalization approach, where $\alpha - \alpha_{\text{low}}$ accounts for energy converted into mass and other forces, we extend Dirac's Hamiltonian to include all Standard Model forces, with energy conserved in QFT. Integrated into Woit's framework, this offers a geometric interpretation of energy sharing, mass generation, and flavor mixing, providing a unified description of particle interactions.

References

[1] N. Cook, "Axiomatic Basis for Quantum Field Theory," Substack, March 29, 2025.

[2] N. Cook, "A Recalculation of the Electromagnetic, Weak and Strong Coupling Constants via a Geometric Mean Method Based on the Fine Structure Constant, with a Discussion of the Implications for the Standard Model and the Nature of the Pion," viXra:1111.0111v1, 2011.

[3] P. Woit, "Quantum Field Theory and Representation Theory: A Sketch," arXiv:hep-th/0206135, 2002.

[4] J. Schwinger, "On Quantum-Electrodynamics and the Magnetic Moment of the Electron," *Physical Review*, vol. 73, pp. 416–417, 1948.